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# AN INCLUSIVE LOCAL IRREGULARITY COLORING OF GRAPHS

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ABSTRACT. All graph in this paper are connected and simple. Let $G = (V, E)$ be a simple graph, where $V(G)$ is vertex set and $E(G)$ is edge set. The local irregularity vertex coloring of $G$ is $l : V(G) \to \{1, 2, \cdots, k\}$ and $w : V(G) \to N$ where $w(u) = \Sigma_{v \in N(u)} l(v)$ such that $opt(l) = \min\{\max\{l_i\}$ and for every $uv \in E(G), w(u) \neq w(v)$, $w$ is a local irregularity vertex coloring. The minimum of color set is called the local irregular chromatic number, denoted by $\chi_{lis}(G)$. In this paper, we determine the local irregular chromatic number of graphs.

## 1. INTRODUCTION

All graph in this paper are connected and simple. Let $G = (V, E)$ be a simple graph, where $V(G)$ is vertex set and $E(G)$ is edge set. Kristiana, et.al in [1] defined of local irregularity vertex coloring of $G$ is $l : V(G) \to \{1, 2, \cdots, k\}$ and $w : V(G) \to N$ where $w(u) = \Sigma_{v \in N(u)} l(v)$ such that $opt(l) = \min\{\max\{l_i\}$ and for every $uv \in E(G), w(u) \neq w(v)$, $w$ is a local irregularity vertex coloring. Furthermore, Kristiana, et.al introduced chromatic number local irregular, denoted by $\chi_{lis}(G)$ is cardinality of color of local irregularity vertex coloring. Kristiana et. al [2] found local irregularity vertex coloring of related wheel graphs and [3] found local irregularity of some families graph. Some of the propositions used in this paper:

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**Proposition 1.1.** *Let $P_n$ be a path graph for $n \geq 2$,*

$$\chi_{lis}(P_n) = \begin{cases} 2, & \textit{for } i = 2, 3 \\ 3, & \textit{for } i \geq 4 \end{cases}.$$

**Proposition 1.2.** *Let $C_n$ be a cycle graph for $n \geq 3$,*

$$\chi_{lis}(C_n) = \begin{cases} 2, & \textit{for } i \textit{ even} \\ 3, & \textit{for } i \textit{ odd} \end{cases}.$$

**Proposition 1.3.** *Let $S_n$ be a star graph for $n \geq 2$, $\chi_{lis}(S_n) = 2$.*

An inclusive vertex irregular $d$-distance of graph $G$ was studied by Baca, et.al [4]. Furthermore, Dafik, et.al [5] defined an inclusive distance antimagic labeling of graph $G$. In this paper, we develop new concept in local irregularity vertex coloring, we called an inclusive local irregularity vertex coloring.

## 2. Results

In this paper, we define inclusive local irregularity vertex coloring of graph and we discuss inclusive local irregularity vertex coloring of some special graphs, namely path graph, cycle graph, star graph and complete graph.

**Definition 2.1.** *A function $w : V(G) \to N$ is local irregularity vertex coloring, if $w^i(v) = l(v) + \sum_{u \in N} l(u)$ then $w^i$ is inclusive local irregularity vertex coloring.*

**Definition 2.2.** *The minimum $k$ of inclusive local irregularity vertex coloring of graph $G$ is called inclusive chromatic number local irregularity, denoted by $\chi^i_{lis}(G)$.*

**Lemma 2.1.** *Let $G$ be a graph connected and simple, $\chi^i_{lis}(G) \geq \chi_{lis}(G)$.*

*Proof.* We have $l : V(G) \to \{1, 2, \ldots, k\}$ as label function of vertex such that $uv \in E(G)$, $l(u) \neq l(v)$ and $\chi_{lis}(G)$ denoted minimum of cardinality local irregularity vertex coloring. based on Definition 2.1, there is a minimum equal to or greater than chromatic number local irregularity. □

We study an exact value inclusive chromatic number local irregularity of some special graph, namely path graph, cycle graph, and star graph.

**Theorem 2.1.** *Let $P_n$ be a path graph for $n \geq 4$, $\chi^i_{lis}(P_n) = 3$.*

*Proof.* $V(P_n) = \{x_i : 1 \leq i \leq n\}$ and $E(P_n) = \{x_i x_{i+1} : 1 \leq i \leq n-1\}$. For every $v \in V(P_n)$ is labeled by 1 then $w^i(x_i) = l(x_{i-1}) + l(x_i) + l(x_{i+1}) = 3$ and $w^i(x_{i+1}) = l(x_i) + l(x_{i+1}) + l(x_{i+2}) = 3$. It is contradiction by definition, $w^i(x_i) \neq w^i(x_{i+1})$ such that $opt(l) = 2$. Based on Lemma 2.1 and Proposition 1.1 that $\chi^i_{lis}(P_n) \geq \chi_{lis}(P_n) = 3$. To show the upper bound, we define $l : V(G) \to \{1, 2\}$ as follows:

$$l(x_i) = \begin{cases} 1, & \text{for } i \text{ odd} \\ 2, & \text{for } i \text{ even} \end{cases},$$

so that $opt(l) = 2$ and weight function as follows:

$$w^i(x_i) = \begin{cases} 3, & \text{for } i = 1, i \\ 4, & \text{for } i \text{ odd} \\ 5, & \text{for } i \text{ even} \end{cases}.$$

We have $|w^i(V(P_n))| = 3$, hence $\chi^i_{lis}(P_n) \leq 3$. The proof is complete. $\square$

Figure 1 is the illustration of the inclusive local irregularity coloring of a path graph $P_n$.

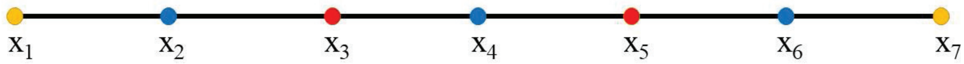


FIGURE 1. Inclusive Local Irregularity Vertex Coloring of Path Graph, $\chi^i_{lis}(P_n) = 3$

**Theorem 2.2.** *Let $C_n$ be a cycle graph for $n \geq 4$ and $n$ is even, $\chi^i_{lis}(C_n) = 2$.*

*Proof.* : $V(C_n) = \{x_i; 1 \leq i \leq n\}$ and $E(C_n) = \{x_i x_i + 1; 1 \leq i \leq n-1\} \cup \{x_1 x_n\}$. For every $v \in V(C_n)$ is labeled by 1 then $w^i(x_i) = l(x_{i-1}) + l(x_i) + l(x_{i+1}) = 3$ and $w^i(x_{i+1}) = l(x_i) + l(x_{i+1}) + l(x_{i+2}) = 3$. It is contradiction by definition, $w^i(x_i) \neq w^i(x_{i+1})$ such that $opt(l) = 2$. Based on Lemma 2.1 and Proposition 1.2, $\chi^i_{lis}(C_n) \geq \chi_{lis}(C_n) = 2$. To show the upper bound, we define $l : V(G) \to \{1, 2\}$ as follows:

$$lx_i) = \begin{cases} 1, & \text{for } i \text{ odd} \\ 2, & \text{for } i \text{ even} \end{cases},$$

so that $opt(l) = 2$ and weight function as follows:

$$w^i(x_i) = \begin{cases} 4, & \text{for } i \text{ even} \\ 5, & \text{for } i \text{ odd} \end{cases}.$$

We have $|w^i(V(C_n))| = 2$, hence $\chi^i_{lis}(C_n) = 3$. The proof is complete. $\square$

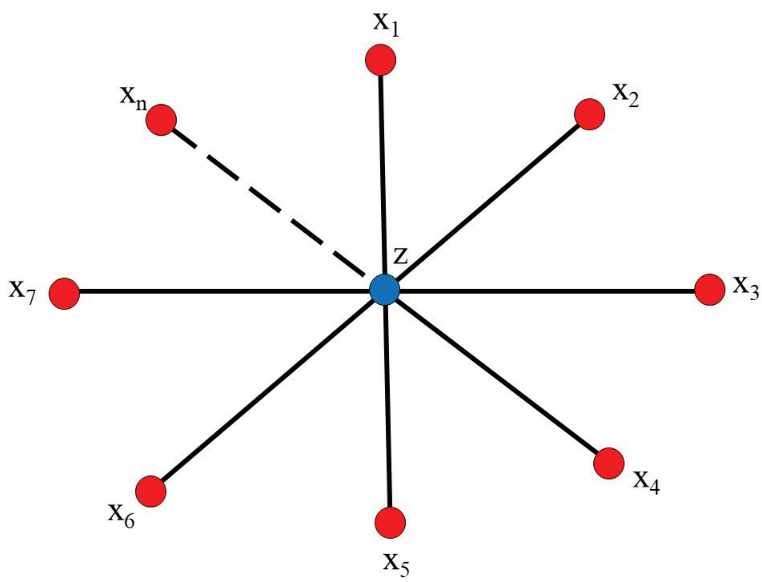


FIGURE 2. Inclusive Local Irregularity Vertex Coloring of Star Graph, $\chi^i_{lis}(S_n) = 2$

**Theorem 2.3.** *Let $S_n$ be a star graph for $n \geq 3$, $\chi^i_{lis}(S_n) = 2$.*

*Proof.* $V(S_n) = \{x, x_i, 1 \leq i \leq n\}$ where the vertex $x$ is the central vertex and $x_i$ is pendant vertex. If we give label $1$ in every vertices in $S_n$, then we get the vertex weight $w^i(x) = n + 1$ and $w^i(x_i) = 2$ such that $opt(l) = 1$. Based on Lemma 2.1 and Proposition 1.3 that the lower bound of $\chi^i_{lis}(S_n) \geq \chi_{lis}(S_n) = 2$. Furthermore, we prove that the upper bound of the chromatic number $\chi^i_{lis}(S_n) \leq 2$, we define $l : V(S_n) = \{1, 2, \ldots, n\}$ as follows:

$$l(x) = 1$$
$$l(x_i) = 1, \text{ for } 1 \leq i \leq n.$$

The weight function of inclusive local irregularity coloring on a star graph $S_n$ as follows:

$$w^i(x) = i + 1$$
$$w^i(x_i) = 2, \text{ for } i \leq i \leq n.$$

It easy to show $\chi^i_{lis}(S_n) \leq 2$. Hence, $\chi^i_{lis}(S_n) = 2$. This proof is complete. $\square$

Figure 2 is the illustration of inclusive local irregularity coloring of a star graph $S_n$.

## 3. Conclusion

We get the bounds of the local irregular chromatic number of graphs and determine the exact value of its concept of particular graphs namely path, cycle, and star

## Acknowledgment

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