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A NEW CLASS OF IDEAL BINARY TOPOLOGICAL SPACES

S. MEENA PRIYADARSHINI[1] AND V. KOKILAVANI

ABSTRACT. The notion of mildly α generalized (mαg) binary closed sets is introduced in ideal binary topological spaces. Characterizations and properties of mαIg closed sets are given.

1. INTRODUCTION

Nithyanantha jothi and Thangavelu [6] introduced the concept of binary topology and discussed some of its basic properties in 2011. In 2018 [10], a new notion of generalized binary closed sets in binary topological space was studied by Santhini and Dhivya. In 2018, Shymapada modak and Al.omari [2] introduced generalized closed sets in binary ideal topological spaces.

2. PRELIMINARIES

In this section, basics of binary topology and of binary ideal topological spaces are given. Throughout the paper mildly α generalized binary closed sets is denoted as mαg binary closed sets.

Definition 2.1 ([2]). *Let X and Y be two nonempty sets and let (A, B) $\in \wp$(X)$\times$ $\wp$(Y)and (C,D) $\in \wp$(X)$\times$ $\wp$(Y) respectively. Then:*

[1]*corresponding author*

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(i) *(A,B) $\subseteq$ (C,D) if and only if A $\subseteq$ C and B $\subseteq$ D.*
(ii) *A, B) = (C,D) if and only if A=C and B=D.*
(iii) *(A,B)$\cup$(C,D) = (H,K) if and only if (A$\cup$C) = H and (B$\cup$D) = K.*
(iv) *(A,B)$\cap$(C,D) = (H,K) if and only if (A$\cap$C) = H and (B$\cap$D) = K.*
(v) *(A,B)^c = (X $\backslash$A,Y $\backslash$B).*
(vi) *(A,B)$\backslash$(C,D) = (A,B)$\cap$(C,D)^c.*

Definition 2.2. [2] *Let X and Y be any two nonempty sets . A binary topology from X to Y is a binary structure M$\subseteq$ $\wp$(X)$\times$ $\wp$ (Y) that satisfies the axioms namely*

(i) *(ϕ,ϕ) and (X, Y) $\in$ M;*
(ii) *(A$\cap$B , C$\cap$ D) $\in$ M whenever (A, C) $\in$ M and (B, D) $\in$M;*
(iii) *If (Aα, Bα): $\alpha\in$ Δ is a family of members of M , then α, A , B $\in$ M.*

If M is a binary topology from X to Y then the triplet (X, Y, M) is called a binary topological space and the members of M are called the binary open subsets of the binary topological space (X,Y, M).

The elements of X $\times$Y are called the binary points of the binary topological space(X, Y, M).

If Y=X then M is called a binary topology on X in which case we write (X, M) as a binary topological space. The examples of binary topological spaces are given in [2].

Definition 2.3. *Let (X, Y,M) be a binary topological space. Let (A, B) $\subseteq$ (X, Y).Then(A, B) is called*

(i) ([10]) *binary semi open if there exists a binary open set (U, V) such that (U,V) $\subseteq$ (A, B)$\subseteq$b-cl(U, V).*
(ii) ([5]) *binary α closed if b-cl(b-int(b-cl(A, B))$\subseteq$ (A, B) and binary α open if b-int(b-cl(b-int(A, B)) $\subseteq$ (A, B).*
(iii) ([8]) *generalized binary closed set, if b-cl(A,B) $\subseteq$(U,V) whenever (A,B) $\subseteq$(U,V) and (U,V) is binary open in(X, Y, M).*
(iv) ([5]) *α generalized binary closed sets, if b- αcl(A,B) $\subseteq$(U,V) whenever (A,B) $\subseteq$(U,V) and (U,V) is binary open.*
(v) ([5]) *mildly α generalized(mαg) binary closed sets, if b-cl(b-int(A, B)) $\subseteq$(U,V) whenever (A,B) $\subseteq$(U,V) and (U,V) is αg binary open.*

Definition 2.4. [2] *Let X and Y be any two non empty sets. A binary ideal from X to Y is a binary structure I $\subseteq$ $\wp$(X)$\times$ $\wp$(Y) that satisfies the following axioms:*

(i) *(A,B) ∈I and (C,D) ⊆ (A,B) implies (C,D) ∈I (hereditary).*

(ii) *(A, B)∈ I and (C, D) ∈ I implies (A ∪ C, B ∪ D) ∈ I (finite additivity).*

Definition 2.5. [2] *Let (X,Y,M) be a binary topological space with an binary ideal I on ℘(X)× ℘(Y)is called ideal binary topological space and it is denoted as (X,Y,M,I).*

For a binary subset (A,B) of $X\times$ Y, we define the following set operator: (,)∗ : ℘(X)× ℘(Y)⟶ ℘(X)× ℘(Y), is called a binary local function with respect to M and I is defined as follows: for (A,B)⊆(X,Y),$(A,B)^*$(I,M) = (x,y) ⊆ (X,Y) \(U∩A,V∩B) ∉I for every (U,V)∈ M(x,y) where M(x,y) = (U,V)∈M: (x,y) ∈ (U,V).

Here (A,B) * (I,M) is briefly denoted by $(A,B)^*$ and is called Binary local function of (A,B) with respect to I and M. From [2] *we have C^*: ℘(X)× ℘(Y)⟶ ℘(X)× ℘(Y) is a Kuratowski closure operator.*

Therefore (U,V) ⊆ (X,Y) : C^* [(X,Y)\(U,V)] =(X,Y)\(U,V), forms a binary topology on $X\times Y$, and it is denoted as M^*.

Lemma 2.1. [2] *Let (X, Y, M, I) be an ideal binary topological space. Then β (M, I) = (V_1, V_2)\I: (V_1, V_2) is a binary open set of (X, Y, M, I) ∈I is a basis for M.*

Definition 2.6. [2] *Let (X, Y, M) be a binary topological space. Then the generalized kernel of (A,B) ⊆ (X,Y) is denoted by g-ker(A,B) and defined as g-ker(A,B) = ∩(U,V) ∈ M : (A,B) ⊆ (U,V).*

Definition 2.7. [2] *Let (X, Y, M) be a binary topological space and (A, B) ⊆ (X, Y). Then g-ker(A,B) = (x,y) ∈ X ×Y : b-Cl((x,y))∪(A,B) ≠(ϕ,ϕ).*

Definition 2.8. [2] *A subset (A,B) of an ideal binary topological space (X,Y,M,I) is called I_g-closed if $(A,B)^*$ ⊆ (U,V) whenever (U,V) is binary open and (A,B) ⊆ (U,V). A subset (A,B) of a binary ideal topological space (X,Y, M,I) is called I_g-open if (X,Y)\(A,B) is I_g closed.*

Definition 2.9. [2] *Let (X, Y, M, I) be an ideal binary topological space. Then the subset (A,B) of X × Y is said to be ∗-dense in itself if $(A,B)^*$ = (A,B).*

Lemma 2.2. [2] *Let (X, Y, M, I) be an ideal binary topological space and (A, B) ⊆ X ×Y. If (A, B) is ∗-dense in itself, then (A, B)* =b-Cl ((A, B) *) = b-Cl(A,B) = C^*(A,B).*

3. ${}_{m\alpha}\mathrm{I}_g$ BINARY CLOSED SETS

In this section we introduce ${}_{m\alpha}\mathrm{I}_g$ binary generalized closed sets and study some of their properties.

Definition 3.1. *A subset (A, B) of an ideal binary topological space (X, Y,M,I) is said to be ${}_{\alpha}I_g$ binary closed if $(A,B)^* \subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is binary α-open.*

Definition 3.2. *A subset (A,B) of an ideal binary topological space (X,Y, M,I) is called ${}_{m\alpha}I_g$-closed if $(A,B)^* \subseteq$ (U,V) whenever (U,V) is αg binary open and (A,B) $\subseteq$ (U,V). A subset (A,B) of a binary ideal topological space (X,Y, M,I) is called ${}_{m\alpha}I_g$-open if (X,Y)$\setminus$(A,B) is ${}_{m\alpha}I_g$ closed.*

Definition 3.3. *Let(X, Y,M) be a binary topological space and (A, B) $\subseteq$(X, Y). Then αg-ker(A,B) = (x,y) $\in X\times Y$: b-αgCl((x,y))$\cap$(A,B) $\neq$(ϕ,ϕ).*

Theorem 3.1. *If (X, Y, M,I) is any ideal binary topological space, then the following are equivalent*

- (i) *(A, B) is ${}_{m\alpha}I_g$ closed.*
- (ii) *C^*(A,B) $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is αg binary open in $X \times Y$.*
- (iii) *for all (x,y) $\in$ C^*(A,B), b-αgCl((x,y))$\cap$(A,B) $\neq$(ϕ,ϕ).*
- (iv) *C^* (A, B)$\setminus$(A,B) contain no nonempty αg binary -closed set.*
- (v) *$(A, B)^* \setminus$ (A, B) contains no nonempty αg binary-closed set.*

Proof.

(i)$\Rightarrow$(ii): If (A,B) is ${}_{m\alpha}\mathrm{I}_g$-closed, then (A,B) $^* \subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is αg binary open in X$\times$Y and so C * (A,B) =(A,B)$\cup$(A,B)* $\subseteq$ (U,V) whenever (A,B) $\subseteq$ (U,V) and (U,V) is αg binary open in X $\times$Y .

(ii)$\Rightarrow$(iii): Suppose (x,y) $\in$ C*(A,B) and (x,y)$\notin$ αg -ker(A,B). Then b-αgCl((x,y))$\cap$(A,B) = (ϕ,ϕ) (from definition 3.8) implies that (A,B) $\subseteq$(X,Y)$\setminus$b-αgCl((x,y))). By (ii), a contradiction, since (x,y) $\in$ C* (A,B).

(iii)$\Rightarrow$(iv): Suppose (G,H) $\subseteq$ C*(A,B) $\setminus$ (A,B), (G,H) is αg binary closed and (x,y) $\in$ (G,H). Since (G,H) $\subseteq$ C* (A,B)$\setminus$(A,B), (G,H) $\cap$(A,B) =(ϕ,ϕ). We have b-Cl((x,y)) $\cap$ (A,B) = (ϕ,ϕ) because (G,H) is αg binary closed and (x,y) $\in$ (G,H).It is a contradiction.

(iv)$\Rightarrow$(v): Since C * (A,B)$\backslash$ (A,B) =(A,B)$\cup$(A,B)$^*\backslash$ (A,B)= (A,B)$\cup$(A,B) * $\cap$ (A,B)^c =((A,B)$\cap$ (A,B)^c))$\cup$(A,B) $^*\cap$ (A,B)^c = (A,B) * $\cap$ (A,B)^c =(A,B)$^*\backslash$(A,B). Therefore (A, B) * $\backslash$ (A, B) contains no nonempty αg binary-closed set.

(v)$\Rightarrow$(i): Let (A,B) $\subseteq$ (U,V) where (U,V) be αg binary open subset containing (A,B). Therefore ((X,Y)$\backslash$(U,V)) $\subseteq$ ((X,Y)$\backslash$(A,B)) and so (A,B)$^*\cap$((X,Y)$\backslash$(U,V)) $\subseteq$(A,B)* $\cap$((X,Y)$\backslash$(A,B)) =(A,B) * $\backslash$(A,B). Therefore (A,B)$^*\cap$((X,Y)$\backslash$(U,V)) $\subseteq$ (A,B)* $\backslash$(A,B). Since (A, B)* is always αg binary closed set. (A, B)$^*\cap$((X,Y)$\backslash$(U,V)) is a αg binary closed set contained in (A,B)* $\backslash$(A,B). By assumption, (A, B) * $\cap$ ((X,Y) $\backslash$ (U,V)) = (ϕ,ϕ). Hence we have (A,B*) $\subseteq$ (U,V). Therefore (A, B) is $_{m\alpha}$I$_g$ closed. □

Theorem 3.2. *Let (X,Y, M,I) be an ideal binary topological space, for every (A,B) $\in$I,(A,B) is $_{m\alpha}I_g$ binary closed.*

Proof. Let (A,B) $\subseteq$(U,V) where (U,V) is αg binary open set.since (A,B)*= (ϕ,ϕ) for every (A,B) $\in$I,Then C*(A,B) =(A,B)$\cup$(A,B)* $\subseteq$(A,B) $\subseteq$ (U,V).Therefore by theorem 4.1, (A,B) is $_{m\alpha}$I$_g$ binary closed. □

Theorem 3.3. *Let (X,Y, M,I) be an ideal binary topological space, for every $_{m\alpha}I_g$ binary closed, αg binary open set isC *closed set.*

Proof. Since (A,B) is $_{m\alpha}$I$_g$ binary closed, αg binary open. Then (A,B)* $\subseteq$(A,B) whenever (A,B) $\subseteq$(A,B) and (A,B) is αg binary open. Hence (A,B) isC * closed set. □

Theorem 3.4. *Every I_g binary closed is $_{\alpha}I_g$ binary closed*

Proof. Let (A, B) $\subseteq$ (U,V) and (U,V) is binary α open. Clearly every binary open set is binary α open. Since (A,B) is I$_g$ binary closed set (A,B)* $\subseteq$ (U,V) which implies that (A,B) is an $_{\alpha}$I$_g$ binary closed set. The converse of the theorem need not be true as seen from the following example. □

Example 1. *Let X={0,1},Y={a,b,c}, M ={(ϕ, ϕ),({0},{a}),({1},{b}),(X,{a,b}), (X,Y)} is a binary topology from X to Y, let I={({0},{a}),({1},{b}),({1},ϕ), (ϕ,{a,c}),(ϕ,ϕ)} Clearly the set (A,B) =(X,ϕ) is $_{\alpha}I_g$s binary closed but not an I_g binary closed set.*

Theorem 3.5. *Every $_{\alpha}I_g$ binary closed is a $_{m\alpha}I_g$ binary closed.*

Proof. Let (A,B) $\subseteq$ (U,V) and (U,V) is mαg binary open. Clearly by [5] every αg binary open set is mαg binary open. Since (A,B) is ${}_{\alpha}$I$_g$ binary closed set (A,B)* $\subseteq$ (U,V) which implies that (A,B) is an ${}_{m\alpha}$I$_g$ binary closed set. The converse of the theorem need not be true as seen from the following example □

Example 2. *Let X={0,1},Y={a,b,c},M ={(ϕ, ϕ),({0},{a}),({1},{b}),(X,{a,b}), (X,Y)} is a binary topology from X to Y, let I={({0},{a}),({1},{b}),({1}, ϕ), (ϕ,{a,c}),(ϕ, ϕ)} Clearly the set (A,B)=(X,{a,b}) is ${}_{m\alpha}I_g$ binary closed but not an ${}_{\alpha}I_g$ binary closed set.*

Theorem 3.6. *Let (X, Y, M, I) be an ideal binary topological space and (A, B) $\subseteq$ X $\times$Y. If (A, B) is a ${}_{m\alpha}I_g$-closed set, then the following are equivalent*

(i) *(A, B) is a C *-closed set.*
(ii) *C * (A, B)$\setminus$ (A,B) is a αg binary closed set.*
(iii) *(A,B) * $\setminus$ (A,B) is a αg binary closed set.*

Proof.
(i)$\Rightarrow$(ii): If (A,B) is C *-closed, then(A,B) * $\subseteq$ (A,B).So C * (A,B)$\setminus$(A,B) =(A,B)$\cup$ (A,B)$^*\setminus$ (A,B)= (ϕ,ϕ) and so C * (A,B)$\setminus$(A,B) is αg binary closed.

(ii)$\Rightarrow$(iii): This follows from the fact that C * (A,B)$\setminus$(A,B) = (A,B)^{b*} $\setminus$(A,B) is αg binary closed

(iii)$\Rightarrow$(i): If (A,B) * $\setminus$(A,B) is αg binary closed closed and (A,B) is ${}_{m\alpha}$I$_g$ closed and αg binary open then (A,B) is C*-closed set.-closed, from Theorem 2.4[4], (A,B)* $\setminus$(A,B) = (ϕ,ϕ) and so (A,B) is C *-closed. □

Theorem 3.7. *Let (X,Y, M,I) be an ideal binary topological space. Then every subset of X$\times$Y is ${}_{m\alpha}I_g$ -closed if and only if every M - αg open set is C*-closed.*

Proof. Suppose every subset of X $\times$Y is ${}_{m\alpha}$I$_g$ -closed. If (U,V) is *M* - αg open, then (U,V) is ${}_{m\alpha}$I$_g$ -closed and so (U,V)* $\subseteq$ (U,V). Hence (U,V) is C *-closed. Conversely, suppose that every *M*- αg set is C *-closed. If (A,B) $\subseteq$X $\times$ Y and (U,V) is a *M* - αg set such that (A,B) $\subseteq$ (U,V), then(A,B) $^*\subseteq$ (U,V) $^*\subseteq$ (U,V) and so (A,B) is ${}_{m\alpha}$I$_g$ -closed. □

Lemma 3.1. *Let (X,Y,M,I) be an ideal binary topological spaceand (A,B) $\subseteq$ X $\times$Y . If (A,B) is *-dense in itself, then (A,B)* = b-αgCl ((A,B)*)= b-αgCl(A,B) = C*(A,B).*

Proof. Let (A,B) be*-dense in itself. Then we have (A,B) $\subseteq$ (A,B)* and hence b-αgCl(A,B) $\subseteq$ b-αgCl((A,B)*). We know that (A,B)* = b-αgCl((A,B)*) $\subseteq$ b-αgCl(A,B) from Theorem 2.4 [3] . In this case b-αgCl(A,B) = b-αgCl((A,B)*) = (A,B)*. Since (A,B)* = b-αgCl(A,B), we have C*(A,B) = b-αgCl(A,B) □

Theorem 3.8. *If (X,Y,M,I) is an ideal binary topological space and (A,B) is *-dense in itself, $_{m\alpha}I_g$-closed subset of $X{\times}Y$, then (A,B) is mαg binary closed*

Proof. Suppose (A,B) is a b*-dense in itself, mαIg-closed subset of X×Y.If (U,V) is any αg binary open set containing (A,B), then by Theorem 4.1 ,C *(A,B) $\subset$ (U,V). Since (A,B) is *-dense in itself, by above lemma 3.1 ,b-αgCl(A,B)$\subseteq$(U,V) and so (A,B) is mαg binary closed. □

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DEPARTMENT OF MATHEMATICS, KUMARAGURU COLLEGE OF TECHNOLOGY, COIMBATORE-641 049, TAMILNADU, IINDIA

Email address: meenapriyadarshini.s.sci@kct.ac.in

DEPARTMENT OF MATHEMATICS, KONGUNADU ARTS AND SCIENCE COLLEGE, COIMBATORE-641 049, TAMILNADU, IINDIA

Email address: vanikasc@yahoo.co.in