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# MULTI GRANULATION ON NANO SOFT TOPOLOGICAL SPACE

S.P.R. PRIYALATHA<sup>1</sup> AND WADEI. F. AL-OMERI

ABSTRACT. In this paper, we explore a nano soft topological space with a multi granulation is known as "Multi-Nano Soft Topological Space (MNSTS)". We study the characterization and properties of soft approximation space in MN-STS. Further, we define multi-nano soft interior and multi-nano soft closure an investigation is done on properties with a model.

## 1. INTRODUCTION

Molodtsov [5], in 1999, introduced the concept of soft set theory as a mathematical model for handling ambiguities that a known mathematical model can't hold. He has indicated a few applications of soft set theory for discovering answers for some practical problems, for example, financial matters, sociology, designing, clinical science, and so forth. In 1998, Lin [4] considered granular computing utilizing neighbourhood frameworks for the understanding of granules. Lellis Thivagar et.al [3] introduced a nano topological space. The notion of soft topological space which was formulated by Shabir and Naz [7], which is defined over an initial universe with a fixed set of parameters. Recently, the author has studied the soft set and nano topology [2]. The soft set relation was developed by Babitha.et.al. [1] and authors are studied and developed by the concept [8]. This paper we define nano soft topological space using a soft

<sup>&</sup>lt;sup>1</sup>corresponding author

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set based on multi granulation. We also derive the soft approximation space  $(\mathcal{U}, F_A)$  in multi-nano soft topological space. The properties of multi-soft lower and multi-soft upper approximations are discussed with an example. Further, we discuss the properties of multi-nano soft interior and multi-nano soft closure based on MNSTS with examples are given.

# 2. Preliminaries

In the current section, we recollect the some basic definitions of nano topology and soft set.

**Definition 2.1.** [5], [6] A soft set (F, A) denoted by  $F_A$  on the universe  $\mathcal{U}$  is defined by the set of ordered pairs  $F_A = \{(e, F(e)) : e \in E, F(e) \in P(\mathcal{U})\}$ , where  $F : E \rightarrow P(\mathcal{U})$  such that  $F(e) = \emptyset$  if  $e \notin A$ . Here, F is called an approximate function of the soft set  $F_A$ . The set F(e) is called e-approximate value set or e-approximate set which consists of related objects of the parameter  $e \in E$ .

**Definition 2.2.** [1] Let  $F_A$  and  $G_B$  be two soft sets over  $\mathcal{U}$ , then the Cartesian product of  $F_A$  and  $G_B$  is defined as,  $F_A \times G_B = (H, A \times B)$ , where  $H : A \times B \rightarrow P(\mathcal{U} \times \mathcal{U})$  and  $H(a, b) = F(a) \times G(b)$ , where  $\forall (a, b) \in A \times B$ , i.e.,  $H(a, b) = \{(h_i, h_j) : h_i \in F(a) \text{ and } h_j \in G(b)\}$ .

**Definition 2.3.** [8] Let R be a soft equivalence relation on  $F_A$ , then

- (i) soft reflexive if  $F(a) \times F(a) \in R, \forall a \in A$ .
- (*ii*) soft symmetric if  $F(a) \times F(b) \in R \Rightarrow F(b) \times F(a) \in R, \forall a, b \in A$ .
- (iii) soft transitive if  $F(a) \times F(b) \in R, F(b) \times F(c) \in R \Rightarrow F(a) \times F(c) \in R, \forall a, b, c \in A.$

**Definition 2.4.** [1], [8] Let  $F_A$  be a soft set, then soft equivalence class of F(a) denoted by [F(a)] is defined as  $[F(a)] = \{F(b) : F(a) \times F(b) \in R, \forall a, b \in A\}.$ 

**Definition 2.5.** [3] Let  $\mathcal{U}$  be a non-empty finite set of objects called the universe,  $\mathcal{R}$  be an equivalence relation on  $\mathcal{U}$  named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair  $(\mathcal{U}, R)$  is said to be approximation space. Let  $X \subseteq \mathcal{U}$ .

(1) The Lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by

 $L_R(X)$ . That is,  $L_R(X) = \left\{ \bigcup_{x \in \mathcal{U}} \{R(x) : R(x) \subseteq X\} \right\}$ , where R(x) denotes the equivalence class determined by x.

- (2) The Upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by  $U_R(X) = \left\{ \bigcup_{X \in \mathcal{U}} \{R(x) : R(x) \cap X \neq \emptyset\} \right\}.$
- (3) The Boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not -X with respect to R and it is denoted by  $B_R(X) = U_R(X) - L_R(X)$ .

**Definition 2.6.** [3] Let  $\mathcal{U}$  be the universe, R be an equivalence relation on  $\mathcal{U}$  and  $\tau_R(X) = {\mathcal{U}, \emptyset, L_R(X), U_R(X), B_R(X)}$  where  $X \subseteq \mathcal{U}$ .  $\tau_R(X)$  satisfies the following axioms:

- (1)  $\mathcal{U}$  and  $\emptyset \in \tau_R(X)$
- (2) The union of elements of any sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .
- (3) The intersection of the elements of any finite sub collection of  $\tau_R(X)$  is in  $\tau_R(X)$ .

That is,  $\tau_R(X)$  forms a topology on  $\mathcal{U}$  called as the nano topology on  $\mathcal{U}$  with respect to X. We call  $\{\mathcal{U}, \tau_R(X)\}$  as the nano topological space.

# 3. BI-GRANULARITY SOFT APPROXIMATION SPACE BASED ON NANO SOFT TOPOLOGY

In this section, we define the nano soft topological space induced by multigranulation is said to be "Multi-Nano Soft Topological Space(MNSTS)" and their characterization are investigated.

**Definition 3.1.** Let  $\mathcal{U}$  be non-empty finite universe,  $F_A$  be a soft set over  $\mathcal{U}$ .Let M, N be a soft equivalence relation on  $F_A \subseteq F_E$ . Elements belonging to the soft equivalence class of F(a) denoted by [F(a)] are said to be soft indiscernible with one another. The ordered pair  $(\mathcal{U}, F_A)$  is said to be soft approximation space .Let  $G_B \subseteq F_A$ .

(i) If  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) = \bigcup_{a \in A} \{ [F(a)] : [F(a)]_M \subseteq G_B \text{ or } [F(a)]_N \subseteq G_B \}$  is a multisoft lower approximation of  $F_A$  with respect to  $G_B$ .

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- (*ii*) If  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) = \bigcup_{a \in A} \{ [F(a)] : [F(a)]_M \cap G_B \neq \tilde{\emptyset} \}$  and  $[F(a)]_N \cap G_B \neq \tilde{\emptyset} \}$ is a multi-soft upper approximation of  $F_A$  with respect to  $G_B$ .
- (*iii*) If  $\mathcal{B}_{\mathcal{M}+\mathcal{N}}(G_B) = \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B)$  is a multi-soft boundary region of  $F_A$  with respect to  $G_B$ .

**Definition 3.2.** Let  $\mathcal{U}$  be a non empty finite set of objects called the universe,  $F_A \subseteq F_E$  is an soft set over  $\mathcal{U}$ . Then  $(\mathcal{U}, F_A)$  is an ordered pair of soft approximation space and  $\tilde{\tau}_{M+N}(G_B) = {\tilde{\mathcal{U}}, \tilde{\phi}, \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B), \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B), \mathcal{B}_{\mathcal{M}+\mathcal{N}}(G_B)}})$ , where  $G_B \subseteq F_A$ . That is,  $\tilde{\tau}_{M+N}(G_B)$  forms a multi-nano soft topology on  $\mathcal{U}$  having the at most five elements of soft set and triple ordered pair of  $(\mathcal{U}, \tilde{\tau}_{M+N}, E)$  is called a multi-nano soft topological space over  $\mathcal{U}$  with respect to  $G_B$ , then the members of  $\tilde{\tau}_{M+N}$  are said to be multi-nano soft open sets in  $\mathcal{U}$ .

**Example 1.** Let  $\mathcal{U} = \{k_1, k_2, k_3, k_4, k_5, k_6\}$  be the universe,  $E = \{d_1, d_2, d_3, d_4, d_5\}$ and  $A = \{d_1, d_2, d_3, d_4\}$  be a set of parameters. Also, let  $F_A = \{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\}), (d_3, \{k_2, k_3\}), (d_4, \{k_4, k_5\})\}$  and  $G_B \subseteq F_A$ , where  $G_B = \{(d_1, \{k_1, k_2\}), (d_2, \{k_3\}), (d_3, \{k_2, k_3\})\}$  such that  $F(d_1) = \{k_1, k_2\}, F(d_2) = \{k_3\}, F(d_3) = \{k_2, k_4\}, F(d_4) = \{k_4, k_5\}$  is a soft set over  $\mathcal{U}$  and  $M = \{F(d_1) \times F(d_1), F(d_2) \times F(d_2), F(d_3) \times F(d_3), F(d_4) \times F(d_4), F(d_1) \times F(d_2), F(d_2) \times F(d_1)\}$ and  $N = \{F(d_1) \times F(d_1), F(d_2) \times F(d_2), F(d_3) \times F(d_3), F(d_4) \times F(d_3), F(d_4) \times F(d_4), F(d_2) \times F(d_4), F(d_2) \times F(d_3), F(d_3) \times F(d_3), F(d_3) \times F(d_3), F(d_3) \times F(d_3), F(d_4) \times F(d_3), F(d_4) \times F(d_4), F(d_2) \times F(d_4), F(d_2) \times F(d_3), F(d_3) \times F(d_3), F(d_4) \times F(d_4), F(d_4) \times F(d_4), F(d_2) \times F(d_3), F(d_3) \times F(d_3), F(d_3) \times F(d_3), F(d_4) \times F(d_4), F(d_4) \times F(d_4), F(d_2) \times F(d_3), F(d_3) \times F(d_3), F(d_3) \times F(d_3), F(d_3) \times F(d_3), F(d_3) \times F(d_3), F(d_4) \times F(d_4), F(d_4) \times F(d_4), F(d_2) \times F(d_3), F(d_3) \times F(d_3) \times F(d_3), F(d_4) \times F(d_4), F(d_5) \times F(d_5)$  is a two soft equivalence relation.

Then  $[F(d_1)]_M = \{F(d_1), F(d_2)\}, [F(d_2)]_M = \{F(d_1), F(d_2)\}, [F(d_3)]_M = \{F(d_3)\}, [F(d_4)]_M = \{F(d_4)\} \text{ and } [F(d_1)]_N = \{F(d_1)\}, [F(d_2)]_N = \{F(d_2), F(d_3)\}, [F(d_4)]_N = \{F(d_4)\}.$ 

So, we have  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) = \{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\})\}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) = \{\{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\})\}, (d_2, \{k_2, k_3\})\}, (d_3, \{k_2, k_3\})\}, \{(d_2, \{k_2, k_3\}), (d_3, \{k_2, k_3\})\}$  and  $\mathcal{B}_{\mathcal{R}}(G_B) = \{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\})\}, (d_3, \{k_2, k_3\})\}, (d_3, \{k_2, k_3\})\}, \{(d_2, \{k_2, k_3\}), (d_3, \{k_2, k_3\})\}.$  Hence

$$\begin{split} \tilde{\tau}_{M+N}(G_B) &= \{\mathcal{U}, \phi, \{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\})\}, \{\{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\})\}, \\ &\{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\}), (d_3, \{k_2, k_3\})\}, \{(d_2, \{k_2, k_3\})\}, \\ &(d_3, \{k_2, k_3\})\}, \{(d_1, \{k_1, k_2\}), (d_2, \{k_2, k_3\}), (d_3, \{k_2, k_3\})\}, \\ &\{(d_2, \{k_2, k_3\}), (d_3, \{k_2, k_3\})\}\} \end{split}$$

is a multi-nano soft topological space.

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**Definition 3.3.** Let  $F_A \subseteq F_E$  be a soft set over  $\mathcal{U}$  and  $(\mathcal{U}, F_A)$  be a soft approximation space and  $G_B \subseteq F_A$ . We define the characterization of five basic types of multi-nano soft topological space as follows as:

- (i) If  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) = \tilde{\phi}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) = \tilde{\mathcal{U}}$ , then  $\tilde{\tau}_{M+N}(G_B) = {\tilde{\mathcal{U}}, \tilde{\phi}}$  is called as multi-nano indiscrete soft topology on  $\mathcal{U}$ .
- (*ii*) If  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) = \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) = \hat{\mathcal{U}}$ , then the multi-nano soft topology  $\tilde{\tau}_{\mathcal{M}+\mathcal{N}}(G_B) = \{\tilde{\mathcal{U}}, \tilde{\phi}, \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B)\}.$
- (*iii*) If  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) = \tilde{\phi}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) \neq \tilde{\mathcal{U}}$ , then multi-nano soft topology  $\tilde{\tau}_{\mathcal{M}+\mathcal{N}}(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B)\}.$
- (iv) If  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) \neq \tilde{\emptyset}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) = \tilde{\mathcal{U}}$ , then multi-nano soft topology  $\tilde{\tau}_{M+\mathcal{N}}(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B), \mathcal{B}_{\mathcal{M}+\mathcal{N}}(G_B)\}.$
- (v) If  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) \neq \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B)$ , where  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) \neq \tilde{\emptyset}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B) \neq \tilde{\mathcal{U}}$ , then multi-nano discrete soft topology on  $\tilde{\tau}_{M+N}(G_B) = \{\tilde{\mathcal{U}}, \tilde{\emptyset}, \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B), \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B), \mathcal{B}_{\mathcal{M}+\mathcal{N}}(G_B)\}$ .

**Proposition 3.1.** Let  $(\mathcal{U}, F_A)$  be a soft approximation space and let  $G_B, H_C \in F_A$ and  $M, N \in R$ . Then

(i)  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(F_{\phi}) = F_{\phi}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(F_{\phi}) = F_{\phi}$ . (ii)  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(F_{A}) = F_{A}$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(F_{A}) = F_{A}$ . (iii) If  $G_{B} \subseteq H_{C}$  then  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_{B}) \subseteq \mathcal{L}_{\mathcal{M}+\mathcal{N}}(H_{C})$  and  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_{B}) \subseteq \mathcal{U}_{\mathcal{M}+\mathcal{N}}(H_{C})$ . (iv)  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_{B}) \subseteq [\mathcal{U}_{\mathcal{M}+\mathcal{N}}(H_{C})^{c}]^{c}$ . (v)  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_{B} \cap H_{C}) = \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_{B}) \cap \mathcal{L}_{\mathcal{M}+\mathcal{N}}(H_{C})$ . (vi)  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_{B} \cap H_{C}) = \mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_{B}) \cap \mathcal{L}_{\mathcal{M}+\mathcal{N}}(H_{C})$ . (vii)  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_{B} \cup H_{C}) = \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_{B}) \cup \mathcal{U}_{\mathcal{M}+\mathcal{N}}(H_{C})$ . (ix)  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_{B} \cap H_{C}) \subseteq \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_{B}) \cap \mathcal{U}_{\mathcal{M}+\mathcal{N}}(H_{C})$ .

**Proposition 3.2.** Let  $(\mathcal{U}, \tilde{\tau}_{M+N}, E)$  be a multi-nano soft topological space and let  $G_B \in F_A$  and  $M, N \in R$ . Then

(i) 
$$G_B \subseteq \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B)$$
  
(ii)  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) \subseteq G_B$   
(iii)  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(\mathcal{L}_{\mathcal{M}+\mathcal{N}})(G_B) \subseteq G_B$ .  
(iv)  $G_B \subseteq \mathcal{L}_{\mathcal{M}+\mathcal{N}}(\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B))$ .  
(v)  $\mathcal{U}_{\mathcal{M}+\mathcal{N}}(\mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B)) \subseteq \mathcal{U}_{\mathcal{M}+\mathcal{N}}(G_B)$ .  
(vi)  $\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B) \subseteq \mathcal{L}_{\mathcal{M}+\mathcal{N}}(\mathcal{L}_{\mathcal{M}+\mathcal{N}}(G_B))$ 

**Definition 3.4.** Let  $(\mathcal{U}, \tilde{\tau}_{M+N}, E)$  be a multi- nano soft topological space over  $\mathcal{U}$ . Then multi-nano soft interior of soft set  $H_C \subseteq F_A$  over  $\mathcal{U}$  is denoted by  $H_C^\circ$ . Thus  $H_C^\circ$  is the largest multi nano soft open set contained in  $H_C$  and is defined as the union of all multi-nano soft open sets contained in  $H_C$ .

**Example 2.** From Example 3.4 Let  $H_C = \{(d_3, \{k_2, k_3\})\}$ , then multi-nano soft interior  $H_C^{\circ} = \{(d_3, \{k_2, k_3\}), (d_2, \{k_2, k_3\})\}$ .

**Theorem 3.1.** Let  $(\mathcal{U}, \tau_{M+N}, E)$  be a multi-nano soft topological space over  $\mathcal{U}$  and  $H_C \subseteq F_A$  and  $H_C$  is an multi-nano soft open set if and only if  $H_C = H_C^{\circ}$ .

*Proof.* If  $H_C$  is an multi-nano soft open set, then the largest multi-nano soft open set that is contained by  $H_C$  is equal to  $H_C$ . Therefore  $H_C = H_C^{\circ}$ . Conversely, It is know that  $H_C^{\circ}$  is a multi-nano soft open set, and if  $H_C^{\circ} = H_C$ , then  $H_C$  is an multi-nano soft open set.

**Theorem 3.2.** Let  $(\mathcal{U}, \tau_{M+N}, E)$  be a multi-nano soft topological space and  $H_C, I_D \subseteq F_A$ . Then

- (a)  $[H_C^{\circ}]^{\circ} = H_C^{\circ}$ . (b)  $H_C \subseteq I_D \Rightarrow H_C^{\circ} \subseteq I_D^{\circ}$ . (c)  $H_C^{\circ} \cap I_D^{\circ} = [H_C \cap I_D]^{\circ}$ .
- $(d) \quad H^{\circ} \mapsto H^{\circ} \subset [H \cap L]^{\circ}$

(d)  $H_C^{\circ} \cup I_D^{\circ} \subseteq [H_C \cap I_D]^{\circ}$ .

**Definition 3.5.** Let  $(\mathcal{U}, \tau_{M+N}, E)$  be a multi-nano soft topological space over  $\mathcal{U}$ . Then multi-nano soft closure of soft set  $H_C \subseteq F_A$  over  $\mathcal{U}$  is denoted by  $\overline{H}_C$ . Thus  $\overline{H}_C$  is the smallest multi-nano soft closed set which containing  $H_C$  and is defined as the intersection of all multi-nano soft closed supersets of  $H_C$ .

**Example 3.** By example 3.4 and the complement of multi-nano soft topological space

$$\begin{split} &[\tilde{\tau}_{M+N}(G_B)]^c = \{\tilde{\mathcal{U}}, \tilde{\phi}, \{(d_1, \{k_3, k_4, k_5, k_6\}), (d_2, \{k_1, k_4, k_5, k_6\})\}, \\ &\{\{(d_1, \{k_3, k_4, k_5, k_6\}), (d_2, \{k_1, k_4, k_5, k_6\})\}, \{(d_1, \{k_3, k_4, k_5, k_6\}), \\ &(d_2, \{k_1, k_4, k_5, k_6\}), (d_3, \{k_1, k_4, k_5, k_6\})\}, \{(d_2, \{k_1, k_4, k_5, k_6\}), \\ &(d_3, \{k_1, k_4, k_5, k_6\})\}, \{(d_1, \{k_3, k_4, k_5, k_6\}), (d_2, \{k_1, k_4, k_5, k_6\}), \\ &(d_3, \{k_1, k_4, k_5, k_6\})\}, \{(d_2, \{k_1, k_4, k_5, k_6\}), (d_3, \{k_1, k_4, k_5, k_6\})\}, \\ \end{split}$$

Let  $H_C = \{(d_2, \{k_1, k_4, k_5, k_6\})\}$  is a nano multi-soft closed set, then multi-nano soft closure  $\overline{H_C} = \{(d_2, \{k_1, k_4, k_5, k_6\})\} = H_C$ .

**Theorem 3.3.** Let  $(\mathcal{U}, \tau_{M+N}, E)$  be a multi-nano soft topological space over  $\mathcal{U}, H_C, I_D \subseteq F_A$ . Then

- (a)  $\overline{\phi} = \phi$  and  $\overline{\mathcal{U}} = \mathcal{U}$ .
- (b)  $H_C \subseteq \overline{H_C}$ .
- (c)  $H_C$  is a multi-nano soft closed set if and only if  $H_C = \overline{H_C}$ .
- (d)  $\overline{H_C} = \overline{H_C}$ .

(e) 
$$I_D \subseteq H_C \Rightarrow \overline{I}_D \subseteq \overline{H}_C$$

- (f)  $\overline{H}_C \cap \overline{I}_D \subseteq \overline{[H_C \cap I_D]}.$
- (g)  $\overline{H}_C \cup \overline{I}_D = \overline{[H_C \cup I_D]}.$

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DEPARTMENT OF MATHEMATICS KONGUNADU ARTS AND SCIENCE COLLEGE COIMBATORE-641 029,TAMIL NADU,INDIA. *Email address*: spr.priyalatha@gmail.com

MATHEMATICS DEPARTMENT, FACULTY OF SCIENCE AL-BALQA APPLIED UNIVERSITY SALT 19117, JORDAN. *Email address*: wadeialomeri@bau.edu.jo 7717