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PYTHAGOREAN FUZZY MAGIC LABELING GRAPHS

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ABSTRACT. A new concept of Pythagorean fuzzy magic, bimagic and antimagic graphs are introduced. Pythagorean fuzzy magic labeling and Pythagorean fuzzy magic constants for star, path, cycle and fan graphs are newly defined. Further, Pythagorean fuzzy bridge, Pythagorean fuzzy cut node and degrees of Pythagorean Fuzzy Magic Graphs are discussed.

1. INTRODUCTION

Fuzzy graphs are implemented where there exists vagueness in nodes and arcs. Yet, fuzzy labeling graphs give more compatibilty, flexibility and accuracy to the network compared to fuzzy graphs and have various application in the field of Computer Science, Chemistry, Physics and other divisions of Mathematics.

Nagoorgani and Rajalakshmi [7] were the first to apply the concept of labeling in fuzzy graph and introduced fuzzy magic graph. They proved that any path graph and cycle graph of odd length are fuzzy magic graph. The idea of interval valued fuzzy magic labeling was discussed by Mishra and Anita pal [6]. The notion of interval valued intuitionistic fuzzy graph and interval valued intuitionistic fuzzy magic graph was introduced by Kishore Kumar et.al [4]. Ameenal Bibi and Devi [2] extended the concept of fuzzy magic labeling to fuzzy

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bi-magic labeling and defined it on interval valued fuzzy graph. They discussed the bi-magic labeling for cycle graph of odd length and star graph. The definition of fuzzy anti magic graph was given by Shajila and Vimala [11] where they investigated the fuzzy anti magic labeling for path and butterfly graph.

Rifayatahli et.al [10] proposed the idea of Hesitancy Fuzzy Magic Labeling (HFML) and Fathalian et.al [3] proved that any connected and complete graph admits HFML. The concept of Pythagorean Fuzzy Graph (PFG) was introduced by Naz et.al [8] and they developed a series of operational laws for PFGs. The idea of Pythagorean Fuzzy Labeling Graph Ω_{PFLG} are introduced by Bharathi T [9] and Ashwini Sibiya Rani P in which they discussed the sub graphs and union of Ω_{PFLG}.

2. Preliminaries

A fuzzy labeling graph $G = (V, E)$ is said to be a *fuzzy magic graph* [7] if $\mu_1 : V \to [0, 1]$ and $\mu_2 : V * V \to [0, 1]$ such that for all $b_i, b_{i+1} \in V$, $\mu_2(b_i, b_{i+1}) < min(\mu_1(b_i), \mu_1(b_{i+1}))$ and $\mu_1(b_i) + \mu_2(b_i, b_{i+1}) + \mu_1(b_{i+1}) = k_1$ where k_1 is the constant value. This constant value is called as fuzzy magic constant and denoted as $m_0(G)$. G is said to be a *fuzzy bi-magic graph* [2] if $\mu_1(b_i)+\mu_2(b_i, b_{i+1})+\mu_1(b_{i+1}) = k_1$ or k_2 for all $b_i, b_{i+1} \in V$ and are denoted as $Bm_0(G)$. G is said to be a *fuzzy anti-magic graph* [11] if $\mu_1(b_i) + \mu_2(b_i, b_{i+1}) + \mu_1(b_{i+1})$ have distinct values for all $b_i, b_{i+1} \in V$.

A *fuzzy star graph* [12] comprises of two set of nodes A and B with $|A| = 1$ and $|B| > 1$ such that $\mu_2(a, b_i) > 0$ for all $1 \leq i \leq m$ and $\mu_2(b_i, b_{i+1}) = 0$ for all $1 \leq i \leq m - 1$. A *fuzzy path* P [5] is defined as a sequence of distinct nodes $b_1, b_2, \ldots, b_m$ such that $\mu_2(b_i, b_{i+1}) > 0$ for all $1 \leq i \leq m - 1$. A path is a *cycle* if $b_1 = b_m$ and $m \geq 3$, where m is the length. A *fuzzy fan graph* $F_{n,m}$ is defined as the join of two graphs A and B where A is the empty graph $\overline{K_n}$ and B is the path graph. The case $n = 1$ denotes the usual fan graph and the case $n = 2$ denotes the double fan graph.

A Pythagorean Fuzzy Graph $G_1 = (V_1, E_1)$ is said to be a *Pythagorean Fuzzy Labeling Graph* Ω_{PFLG} [9] if $\mu_1 : V_1 \to [0, 1]$, $v_1 : V_1 \to [0, 1]$ and $\mu_2 : V_1 * V_1 \to [0, 1]$, $v_2 : V_1 * V_1 \to [0, 1]$ are bijective such that the degree of membership and degree of non membership of the nodes and arcs are distinct. And for every arc (b_i, b_{i+1}), $\mu_2(b_i, b_{i+1}) < \min(\mu_1(b_i), \mu_1(b_{i+1}))$, $v_2(b_i, b_{i+1}) > \max(v_1(b_i), v_1(b_{i+1}))$

such that $0 \leq \mu_1^2(b_i) + v_1^2(b_i) \leq 1$ and $0 \leq \mu_2^2(b_i, b_{i+1}) + v_2^2(b_i, b_{i+1}) \leq 1$. The *degree* [8] of a nodes b_i in a G_1 is defined as $d_{\mu_1}(b_i) = \sum_{b_i, b_{i+1} \neq b_i \in V_1} \mu_2(b_i, b_{i+1})$ and $d_{v_1}(b_i) = \sum_{b_i, b_{i+1} \neq b_i \in V_1} v_2(b_i, b_{i+1})$.

The *strength of connectedness* $CONN_{G_1}$ [1] of a pair of nodes $b_i, b_j \in V_1$ in G_1 is defined as maximum of μ-strength and minimum of v-strength. An arc (b_i, b_{i+1}) is called a *fuzzy bridge* [5] if its deletion, decrease the $CONN_{G_1}$ between some set of nodes in G_1 . A node b_i is said to be a *fuzzy cut node* [5] if its deletion, decrease the $CONN_{G_1}$ between some other set of nodes in G_1.

3. Pythagorean fuzzy magic labeling graph

A Pythagorean fuzzy labeling graph $G_P = (V_P, E_P)$ is said to be a *Pythagorean fuzzy magic labeling graph* if $\mu_1 : V_P \to [0,1]$, $v_1 : V_P \to [0,1]$ and $\mu_2 : V_P * V_P \to [0,1]$, $v_2 : V_P * V_P \to [0,1]$ are bijective such that $\mu_1(b_i) + \mu_2(b_i, b_{i+1}) + \mu_1(b_{i+1})$ have constant value say k_1 and $v_1(b_i) + v_2(b_i, b_{i+1}) + v_1(b_{i+1})$ have constant value say k_3 for all $b_i, b_{i+1} \in V_P$. The Pythagorean fuzzy membership magic constant k_1 is denoted by $M\omega_1(G_P)$ and the Pythagorean fuzzy non membership magic constant k_3 is denoted as $M\omega_3(G_P)$. A graph that admits Pythagorean fuzzy magic labeling are known as *Pythagorean fuzzy magic graph*, Ω_{PFLG}^{M}.

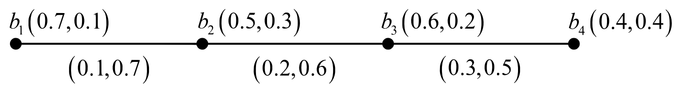

FIGURE 1. Pythagorean fuzzy magic path graph P_4

Example 1.

G_P is said to be a *Pythagorean fuzzy bi-magic graph*, $\Omega_{PFLG}^{B(M)}$ if $\mu_1(b_i)+\mu_2(b_i, b_{i+1})+\mu_1(b_{i+1})$ is either one of the constant k_1 or k_2 and $v_1(b_i) + v_2(b_i, b_{i+1}) + v_1(b_{i+1})$ is either one of the constant k_3 or k_4 for all $b_i, b_{i+1} \in V_P$. The Pythagorean fuzzy membership magic constant k_1 and k_2 are denoted by $M\omega_1(G_P)$ and $M\omega_2(G_P)$. The Pythagorean fuzzy non membership magic constant k_3 and k_4 are denoted by $M\omega_3(G_P)$ and $M\omega_4(G_P)$. G_P is said to be a *Pythagorean fuzzy anti magic graph*, $\Omega_{PFLG}^{A(M)}$ if $\mu_1(b_i) + \mu_2(b_i, b_{i+1}) + \mu_1(b_{i+1})$ and $v_1(b_i) + v_2(b_i, b_{i+1}) + v_1(b_{i+1})$ have distinct values for all $b_i, b_{i+1} \in V_P$.

A *Pythagorean fuzzy star graph* comprises of two set of nodes A and B with $|A| = 1$ and $|B| > 1$ such that $\mu_1^2(a) + v_1^2(a) > 0$, $\mu_1^2(b_i) + v_1^2(b_i) > 0$, $\mu_2^2(a, b_i) + v_2^2(a, b_i) > 0$ and satisfies the following condition, $\mu_2(a, b_i) \leq min(\mu_1(a), \mu_1(b_i))$, $v_2(a, b_i) \geq max(v_1(a), v_1(b_i))$.

A *Pythagorean fuzzy path* P_m in $G_P = (V_P, E_P)$, is defined as a sequence of distinct nodes $b_1, b_2, \ldots, b_m$ such that $\mu_1^2(b_i) + v_1^2(b_i) > 0$ for all $1 \leq i \leq m$ and $\mu_2^2(b_i, b_{i+1}) + v_2^2(b_i, b_{i+1}) > 0$ for all $1 \leq i \leq m-1$ and satisfies the following condition, $\mu_2(b_i, b_{i+1}) \leq \min(\mu_1(b_i), \mu_1(b_{i+1}))$, $v_2(b_i, b_{i+1}) \geq \max(v_1(b_i), v_1(b_{i+1}))$. A path P_m is a cycle if $b_1 = b_m$ and $m \geq 3$ where m is the length of the path P_m .

A fan graph $F_{1,m}$ is said to be a *Pythagorean fuzzy fan graph* if $|A| = 1$ and $|B| > 1$ such that for all $1 \leq i \leq m$, $\mu_1^2(a) + v_1^2(a) > 0$, $\mu_1^2(b_i) + v_1^2(b_i) > 0$, $\mu_2^2(a, b_i) + v_2^2(a, b_i) > 0$ and for all $1 \leq i \leq m-1$, $\mu_2^2(b_i, b_{i+1}) + v_2^2(b_i, b_{i+1}) > 0$. It also satisfies the following conditions $\mu_2(a, b_i) \leq min(\mu_1(a), \mu_1(b_i))$, $\mu_2(b_i, b_{i+1}) \leq min(\mu_1(b_i), \mu_1(b_{i+1}))$, and $v_2(a, b_i) \geq max(v_1(a), v_1(b_i))$,
$v_2(b_i, b_{i+1}) \geq max(v_1(b_i), v_1(b_{i+1}))$.

A bridge $(b_i, b_{i+1}) \in E_P$ is said to be a μ-bridge if its deletion, decrease the $\mu - CONN_{G_P}$ between some pair of nodes. A bridge $(b_i, b_{i+1}) \in E_P$ is said to be a v-bridge if its deletion, increase the $v - CONN_{G_P}$ between some pair of nodes. A bridge $(b_i, b_{i+1}) \in E_P$ is said to be a *Pythagorean fuzzy bridge* if it is $\mu-$bridge and $v-$ bridge.

A node $b_i \in V_P$ is said to be a μ-cut node if its deletion, decrease the $\mu - CONN_{G_P}$ between some other pair of nodes. A node $b_i \in V_P$ is said to be a $v-$cut node if its deletion, increase the $v - CONN_{G_P}$ between some other pair of nodes. A node $b_i \in V_P$ is said to be a *Pythagorean fuzzy cut node* if it is $\mu-$cut node and $v-$ cut node.

Theorem 3.1. *Any Pythagorean fuzzy star graph $S_{1,m}$ of order $m \geq 3$ is a Pythagorean fuzzy magic graph.*

Proof. Let $S_{1,m}$ be a Pythagorean fuzzy star graph with $m+1$ nodes and m arcs. Let $\{a, b_1, b_2, \ldots, b_m\}$ be the set of nodes and $\{ab_1, ab_2, \ldots, ab_m\}$ be the set of arcs where $|A| = a$ and $|B| = b_i$. Choose $\omega \to (0, 1]$ such that $\omega = 10^{-1}$ if $m \leq 4$, $\omega = 10^{-2}$ if $5 \leq m \leq 49$, $\omega = 10^{-3}$ if $50 \leq m \leq 499$ and so on. The Pythagorean fuzzy magic labeling for star graph are defined as follows

$$\begin{aligned}
\mu_1(a) &= (2m+1)*\omega \\
\mu_1(b_i) &= (m+i)*\omega & ;1 \leq i \leq m \\
v_1(a) &= (m+1)*\omega \\
v_1(b_i) &= i*\omega & ;1 \leq i \leq m \\
\mu_2(a,b_i) &= \mu_1(a) - [(m+i)*\omega] & ;1 \leq i \leq m \\
v_2(a,b_i) &= 2(v_1(a)) - (i*\omega) & ;1 \leq i \leq m.
\end{aligned}$$

The Pythagorean fuzzy magic constant for Star Graph $S_{1,m}$ is given as

$$\begin{aligned}
M_{\omega_1}(S_{1,m}) &= [2(|V|+|E|)]*\omega \;=\; [2(2m+1)]*\omega \\
M_{\omega_2}(S_{1,m}) &= M_{\omega_1}(S_{1,m}) - [(m-1)*\omega] \;=\; [3(m+1)]*\omega.V
\end{aligned}$$

Thus, any Pythagorean fuzzy star graph $S_{1,m}$ is a Ω^M_{PFLG} for all $m \geq 3$. □

Theorem 3.2. *Any Pythagorean fuzzy path graph P_m of length $m \geq 3$ is a Pythagorean fuzzy magic graph.*

Proof. Let P_m be a path graph with m nodes and $m-1$ arcs. Let $\{b_1, b_2, ..., b_m\}$ be the set of nodes and $\{b_1b_2, b_2b_3, ..., b_{m-1}b_m\}$ be the set of arcs. Choose $\omega \to (0,1]$ such that $\omega = 10^{-1}$ if $m \leq 5$, $\omega = 10^{-2}$ if $6 \leq m \leq 50$, $\omega = 10^{-3}$ if $51 \leq m \leq 500$ and so on. For a path graph P_m, the Pythagorean fuzzy magic labeling for the arcs $(b_i, b_{i+1}) \in E_P$ are defined as

$$\begin{aligned}
\mu_2(b_i,b_{i+1}) &= i*\omega & ;1 \leq i \leq m-1 \\
v_2(b_i,b_{i+1}) &= (2m-i)*\omega & ;1 \leq i \leq m-1.
\end{aligned}$$

The Pythagorean fuzzy magic labeling for the nodes $b_i, b_{i+1} \in V_P$ in P_m are defined as

Case (i): When m is odd

$$\begin{aligned}
\mu_1(b_{2i-1}) &= (2m-i)*\omega & ;1 \leq i \leq \frac{m+1}{2} \\
\mu_1(b_{2i}) &= \mu_1(b_m) - (i*\omega) & ;1 \leq i \leq \frac{m-1}{2} \\
v_1(b_{2i-1}) &= i*\omega & ;1 \leq i \leq \frac{m+1}{2} \\
v_1(b_{2i}) &= v_1(b_m) + (i*\omega) & ;1 \leq i \leq \frac{m}{2}.
\end{aligned}$$

Case (ii): when m is even

$$\begin{aligned}
\mu_1(b_{2i-1}) &= (2m-i)*\omega && ;1 \leq i \leq \frac{m}{2} \\
\mu_1(b_{2i}) &= \mu_1(b_{m-1}) - (i*\omega) && ;1 \leq i \leq \frac{m}{2} \\
v_1(b_{2i-1}) &= i*\omega && ;1 \leq i \leq \frac{m}{2} \\
v_1(b_{2i}) &= v_1(b_{m-1}) + (i*\omega) && ;1 \leq i \leq \frac{m}{2}.
\end{aligned}$$

The Pythagorean fuzzy magic constant for path P_m of odd length is given as

$$\begin{aligned}
M_{\omega_1}(P_m) &= \left[2\Big(|V|+|E|\Big) - \left(\frac{m-1}{2}\right)\right]*\omega = \left(\frac{7m-3}{2}\right)*\omega \\
M_{\omega_2}(P_m) &= \left[\Big(|V|+|E|+m\Big) - \left(\frac{m-5}{2}\right)\right]*\omega = \left(\frac{5m+3}{2}\right)*\omega.
\end{aligned}$$

The Pythagorean fuzzy magic constant for path P_m of even length is given as

$$\begin{aligned}
M_{\omega_1}(P_m) &= \left[2(|V|+|E|) - \left(\frac{m-2}{2}\right)\right]*\omega = \left(\frac{7m-2}{2}\right)*\omega \\
M_{\omega_2}(P_m) &= \left[(|V|+|E|+m) - \left(\frac{m-4}{2}\right)\right]*\omega = \left(\frac{5m+2}{2}\right)*\omega.
\end{aligned}$$

Thus, any Pythagorean fuzzy path graph P_m is a Ω^M_{PFLG} for all $m \geq 3$. □

Theorem 3.3. *A Pythagorean fuzzy cycle graph, C_m of odd length $m \geq 3$ is a Pythagorean fuzzy magic graph and C_m of even length $m \geq 4$ is a Pythagorean fuzzy bi-magic graph.*

Proof. Let C_m be a cycle graph of odd length with m nodes and m arcs. Let $\{b_1, b_2, \ldots, b_m\}$ be the set of nodes and $\{b_1b_2, b_2, b_3, \ldots, b_mb_1\}$ be the set of arcs. Choose $\omega \to (0,1]$ such that $\omega = 10^{-1}$ if $m \leq 4$, $\omega = 10^{-2}$ if $5 \leq m \leq 49$, $\omega = 10^{-3}$ if $50 \leq m \leq 499$ and so on. For a cycle graph C_m, the Pythagorean fuzzy magic labeling for the arcs (b_i, b_{i+1}) and $(b_i, b_1) \in E_P$ are defined as

$$\begin{aligned}
\mu_2(b_i, b_{i+1}) &= i*\omega && ;\ 1 \leq i \leq m-1 \\
\mu_2(b_i, b_1) &= i*\omega && ;\ i = m \\
v_2(b_i, b_{i+1}) &= [2m-(i-1)]*\omega && ;\ 1 \leq i \leq m-1 \\
v_2(b_i, b_1) &= [2m-(i-1)]*\omega && ;\ i = m.
\end{aligned}$$

The Pythagorean fuzzy magic labeling for the nodes b_i in C_m are defined as
Case (i): When m is odd

$$\begin{aligned}
\mu_1(b_{(m+2)-2i}) &= \mu_2(b_m, b_1) + (i * \omega) && ; 1 \ \leq i \leq \frac{m+1}{2} \\
\mu_1(b_{(m+1)-2i}) &= \mu_1(b_1) + (i * \omega) && ; 1 \leq i \leq \frac{m-1}{2} \\
v_1(b_{2i-1}) &= v_1(b_{m-1}) + (i * \omega) && ; 1 \leq i \leq \frac{m+1}{2} \\
v_1(b_{2i}) &= i * \omega && ; 1 \leq i \leq \frac{m-1}{2}.
\end{aligned}$$

Case (ii): When m is even

$$\begin{aligned}
\mu_1(b_{(m+2)-2i}) &= \mu_2(b_m, b_1) + (i * \omega) && ; 1 \ \leq i \leq \frac{m}{2} \\
\mu_1(b_{(m+1)-2i}) &= \mu_1(b_2) + (i * \omega) && ; 1 \leq i \leq \frac{m}{2} \\
v_1(b_{2i-1}) &= i * \omega && ; 1 \leq i \leq \frac{m}{2} \\
v_1(b_{2i}) &= v_1(b_{m-1}) + (i * \omega) && ; 1 \leq i \leq \frac{m}{2}.
\end{aligned}$$

Pythagorean fuzzy magic constant for the cycle graph with odd length is given as,

$$\begin{aligned}
M_{\omega_1}(C_m) &= \left[2\Big(|V| + |E|\Big) - \left(\frac{m-3}{2}\right)\right] * \omega = \left(\frac{7m+3}{2}\right) * \omega \\
M_{\omega_2}(C_m) &= \left[\Big(|V| + |E| + m\Big) - \left(\frac{m-3}{2}\right)\right] * \omega = \left(\frac{5m+3}{2}\right) * \omega.
\end{aligned}$$

Pythagorean fuzzy bi-magic constant of the cycle graph with even length are given as,

$$\begin{aligned}
M_{\omega_1}(C_m) &= k_1 = \left[2\Big(|V| + |E|\Big) - \left(\frac{m-2}{2}\right)\right] * \omega \ = \left(\frac{7m+2}{2}\right) * \omega \\
M_{\omega_2}(C_m) &= k_2 = \left[2\Big(|V| + |E|\Big) + 1\right] * \omega = (4m+1) * \omega \\
M_{\omega_3}(C_m) &= k_3 = \left[\Big(|V| + |E| + m\Big) - \left(\frac{m-4}{2}\right)\right] * \omega = \left(\frac{5m+4}{2}\right) * \omega \\
M_{\omega_4}(C_m) &= k_4 = \Big[|V| + |E| + m\Big] * \omega = [2(m+1)] * \omega.
\end{aligned}$$

Thus, any Pythagorean fuzzy cycle graph C_m of odd length where $m \geq 3$ is a Ω^{M}_{PFLG} and any C_m of even length where $m \geq 4$ ia a $\Omega^{B(M)}_{PFLG}$. □

Theorem 3.4. *A Pythagorean fuzzy fan graph $F_{1,m}$ of length $m \geq 3$ is a Pythagorean fuzzy anti magic graph.*

Proof. Let $F_{1,m}$ be the fan graph and let $\{a, b_1, b_2, \ldots, b_m\}$ be the set of nodes and $\{ab_1, ab_2, \ldots, ab_m, b_1b_2, b_2b_3, \ldots, b_{m-1}b_m\}$ be the set of arcs. Choose $\omega \to (0, 1]$ such that $\omega = 10^{-1}$ if $m = 3$, $\omega = 10^{-2}$ if $4 \leq m \leq 33$, $\omega = 10^{-3}$ if $34 \leq m \leq 333$ and so on. The Pythagorean fuzzy magic labeling for fan graph are defined as follows

$$\begin{aligned}
\mu_1(a) &= (m+1) * \omega \\
\mu_1(b_i) &= (2m+i) * \omega && ; 1 \leq i \leq m \\
v_1(a) &= 1 * \omega \\
v_1(b_i) &= v_1(a) + (i * \omega) && ; 1 \leq i \leq m \\
\mu_2(a, b_i) &= i * \omega && ; 1 \leq i \leq m \\
\mu_2(b_i, b_{i+1}) &= \mu_1(a) + (i * \omega) && ; 1 \leq i \leq m-1 \\
v_2(a, b_i) &= v_1(b_m) + (i * \omega) && ; 1 \leq i \leq m \\
v_2(b_i, b_{i+1}) &= 2 * v_1(b_m) + [(i-1) * \omega] && ; 1 \leq i \leq m-1.
\end{aligned}$$

The Pythagorean fuzzy magic constant $M_{\omega_1}(F_{1,m})$ and $M_{\omega_2}(F_{1,m})$ differs by $2 * \omega$ and $3 * \omega$ for the star graph and path graph respectively in $F_{1,m}$. Thus, fan graph $F_{1,m}$ is a Pythagorean fuzzy anti magic graph. □

Remark 3.1.

(1) *For any Pythagorean fuzzy magic star graph,* $\sum_{i=1}^{m} \deg[\mu_1(a) + v_1(b_i)] = \sum_{i=1}^{m} [\mu_2(a, b_i) + v_2(a, b_i)]$

(2) *For any Pythagorean fuzzy magic cycle graph,*
$\sum_{i=1}^{m} \mu_1(b_i) = \sum_{i=1}^{m-1} v_2(b_i, b_{i+1}) + v_2(b_m, b_1)$
$\sum_{i=1}^{m} v_1(b_i) = \sum_{i=1}^{m-1} \mu_2(b_i, b_{i+1}) + \mu_2(b_m, b_1)$

(3) *For every Pythagorean fuzzy magic labeling graph,* $M_{\omega_1}(G_P) < M_{\omega_2}(G_P)$ *expect* P_3 *in which* $M_{\omega_1}(G_P) = M_{\omega_2}(G_P)$

(4) *For every Pythagorean fuzzy bi-magic labeling graph,* $M_{\omega_1}(G_P) < M_{\omega_2}(G_P)$ *and* $M_{\omega_3}(G_P) < M_{\omega_4}(G_P)$.

4. Properties of Pythagorean fuzzy magic labeling graph

Properties 4.1. *Every Pythagorean fuzzy magic graph is a Pythagorean fuzzy labeling graph. But the converse is not true.*

Properties 4.2. *For any Pythagorean fuzzy magic graph $G_P^{'} = (V_P^{'}, E_P^{'})$, the following conditions are satisfied*

(i) $\sum_{b_i \in V_P^{'}} \mu_1(b_i) < |V_P^{'}|$ *and* $\sum_{b_i \in V_P^{'}} v_i(b_i) < |V_P^{'}|$.

(ii) $\sum_{(b_i, b_{i+1}) \in E_P^{'}} \mu_2(b_i, b_{i+1}) < |E_P^{'}|$ *and* $\sum_{(b_i, b_{i+1}) \in E_P^{'}} v_2(b_i, b_{i+1}) < |E_P^{'}|$.

(iii) *For all* $b_i \in V_P^{'}$ *and* $(b_i, b_{i+1}) \in E_P^{'}$, $\mu_1(b_i) \cap \mu_2(b_{i+1}) = \emptyset$ *and* $v_1(b_i) \cap v_2(b_{i+1}) = \emptyset$.

Properties 4.3. *Exclusion of Pythagorean fuzzy bridge or Pythagorean fuzzy cut node from a Pythagorean fuzzy magic path is a* Ω_{PFLG}^{M}.

Properties 4.4. *Exclusion of a Pythagorean fuzzy bridge or a Pythagorean fuzzy cut node from a Pythagorean fuzzy magic cycle of odd length is a* Ω_{PFLG}^{M}.

Properties 4.5. *Every* Ω_{PFLG}^{M} *includes exactly one Pythagorean fuzzy bridge and exactly one Pythagorean fuzzy cut node.*

Properties 4.6. *Exclusion of a Pythagorean fuzzy bridge from a Pythagorean fuzzy magic star graph of order* $m \geq 3$ *is also a* Ω_{PFLG}^{M}.

5. Conclusion

Fuzzy magic labeling graph have numerous applications in real life problems such as floor heating systems used in apartments, road network system used to minimize traffic, plumbing system and etc. We introduced the concept of Pythagorean fuzzy magic labeling graph and proved that any cycle graph of odd length, path graph and star graph are Pythagorean fuzzy magic graph. We also proposed the concept of Pythagorean fuzzy bi-magic graph and proved that cycle graph of even length is Pythagorean fuzzy bi-magic graph. We showed a fan graph with $\overline{K_1}$ is a Pythagorean fuzzy anti magic graph. We discuss some properties of Pythagorean fuzzy magic graph along with Pythagorean fuzzy bridge and cut nodes.

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