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HIGHER DIMENSIONAL BIANCHI TYPE-III STRING UNIVERSE WITH BULK VISCOUS FLUID AND CONSTANT DECELERATION PARAMETER(DP)

JITEN BARO AND KANGUJAM PRIYOKUMAR SINGH[1]

ABSTRACT. Here, we have studied a Bianchi type-III string cosmologicl model with bulk viscous fluid and negative constant DP in general relativity considering five dimensional space-time. To get the exact solutions of the survival field equations, we assume that (i) DP is a constant and negative quantity and (ii) the shear scalar and expansion scalar are proportional. Some of the most important parameters of the model are obtained and their behaviors are studied. The model universe obtained here is expanding, shear free throughout the evolution, anisotropic at late time when $n \neq 1$ and the late universe is dominated by the particles.

1. INTRODUCTION

It is now almost proved from observational and theoretical fact that the universe is expanding with acceleration from the big-bang till today. However, no one can guarantee for forever expansion because there is no final conclusion about the expansion or contraction of universe till today. From various literatures and opinions it can be belief that the acceleration of present universe may be accompanied by way of deceleration. But the precise motive of this expanding universe is not known to us till today which inspired all the cosmologists and physicists for similarly research within the area of studies on this field. In the recent past years, several models in cosmology has been proposed by different

[1]*corresponding author*

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authors in order to explain the hidden reasons of expansion of the existing universe with the acceleration in the framework of string theory. The string theory is one of the most important theories in cosmology that study about the unknown facts of the universe. In latest years, the string cosmological problem has attracted huge interest in the field of research because of their great position in the evolution of the universe in early era. Cosmic strings are topologically stable defects, which are probably formed at some stage of the phase transition or earlier the introduction of particles in the early universe. In the field of general relativity, Stachel [1] and Letelier [2] initiated the study on strings. Spatially homogeneous and anisotropic Bianchi type cosmological model plays a great function to describe the large-scale behavior of the universe. Furthermore, from several kinds of literature and findings one can actually locate that the anisotropic model had been taken as possible models to initiate the expansion of the universe.

Bulk viscosity performed a great role in the evolution of the early universe. Its impact on the evolution of universe has been studied by means of several researchers in the frame work of well known concept of general relativity. Misner [3] studied about the consequences of bulk viscosity in the cosmological evolution of the universe. Some of the famous researchers [4-8], who have studied several Bianchi models in the field of general relativity with bulk viscosity.

A cosmological model in higher-dimensions performs a crucial role in different aspects of the early phases of the cosmological evolution of the universe. It is not possible to unify the gravitational forces in nature in typical four-dimensional space-times. So the theory in higher dimensions may be applicable in the early evolution. The study on higher-dimensional space-time gives us an important idea about the universe that our universe was much more smaller at initial epoch than the universe observed in these days. Many researchers motivated to enter into the theory of higher dimensions to discover the hidden phenomenon of the universe. Subsequently, many researchers have already investigated various cosmological models in five dimensional space-time with various Bianchi type models in different aspects [9-12].

Inspired by the above studies, here in this article we have investigated the higher dimensional bulk viscous cosmological model with string in Bianchi type

III space–time considering constant DP. In this paper, Sec. 2 describes the formulation of problem. Sec.3. gives the solutions of the cosmological problems. Some of the important physical and geometrical parameters are derived in sec.4. The results found are discussed in Sec.5. Finally, in last Sec., concluding points are provided.

2. Metric and Field equations

Here Bianchi type-III metric in 5-dimension is considered as

$$ds^2 = a^2dx^2 + b^2(e^{-2x}dy^2 + dz^2) + c^2dm^2 - dt^2. \tag{2.1}$$

Here a, b and c are the functions of t and 'm' is the extra dimensions(space-like). The EFE in general relativity with $8\pi G = 1, C = 1$ is

$$R_{ij} - \frac{1}{2}Rg_{ij} = -T_{ij}. \tag{2.2}$$

The energy-momentum tensor with bulk viscosity is

$$T_{ij} = \rho u_i u_j - \lambda x_i x_j - \xi\theta(g_{ij} + u_i u_j). \tag{2.3}$$

Here, $\rho = \lambda + \rho_p$ is the energy density, λ is the tension density and ρ_p is the particle density, θ is expansion scalar and ξ is the bulk viscosity coefficient. Also $u^i = (0, 0, 0, 0, 1)$ is the five velocity vector of particles and $x^i = (0, 0, c^{-1}, 0, 0)$ represent the unit vector which is space-like and this represents the direction of the strings such that $u_iu^j = -1 = -x_ix^j$ *and* $u_ix^i = 0$.

If R(t)be the average scale factor then the spatial volume is

$$V = ab^2c = R^4. \tag{2.4}$$

Using the equations (2.1)-(2.3) we obtain

$$2\frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{b}^2}{b^2} + 2\frac{\dot{b}\dot{c}}{bc} = \xi\theta, \tag{2.5}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} = \xi\theta, \tag{2.6}$$

$$\frac{\ddot{a}}{a} + \frac{\ddot{b}}{b} + \frac{\ddot{c}}{c} + \frac{\dot{a}\dot{b}}{ab} + \frac{\dot{a}\dot{c}}{ac} + \frac{\dot{b}\dot{c}}{bc} - \frac{1}{a^2} = \lambda + \xi\theta, \tag{2.7}$$

$$\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} + 2\frac{\dot{a}\dot{b}}{ab} + \frac{\dot{b}^2}{b^2} - \frac{1}{a^2} = \xi\theta, \tag{2.8}$$

$$2\frac{\dot{a}\dot{b}}{ab}+\frac{\dot{a}\dot{c}}{ac}+2\frac{\dot{b}\dot{c}}{bc}+\frac{\dot{b}^2}{b^2}-\frac{1}{a^2}=\rho, \tag{2.9}$$

$$\frac{\dot{a}}{a}=\frac{\dot{b}}{b}. \tag{2.10}$$

The overhead dots here denotes the order of derivative w.r.t. time 't'.

3. Solution of the Field Equations

Eqn.(2.10) yields $a = lb$, l is integration constant. We take $l = 1$, then

$$a=b \tag{3.1}$$

By the use of (3.1) in the equations (2.5)-(2.9) we get

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}=\xi\theta, \tag{3.2}$$

$$2\frac{\ddot{b}}{b}+\frac{\ddot{c}}{c}+\frac{\dot{b}^2}{b^2}+2\frac{\dot{b}\dot{c}}{bc}-\frac{1}{b^2}=\lambda+\xi\theta, \tag{3.3}$$

$$3\frac{\ddot{b}}{b}+3\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}=\xi\theta, \tag{3.4}$$

$$3\frac{\dot{b}\dot{c}}{bc}+3\frac{\dot{b}^2}{b^2}-\frac{1}{b^2}=\rho. \tag{3.5}$$

We have 4 highly nonlinear independent differential equations (3.2)-(3.5) with variables b, c, λ, ρ, ξ and θ which are unknown. So to get the exact solutions of above equations we must have two extra conditions.

Berman's [13] suggestion regarding variation of Hubble's parameter H provides us a model universe that expands with constant DP. So for the determinate solution, let us take DP to be a negative constant,

$$q=-\frac{R\ddot{R}}{\dot{R}^2}=h \text{ (constant)}. \tag{3.6}$$

The shear and expansion scalar are proportional ($\sigma \propto \theta$) [Thorne [14], Collins et al. [15]]. This leads to the equation

$$b=c^n, \text{where}, n\neq 0 \text{ is a constant}. \tag{3.7}$$

Solving (3.6), we get

$$R=(\alpha t+\beta)^{\frac{1}{1+h}}\ h\neq -1,\ \alpha \text{ and } \beta \text{ are constants of integration}.$$

Using (2.4), (3.1) and (3.7) we get,

$$a = b = (\alpha t + \beta)^{\frac{4n}{(1+h)(3n+1)}}, \; c = (\alpha t + \beta)^{\frac{4}{(1+h)(3n+1)}}.$$

With the suitable choice of coordinates and constant we take $(\alpha = 1, \; \beta = 0)$

$$a = b = t^{\frac{4n}{(1+h)(3n+1)}}, \; c = t^{\frac{4}{(1+h)(3n+1)}}$$

Which gives the geometry of the metric (2.1)as

$$ds^2 = t^{\frac{8n}{(1+h)(3n+1)}}(dx^2 + e^{-2x}dy^2 + dz^2) + t^{\frac{8}{(1+h)(3n+1)}}dm^2 - dt^2. \tag{3.8}$$

4. Physical and Geometric Parameters

We obtained some of the important physical and geometrical parameters which are useful for the discussion on the evolution of the universe.

$$\rho = \frac{48n(n+1)}{(1+h)^2(3n+1)^2t^2} - t^{-\frac{8n}{(1+h)(3n+1)}}, \tag{4.1}$$

$$\lambda = -t^{-\frac{8n}{(1+h)(3n+1)}},$$

$$\rho_p = \frac{48n(n+1)}{(1+h)^2(3n+1)^2t^2}, \tag{4.2}$$

$$V = t^{\frac{4}{(1+h)}}, \;\; \theta = \frac{4}{(1+h)t},$$

$$H = \frac{1}{(1+h)t},$$

$$\xi = \frac{(2n+1)(3n^2 - 3nq - q) + 3}{(1+h)(3n+1)^2t},$$

$$\sigma = \frac{\sqrt{6}(n-1)}{(1+h)(3n+1)t}. \tag{4.3}$$

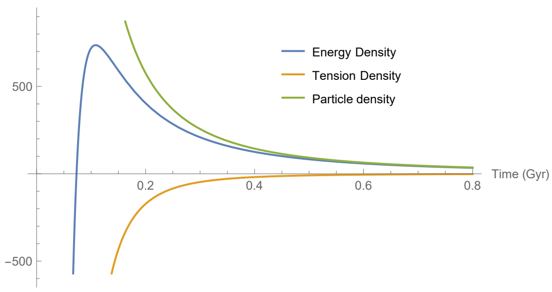

FIGURE 1. Variation of ρ, λ, ρ_p vs. t.

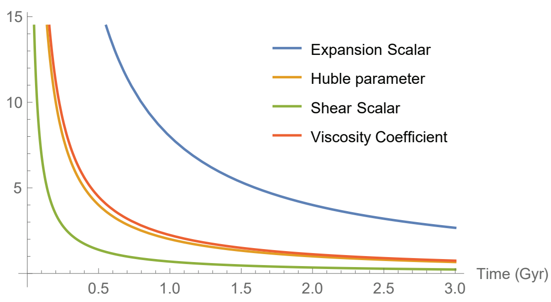

FIGURE 2. Variation of θ, H, σ, V vs. t.

5. Physical Interpretations

The equation (3.8) here represents a Bianchi type-III cosmological model with string in general relativity with constant DP (q=constant) in presence of bulk viscosity in 5-D space-time. The variation of parameters with time for the model are shown above by taking $n = \frac{1}{2}, h = -\frac{1}{2}$.

i. From Fig. 1. and Eqn.(4.1) we have observed that the energy density ρ is negative at the initial epoch and it changes sign from negative to positive after some finite time and finally becomes 0 when $t \to \infty$. Also it is seen that the tension density λ of string is negative(Fig1). It is mentioned by Letelier [2] that λ can be < 0 or > 0. The phase of string disappears when $\lambda < 0$. The strong energy condition $\rho \geq 0$, $\lambda < 0$ as given by Hawking and Ellis [16] are satisfied for the model in the late time universe.

ii. From (4.2), we note $\rho_p \geq 0$ for all time t and ρ_p decreases with time (Fig1). It is also observed that $\frac{\rho_p}{|\lambda|} > 1$ which shows that λ diminishes more faster than ρ_p. This tells us that the late universe is particle dominated.

iii. The bulk viscosity $\xi \to \infty$ when t=0 and it decreases with the increasers of t and finally when $t \to \infty$ bulk viscosity ξ demises (Fig. 2). The function of the bulk viscosity is to retard the expansion of the universe and since bulk viscosity deceases with the time so retardness also decreases which supports in the expansion in faster rate in the late time universe. From the above discussion it can be seen that the bulk viscosity plays a great function in the evolution of the universe.

iv. The volume V=0 at initial epoch t=0 and the volume increases with the increases of the t and it become ∞ when time $t \to \infty$. So the universe is expanding with time. The size of the universe was very small just after the big bang exploitation then the size continuously increasing till now and it will increase late time also.

v. At $t = 0$, the Hubble parameter H and scalar expansion θ, both are infinite and as the time increases gradually they decreases and finally they become 0 at $t \to \infty$ (Fig. 2). Hence the model shows that with the increases of time the universe expands but the expansion rate becomes slower as time increases and at $t \to \infty$, the expansion stops. And since $\frac{dH}{dt} < 0$ which also tells us that our present universe is in the accelerated expanding mode.

vi. In equation (4.3) and Fig. 2. it seen that the value of the shear scalar $\sigma \to \infty$ at initial epoch and it decreases as the time increases and become zero at late universe showing that the universe obtained here is shear free in the late time.

vii. The mean anisotropy parameter $\Delta = \frac{3(n-1)^2}{(3n+1)^2} = constant \neq 0$ for $n \neq 1$ and $\Delta = 0$ for $n = 1$. Also as $t \to \infty$ the value of $\frac{\sigma^2}{\theta^2} = constant \neq 0$ for $n \neq 1$ and $\frac{\sigma^2}{\theta^2} = 0$ for $n = 1$. From both statements we can conclude that this model is anisotropic for large value of t when $n \neq 1$ but it is isotropic for $n = 1$.

6. Conclusion

In this article, we have attempted to present a new solution to the field equations obtained for Bianchi type-III universe with bulk viscosity by using the law of variation of H which yields constant DP. This variational law for H in equation (3.8) explicitly determine the values of the scale factors(R). So here, we have constructed a 5D Bianchi type-III string universe with bulk viscosity and constant DP in general relativity by the use of certain physically plausible assumptions, that agrees with the present day observational data in general relativity. The model is expanding, non shearing, anisotropic for $n \neq 1$ and isotropic for $n = 1$ in the late universe which is also in accordance to the present day observational data made by WMAP and COBE. The present universe starts at initial epoch $t = 0$ with 0 volume and then expand with accelerated motion and the expansion rate slows down with increase of time. The bulk viscosity coefficient plays a great role in the cosmological consequences. The tension density is negative quantity showing that the string phase disappears and present day universe is particle dominated.

REFERENCES

[1] J. STACHEL: *Thickening the String. I. The String Perfect Dust*, Phys Rev D, **21** (1980), 2171-2181.

[2] P. S. LETELIER: *Clouds of Strings in General Relativity*, Phys.Rev.D, **20** (1979), 994-1032.

[3] C. W. MISNER: *The isotropy of the universe*, J. Astrophys, **151** (1968), 431-457.

[4] X. X. WANG: *Bianchi type-III string cosmological model with bulk viscosity in general relativity*, Chin. Phys. Lett., **22** (2005), 29-32.

[5] R. BALI, A. PRADHAN: *Bianchi type-III string cosmological models with time dependent bulk viscosity*, Chin. Phys. Lett., **24** (2007), 585-588.

[6] S. P. KANDALKAR, S. P. WASNIK, S. P. GAWANDE: *Some Bianchi type-I string Cosmological models with Bulk Viscosity*, Rom. Jour. Phys, **56** (2011), 289-296.

[7] V. HUMAD, H. NAGAR, S. SHRIMALI: *Bulk viscous fluid Bianchi type-I string cosmological model in general relativity*, IOSR-JM, **12** (2016), 11-15.

[8] P. K. SINGH, J. BARO: *Bulk Viscous Fluid Bianchi Type-I String Cosmological Model with Negative Constant Deceleration Parameter*, Adv. Math. Sci. J., **9**(7) (2020), 4907–4916.

[9] K. D. KRORI, T. CHAUDHURI, C. R. MAHANTA: *Strings in Some Bianchi Type Cosmologies*, Gen. Rel. Grav., **26** (1994), 265-274.

[10] G. S. KHADEKAR, P. VRISHALI: *String dust cosmological model in higher-dimensional space time*, Int. J. Mod. Phys. D, **14** (2005), 1621-1634.

[11] G. MOHANTY, G. C. SAMANTA: *Five dimensional LRS Bianchi type-I String Cosmological model in general relativity*, FIZIKA B, **19** (2010), 239-246.

[12] G. C. SAMANTA, S. K. BISWAL, G. MOHANTY: *Some five dimensional Bianchi type-III String Cosmological models in General Relativity*, Bulg. J. Phys., **38** (2011), 380-389.

[13] M. S. BERMAN: *A special law of variation for Hubble's parameter*, Nuov.Cim.B, **74**(1983), 182-186.

[14] K. S. THORNE: *Primordial Element Formation, Primrdial Magnectic Fields and Isotropy of the Universe*, Astrophy. J., **148** (1967), 51.

[15] C. B. COLLINS, E. N. GLASS, D. A. WILKINSON: *Exact spatially homogeneous cosmologies*. Gen. Relative.Gravit. **12** (1980), 805-823.

[16] S. W. HAWKING, G. F. R. ELLIS: *The large scale structure of space-time*, Cambridge, England, (1974), 88.

DEPARTMENT OF MATHEMATICAL SCIENCES
BODOLAND UNIVERSITY
KOKRAJHAR-783370, BTR, ASSAM, INDIA
Email address: barojiten5@gmail.com

DEPARTMENT OF MATHEMATICS
MANIPUR UNIVERSITY
IMPHAL-795003 MANIPUR,INDIA
Email address: pk_mathematics@yahoo.co.in