ESTIMATION OF SHORTEST PATH USING DYNAMIC PROGRAMMING THROUGH NEUTROSOPHIC ENVIRONMENT

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ABSTRACT. This article explains a method to determine the shortest path for an acyclic network. Here the Dynamic Programming method is applied to find the shortest route. The edge weights of the acyclic network are involved in terms of the single valued Neutrosophic set. And besides the edge weights are considered in terms of Interval valued Neutrosophic set. In deneutrosophication score function formula is applied. Applying the proposed method, the shortest path is estimated.

1. INTRODUCTION

In 1995, Smarandache [2] evidences the subject of Neutrosophic sets(NS) The NS is to be a set of elements having a membership degree, indeterminate membership and also non-membership with the criterion less than or equal to 3. The Neutrosophic number is an remarkable type of Neutrosophic sets that cover the domain of numbers from those of actual numbers to Neutrosophic numbers. By oversimplifying SVNSS [1] Wang et al., Presented the idea of IVNS. The IVNS [3] is a more common database to oversimplify the notion of altered types of sets to define membership degrees truth, indeterminacy and a false degree in terms of intervals. Harish [4] suggested and analyzed an extension of the score function by combining hesistance. A dynamic programming method was

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originated by Richard Bellman in 1950 and has found solicitation in numerous regions. In dynamic programming the vertices are split into phases such that the edges are connecting vertices from one point to the next stage only. This paper consists of the following sections. Section 1 designates the Introduction about the proposed method. Section 2 defines the definition of Neutrosophic set, Single valued Neutrosophic set, Interval valued neutrosophic number and the score function of single valued Neutrosophic set and Interval valued Neutrosophic number. Section 3 explains the proposed method. Section 4 determines the shortest path for the Single valued neutrosophic network using the proposed method. Section 5 determines the shortest path for the Interval valued neutrosophic network using the proposed method. Section 5 concludes this paper.

2. Neutrosophic Preliminaries

Definition 2.1 (Neutrosophic set)**.** *Let η be a space points with generic element in η denoted by y. Then a Neutrosophic set β in η is characterized by a truth membership function T_β, an indeterminacy membership function I_β and a falsity membership function F_β. The functions T_β, I_β and F_β are real standard or non-standard subsets of $[^-0, 1^+]$ that is $T_\beta : \eta \to [^-0, 1^+]$; $I_\beta : \eta \to [^-0, 1^+]$; $F_\beta : \eta \to [^-0, 1^+]$.*

It should be noted that there is no restriction on the sum of $T_\beta(y), I_\beta(y), F_\beta(y)$. That is $0 \leq T_\beta(y) + I_\beta(y) + F_\beta(y) \leq 3$.

Definition 2.2 (single valued Neutrosophic set)**.** *Let η be a universal space points with generic element in η denoted by y. A single valued Neutrosophic set N is characterized by a truth membership function $T_N(y)$, an indeterminacy membership function $I_N(y)$ and a falsity membership function $F_N(y)$ with $T_N(y)$, $I_N(y)$, $F_N(y)$ $\in [0, 1]$ for all y in η.*

Definition 2.3 (single valued Neutrosophic score function (SVNSF))**.** *Single Valued Neutrosophic Score Function is defined as follows:*

$$SVNSF = \frac{2 + T_r - I_r - F_r}{3}$$

Definition 2.4 (Interval valued Neutrosophic number (IVNS))**.** *An IVNS in Y, which is represented by $\bar{C} = [< y : T_C(y), I_C(y), F_C(y) >, y \in Y]$*

$$\bar{C} = \{< y : [T_C^L(y), T_C^U(y)], [I_C^L(y), I_C^U(y)], [F_C^L(y), F_C^U(y)] >, y \in Y\}$$

where $[T_C^L(y), T_C^U(y)], [I_C^L(y), I_C^U(y)], [F_C^L(y), F_C^U(y)] \subseteq [0, 1].$

Definition 2.5 (Interval valued Neutrosophic score function (SVNSF)). *Interval valued Neutrosophic Score function is defined as follows:*

$$IVNSF = 2 + \frac{T_y^L + T_y^U}{2} - \frac{I_y^L + I_y^U}{2} - \frac{F_y^L + F_y^U}{2}$$

3. Procedure to find the shortest path for the acyclic network

Step 1: Consider the acyclic network. The edge weights are considered in terms of Single valued Neutrosophic set and Interval valued Neutrosophic number.

Step 2: Deneutrosophication has done by the score function of single valued Neutrosophic set and Interval valued Neutrosophic number.

Step 3: Here the vertices are divided into four stages such that the edges are connecting vertices from one stage to the next stage only. The first stage and the last stage will have the single vertex to represent the starting point and the ending point. There are various paths from starting point to ending point. We have to select the path which is giving the least cost.

Step 4: We have to start from the fourth stage. In the fourth stage there is only one end node. Take the cost of the end node as 0.

Step 5: Third stage consists of three vertices 6, 7, 5. Estimate the cost of these three vertices in the third stage.

Step 6: Second stage is also consists of three vertices 2,3,4. Calculate the cost of these three vertices in the third stage. Three edges are getting away from the 4th node. In this case, consider the minimum cost. The formula involved in this calculation is as follows

$$cost(i, j) = min[c(j, l) + cost(i + 1, l)],$$

where i denotes the stage, j denotes the vertex. l denotes another vertex.

Step 7: In the first stage there is only one vertex. Three edges are coming out from the node 1. Choose the minimum cost. Now we are going to take a decision. The decision is taken using the forward method starting from the first node. Here the principle of optimality is used. Principal of optimality says that the problem must be solved by a sequence of decisions. According to our decision taken the shortest path is estimated.

4. ESTIMATION OF SHORTEST PATH FOR THE SINGLE VALUED NEUTROSOPHIC NETWORK

Consider the Single valued Neutrosophic network where the edge weights are taken in terms of Single valued Neutrosophic set. The score function formula is used for deneutrosophication. Then using the proposed method the shortest path is determined.
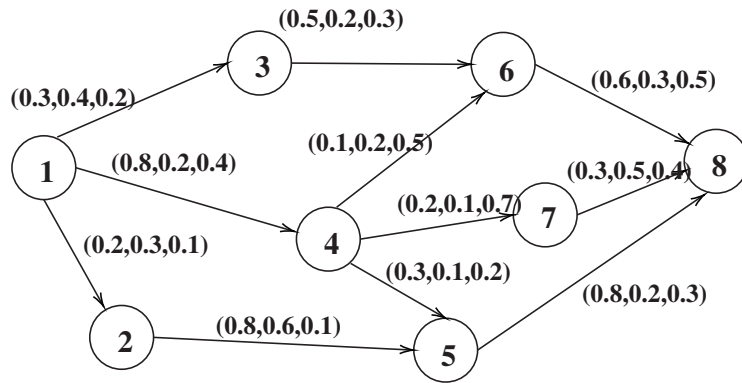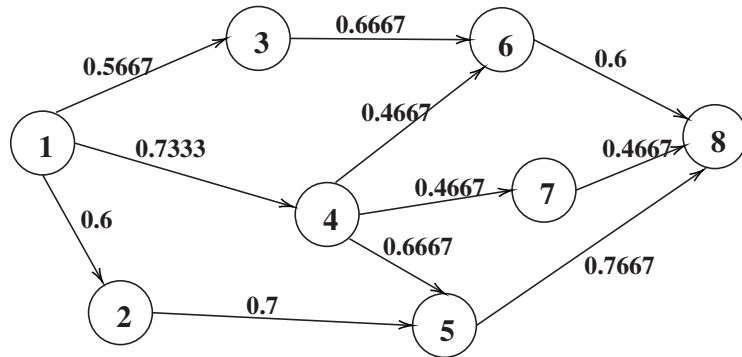

Fig.1 Single valued Neutrosophic Network

Fig.2 Single valued Neutrosophic Network after Deneutrosophication

TABLE 1. Cost of Vertices

Vertex	1	2	3	4	5	6	7	8
Cost	1.6667	1.4667	1.2667	0.9334	0.7667	0.6	0.4667	0
Decision	4	5	6	7	8	8	8	8

Stage 4: Cost (4,8) = 0
Stage 3: Cost (3,5) = 0.7667, Cost (3,7) = 0.4667 , Cost (3,6) = 0.6

Stage 2:

Cost (2,2) = c(2,5)+ Cost(3,5) =0.7+0.7667 =1.4667

Cost (2,3) = c(3,6) +Cost(3,6) = 0.6667+0.6 =1.2667

Cost (2,4) = min c(4,6) + cost (3,6), c(4,7) + cost (3,7), c(4,5)
+ cost (3,5)= min 0.4667+0.6, 0.4667+0.4667, 0.6667+0.7667
= min 1.0667, 0.9334, 1.4334 = 0.9334

Stage 1:

Cost (1,1) = min c(1,3) + cost (2,3), c(1,4) + cost (2,4), c(1,2) + cost(2,2)
= min 0.5667+1.2667, 0.7333+0.9334, 0.6+1.4667
= min 1.8334, 1.6667, 2.0667 = 1.6667

Decisions taken:

d(1,1) = 4;   d(2,4) = 7;   d(3,7) = 8.

From the decision taken it is conferred that the Shortest Path is $1 \rightarrow 4 \rightarrow 7 \rightarrow 8$.

5. ESTIMATION OF SHORTEST PATH FOR THE INTERVAL VALUED NEUTROSOPHIC NETWORK

Consider the Interval valued Neutrosophic network where the edge weights are taken in terms of Interval valued Neutrosophic set. The score function formula is used for deneutrosophication. Then using the proposed method the shortest path is determined
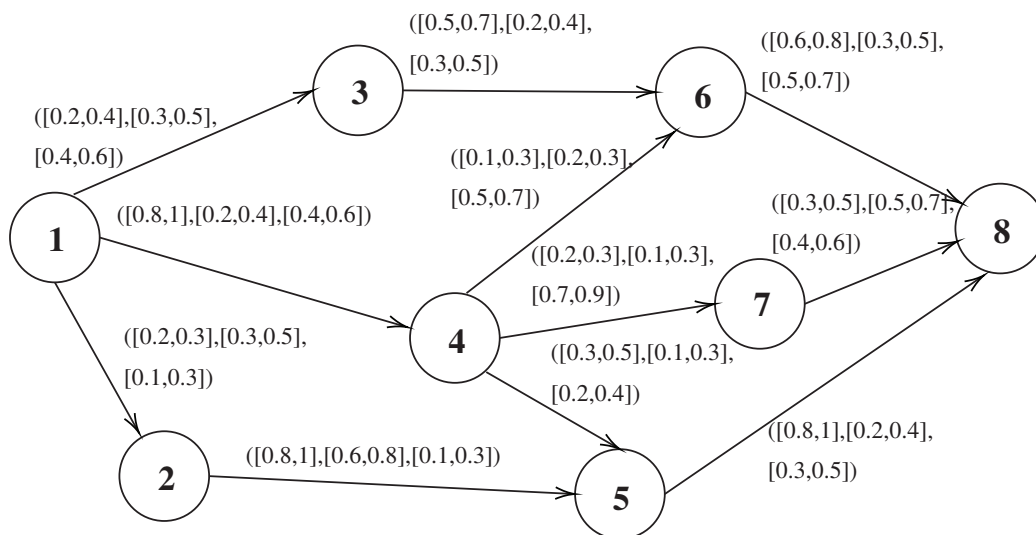

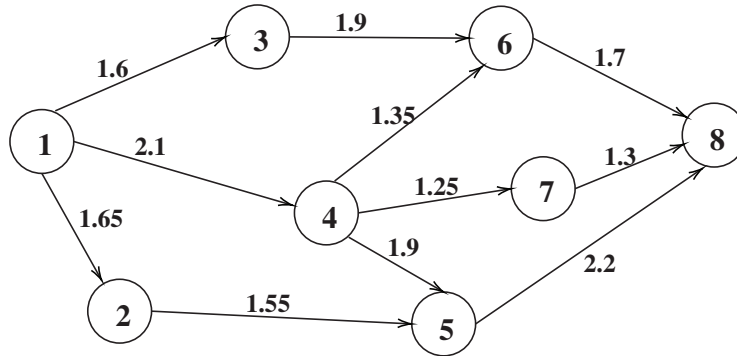Fig.3 interval valued Neutrosophic Network

Fig.4 interval valued Neutrosophic Network after Deneutrosophication

TABLE 2. Cost of Vertices

Vertex	1	2	3	4	5	6	7	8
Cost	4.65	3.75	3.6	2.55	2.2	1.7	1.3	0
Decision	4	5	6	7	8	8	8	8

Stage 4: cost (4, 8) = 0

Stage 3: cost (3, 5) = 2.2, cost (3, 7) = 1.3, cost (3, 6) = 1.7

Stage 2:

cost (2, 3) = c (3, 6)+cost(4,6) = 1.9+1.7=3.6

cost (2, 4) =minc(4,6)+cost(3,6), c(4,7)+ cost(3,7),c(4,5)+cost(3,5)

=min1.35+1.7,1.25+1.3, 1.9+2.2 = min 3.05, 2.55, 4.1= 2.55

Cost (2,2) = c (2,5)+cost(3,5) = 1.55+2.2 = 3.75

Stage 1:

cost (1,1) = minc(1,3) + cost (2,3), c(1,4) + cost(2,4),c(1,2)+cost(2,2)

=min1.6+3.6,2.1+2.55,1.65+3.75 =min5.2,4.65,5.4 = 4.65

Decisions taken:

d(1,1) = 4;   d(2,4) = 7;   d(3,7) = 8.

From the decisions taken it is concluded that the Shortest Path is $1 \rightarrow 4 \rightarrow 7 \rightarrow 8$.

6. CONCLUSION

The Shortest path is determined using dynamic programming through Single valued Neutrosophic set and Interval valued Neutrosophic number. It is conferred that the Neutrosophic Shortest path for the given acyclic network is $1 \rightarrow 4 \rightarrow 7 \rightarrow 8$. In both the method the Neutrosophic shortest path remains

same. This problem can be extended to other Neutrosophic networks with the edge weight as triangular and Trapezoidal Neutrosophic number.

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