

TRANSLATIONS OF INTUITIONISTIC FUZZY SUBALGEBRAS IN BF-ALGEBRAS

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ABSTRACT. This research article explores on, the concepts of IFT to IFS in BF-algebras. The phenomenon of IF-extensions and IF-multiplications of IFS is proposed and several related properties are investigated. In this paper, the interaction between IFTs and IF-extensions of IFSs are investigated.

1. INTRODUCTION AND PRELIMINARIES

Iseki et al. proposed two classes of abstract algebras BCI-algebras and BCK-algebras [3]. It is evident that the class of BCK-algebras is a proper subclass of the class of BCI-Algebras. H.S. Kim et al. [24] proposed a new notion known as a B-algebras, which is a simplification of BCK-algebra. Walendziak [1] defined BF-algebras. In 1965, the notion of fuzzy sets, an extraordinary idea in mathematics, was proposed by Zadeh [25]. Saeid and Rezvani [2] proposed BF-subalgebras based on the above concepts. Atanassov [21,22] was the first researcher who introduced the new idea of “IF-set”, which is depicted as generalized idea of fuzzy set. Satyanarayana et al. [4,6-8]. Proposed fuzzy BF-subalgebras and IFS. Fuzzy Translation of BCK/BCI-algebras, worked out by many researchers such as Lee and Jun [23]. Further some more researchers

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used the concept of fuzzy and fuzzy functions on time scales [9-20]. The aim of this article is applying the notion of the IFTs, IF-extensions and IF-multiplications of IFSs in BF-algebras are investigated.

Definition 1.1. [1] A BF-algebra is a non-empty set Y with a constant 0 and a binary operation satisfying the following axioms:

- (i) $\alpha_1 * \alpha_1 = 0$,
- (ii) $\alpha_1 * 0 = \alpha_1$,
- (iii) $0 * (\alpha_1 * \alpha_2) = \alpha_2 * \alpha_1$ for all $\alpha_1, \alpha_2 \in Y$.

X is considered a BF-algebra in the following conversation.

Example 1. [6] R = The set of real numbers and $A = (R, *, 0)$ be an Algebra given by

$$\alpha_1 * \alpha_2 = \begin{cases} \alpha, & \text{if } \alpha_2 = 0, \\ \alpha_2, & \text{if } \alpha_1 = 0, \\ 0, & \text{otherwise} \end{cases}$$

Then A will become a BF-algebra.

Example 2. [5] The set $X = \{0, p, q, s, t\}$, $*$ is given by the Table:1 is a BF-Algebra

TABLE 1

$*$	0	p	q	s	t
0	0	t	s	q	p
p	p	0	t	s	q
q	q	p	0	t	s
s	s	q	p	0	t
t	t	s	q	p	0

Definition 1.2. [1] $I \subseteq Y$ is know to be subalgebra of Y , if

- (i) $0 \in I$,
- (ii) $y_1 \in I$ and $y_2 \in I \Rightarrow y_1 * y_2 \in I$.

Definition 1.3. An IF-set $A = (X, R_A, J_A)$ is supposed to be IFSs of X if

- (i) $R_A(x_1 * x_2) \geq \min\{R_A(x_1), R_A(x_2)\}$
- (ii) $J_A(x_1 * x_2) \leq \max\{J_A(x_1), J_A(x_2)\}$

for all $x_1, x_2 \in X$.

2. TRANSLATIONS OF IFSs IN BF-ALGEBRAS

The following discussion is on the notation of IFT on X . It evident that, X stands a BF-algebra, and for any IF-set $A = (R_A, J_A)$ of X , $T = 1 - \sup\{R_A(x_1) : x_1 \in X\} = 1 - \inf\{J_A(x_1) : x_1 \in X\}$.

Definition 2.1. $A = (R_A, J_A)$ is an IF-subset of X . Let $\alpha \in [0, T]$. An object having the form $A_\alpha^T = ((R_A)_\alpha^T, (J_A)_\alpha^T)$ is called IF- α -translation of A if $(R_A)_\alpha^T(x_1) = R_A(x_1) + \alpha$ and $(J_A)_\alpha^T(x_1) = J_A(x_1) - \alpha$ for all $x_1 \in X$.

Definition 2.2. Let $A = (R_A, J_A)$ be an IF-Subset of X and let $\alpha \in [0, 1]$. An object having the form $A_\alpha^m = ((R_A)_\alpha^m, (J_A)_\alpha^m)$ is called an IF- α -multiplication of A if $(R_A)_\alpha^m(x_1) = \alpha \cdot R_A(x_1)$ and $(J_A)_\alpha^m(x_1) = \alpha \cdot J_A(x_1)$ for all $x_1 \in X$. For any IF-set $A = (R_A, J_A)$ of X , , an IF-0-multiplication $A_0^m = ((R_A)_0^m, (J_A)_0^m)$ of A is an IFS of X .

Example 3. Consider the BF-algebra $X = \{0, p, q, s, t\}$ in Example 2. Define a IF-subset $A = (R_A, J_A)$ of X by

$$R_A(r) = \begin{cases} 0.4; r \neq q \\ 0.1; r = q \end{cases} \quad \text{and} \quad J_A(r) = \begin{cases} 0.4; r \neq q \\ 0.7; r = q \end{cases},$$

then $A = (R_A, J_A)$ is IFS of X . Here $T = 1 - \sup\{R_A(r) : r \in X\} = 1 - 0.4 = 0.6 = 1 - \inf\{J_A(r) : r \in X\} = 1 - 0.4 = 0.6$. Choose $\alpha = 0.3 \in [0, T]$ and $\beta = 0.2 \in [0, 1]$. Then the mapping $(R_A)_{0.3}^T : X \rightarrow [0, 1]$ $(J_A)_{0.3}^T : X \rightarrow [0, 1]$ are defined by

$$(R_A)_{0.3}^T(r) = \begin{cases} 0.4 + 0.3 = 0.7; r \neq q \\ 0.1 + 0.3 = 0.4; r = q \end{cases}$$

and

$$(J_A)_{0.3}^T(r) = \begin{cases} 0.4 - 0.3 = 0.1; r \neq q \\ 0.7 - 0.3 = 0.4; r = q \end{cases},$$

which satisfies $A_{0.3}^T = ((R_A)_{0.3}^T, (J_A)_{0.3}^T) = (R_A(r) + 0.3, J_A(r) - 0.3)$ for all $r \in X$ is IF-0.3-Translation.

The mappings $(R_A)_{0.2}^m : X \rightarrow [0, 1]$ and $(J_A)_{0.2}^m : X \rightarrow [0, 1]$ are defined by

$$(R_A)_{0.2}^m(r) = \begin{cases} (0.4)(0.2) = 0.08; r \neq q \\ (0.1)(0.2) = 0.02; r = q \end{cases}$$

and

$$(J_A)_{0.2}^m(r) = \begin{cases} (0.4)(0.2) = 0.08; r \neq q \\ (0.7)(0.2) = 0.14; r = q \end{cases},$$

which satisfies $(R_A)_{0.2}^m(r) = 0.2.R_A$ $(J_A)_{0.2}^m(r) = 0.2.J_A$ for all $IF-0.2$ -multiplication.

Theorem 2.1. For all IFS $A = (R_A, J_A)$ of X $\wedge \alpha \in [0, T]$ the ' $IF-\alpha$ -translation'. $A_\alpha^T = ((R_A)_\alpha^T, (J_A)_\alpha^T)$ of $A = (R_A, J_A)$ is a IFS of X .

Proof. Let $r, s \in X$ and $\alpha \in [0, T]$. Then $R_A(r * s) \geq \min\{R_A(r), R_A(s)\}$. Now,

$$\begin{aligned} (R_A)_\alpha^T(r * s) &= R_A(r * s) + \alpha \geq \min\{R_A(r), R_A(s)\} + \alpha \\ &= \min\{R_A(r) + \alpha, R_A(s) + \alpha\} = \min\{(R_A)_\alpha^T(r), (R_A)_\alpha^T(s)\} \end{aligned}$$

and

$$\begin{aligned} (J_A)_\alpha^T(r * s) &= J_A(r * s) - \alpha \leq \max\{J_A(r), J_A(s)\} - \alpha \\ &= \max\{J_A(r) - \alpha, J_A(s) - \alpha\} = \max\{(J_A)_\alpha^T(r), (J_A)_\alpha^T(s)\}. \end{aligned}$$

Hence the theorem follows. $\square$

Theorem 2.2. For all IF-subset $A = (R_A, J_A)$ of X . $\wedge \alpha \in [0, T]$ if the $IF-\alpha$ - Translation $A_\alpha^T = ((R_A)_\alpha^T, (J_A)_\alpha^T)$ of $A = (R_A, J_A)$ is a IFS of X then $A = (R_A, J_A)$ is IFS of X .

Proof. Suppose that $A_\alpha^T = ((R_A)_\alpha^T, (J_A)_\alpha^T)$ is an IFSs of X and $\alpha \in [0, T]$. Let $r, s \in X$, we have

$$\begin{aligned} R_A(r * s) + \alpha &= (R_A)_\alpha^T(r * s) \geq \min\{(R_A)_\alpha^T(r), (R_A)_\alpha^T(s)\} \\ &= \min\{R_A(r) + \alpha, R_A(s) + \alpha\} = \min\{R_A(r), R_A(s)\} + \alpha \end{aligned}$$

and

$$\begin{aligned} J_A(r * s) - \alpha &= (J_A)_\alpha^T(r * s) \leq \max\{(J_A)_\alpha^T(r), (J_A)_\alpha^T(s)\} \\ &= \max\{J_A(r) - \alpha, J_A(s) - \alpha\} = \max\{J_A(r), J_A(s)\} - \alpha, \end{aligned}$$

which implies that $R_A(r * s) \geq \min\{R_A(r), R_A(s)\}$ and $J_A(r * s) \leq \max\{J_A(r), J_A(s)\}$ for all $r, s \in X$. Hence $A = (R_A, J_A)$ is IFS of X . $\square$

Theorem 2.3. $(A_1)_\alpha^T$ and $(A_2)_\alpha^T$ are two IFS of $X \Rightarrow (A_1 \cap A_2)_\alpha^T$ is also a IFS of X .

Proof. $(A_1)_\alpha^T$ and $(A_2)_\alpha^T$ are two IFS of X . Then

$$\begin{aligned} (R_{A_1 \cap A_2})_\alpha^T(x_1 * x_2) &= \min\{(R_{A_1})_\alpha^T(x_1 * x_2), (R_{A_2})_\alpha^T(x_1 * x_2)\} \\ &= \min\{R_{A_1}(x_1 * x_2) + \alpha, R_{A_2}(x_1 * x_2) + \alpha\} \end{aligned}$$

$$\begin{aligned}
&\geq \min\{\min\{R_{A_1}(x_1), R_{A_1}(x_2)\} + \alpha, \min\{R_{A_2}(x_1), R_{A_2}(x_2)\} + \alpha\} \\
&= \min\{\min\{R_{A_1}(x_1) + \alpha, R_{A_1}(x_2) + \alpha\}, \min\{R_{A_2}(x_1) + \alpha, R_{A_2}(x_2) + \alpha\}\} \\
&= \min\{\min\{(R_{A_1})_\alpha^T(x_1), (R_{A_1})_\alpha^T(x_2)\}, \min\{(R_{A_2})_\alpha^T(x_1), (R_{A_2})_\alpha^T(x_2)\}\} \\
&= \min\{\min\{(R_{A_1})_\alpha^T(x_1), (R_{A_2})_\alpha^T(x_1)\}, \min\{(R_{A_1})_\alpha^T(x_2), (R_{A_2})_\alpha^T(x_2)\}\} \\
&= \min\{(R_{A_1 \cap A_2})_\alpha^T(x_1), (R_{A_1 \cap A_2})_\alpha^T(x_2)\} \\
&\quad (R_{A_1 \cap A_2})_\alpha^T(x_1 * x_2) \geq \min\{(R_{A_1 \cap A_2})_\alpha^T(x_1), (R_{A_1 \cap A_2})_\alpha^T(x_2)\} \\
&\quad (J_{A_1 \cap A_2})_\alpha^T(x_1 * x_2) = \max\{(J_{A_1})_\alpha^T(x_1 * x_2), (J_{A_2})_\alpha^T(x_1 * x_2)\} \\
&\quad = \max\{(J_{A_1})(x_1 * x_2) - \alpha, (J_{A_2})(x_1 * x_2) - \alpha\} \\
&\leq \max\{\max\{J_{A_1}(x_1), J_{A_1}(x_2)\} - \alpha, \max\{J_{A_2}(x_1), J_{A_2}(x_2)\} - \alpha\} \\
&= \max\{\max\{J_{A_1}(x_1) - \alpha, J_{A_1}(x_2) - \alpha\}, \max\{J_{A_2}(x_1) - \alpha, J_{A_2}(x_2) - \alpha\}\} \\
&= \max\{\max\{(J_{A_1})_\alpha^T(x_1), (J_{A_1})_\alpha^T(x_2)\}, \max\{(J_{A_2})_\alpha^T(x_1), (J_{A_2})_\alpha^T(x_2)\}\} \\
&= \max\{\max\{(J_{A_1})_\alpha^T(x_1), (J_{A_2})_\alpha^T(x_1)\}, \max\{(J_{A_1})_\alpha^T(x_2), (J_{A_2})_\alpha^T(x_2)\}\} \\
&= \max\{(J_{A_1 \cap A_2})_\alpha^T(x_1), (J_{A_1 \cap A_2})_\alpha^T(x_2)\} \\
&\quad (J_{A_1 \cap A_2})_\alpha^T(x_1 * x_2) \leq \max\{(J_{A_1 \cap A_2})_\alpha^T(x_1), (J_{A_1 \cap A_2})_\alpha^T(x_2)\}
\end{aligned}$$

Hence $(A_1 \cap A_2)_\alpha^T$ is a IFS of X . □

Theorem 2.4. Let $\{A_i/i = 1, 2, 3, \dots\}$ be a family of IFS of X . Then $(\cap A_i)_\alpha^T$ is also a IFS of X , where $(\cap A_i)_\alpha^T = \min\{(A_i)_\alpha^T(x)\}$.

Theorem 2.5. For any IFS $A = (R_A, J_A)$ of X , α is an element in $[0, 1]$, the IF- α -multiplication $A_\alpha^m = ((R_A)_\alpha^m, (J_A)_\alpha^m)$ of $A = (R_A, J_A)$ is an IFS of X .

Proof. Let $r, s \in X$ & $\alpha \in [0, 1]$. Then $R_A(r * s) \geq \min\{R_A(r), R_A(s)\}$. Now

$$\begin{aligned}
&(R_A)_\alpha^m(r * s) = \alpha.R_A(r * s) \geq \alpha.\min\{R_A(r), R_A(s)\} \\
&= \min\{\alpha.R_A(r), \alpha.R_A(s)\} = \min\{(R_A)_\alpha^m(r), (R_A)_\alpha^m(s)\}, (J_A)_\alpha^m(r * s) \\
&= \alpha.J_A(r * s) \leq \alpha.\max\{J_A(r), J_A(s)\} = \max\{\alpha.J_A(r), \alpha.J_A(s)\} \\
&= \max\{(J_A)_\alpha^m(r), (J_A)_\alpha^m(s)\}.
\end{aligned}$$

Hence the theorem follows. □

Theorem 2.6. For any IF-subset $A = (R_A, J_A)$ of X and $\alpha \in [0, 1]$, if the IF- α -multiplication $A_\alpha^m = ((R_A)_\alpha^m, (J_A)_\alpha^m)$ of $A = (R_A, J_A)$ is an of X then $A = (R_A, J_A)$ is IFS of X .

Proof. Assume $A_\alpha^m = ((\mu_A)_\alpha^m, (\lambda_A)_\alpha^m)$ is an IFS of X where $\alpha \in [0, T]$, $x_1, x_2 \in X$. One can have

$$\begin{aligned} \alpha.R_A(x_1 * x_2) &= (R_A)_\alpha^m(x_1 * x_2) \geq \min\{(R_A)_\alpha^m(x_1), (R_A)_\alpha^m(x_2)\} \\ &= \min\{\alpha.R_A(x_1), \alpha.R_A(x_2)\} = \alpha.\min\{R_A(x_1), R_A(x_2)\}, \alpha.J_A(x_1 * x_2) \\ &= (J_A)_\alpha^m(x_1 * x_2) \leq \max\{(J_A)_\alpha^m(x_1), (J_A)_\alpha^m(x_2)\} \\ &= \max\{\alpha.J_A(x_1), \alpha.J_A(x_2)\} = \alpha.\max\{J_A(x_1), J_A(x_2)\}, \end{aligned}$$

which implies that $R_A(x_1 * x_2) \geq \min\{R_A(x_1), R_A(x_2)\}$ and $J_A(x_1 * x_2) \leq \max\{J_A(x_1), J_A(x_2)\}$ for all $x_1, x_2 \in X$ since $\alpha \neq 0$. Hence $A = (R_A, J_A)$ is IFSs of X . $\square$

Theorem 2.7. $(A_1)_\alpha^m$ and $(A_2)_\alpha^m$ be two IFS of $X \Rightarrow (A_1 \cap A_2)_\alpha^m$ is also IFS of X .

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