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BI-MAGIC LABELING OF GRAPHS RELATED TO WHEEL AND CYCLE

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ABSTRACT. Among the various labelings, bi-magic labeling is comparatively intersecting. The labeling of graphs and finding out its limit of gracefulness is a humongous task among researcher communities. In this article, bimagic labelings are obtained for wheel of odd length, wheel extension with one chord, removal one outer edge from double wheel, cell graph, and finite copies of triangles.

1. INTRODUCTION

Ahmad et.al. [2011] obtained results on super edge magic deficiency of unicyclic graphs. Baig et.al. [2014] got super edge-magic labeling of volvox and pancyclic graphs. BaskarBabujee and Jagadesh [2008] analyzed super edge bimagic labeling for some classes of connected graphs derived from fundamental graphs. Bacaet. al. [2006] destroyed super edge-antimagic labelings of the generalized Petersen graph $P(n, \frac{(n-1)}{2})$. Bacaet. al. [2007] studied edge-antimagic graphs, like path and cycle. Figueroa-Centenoet. al. [2005] investigated the edge-magic labelings of certain disjoint unions of paths, stars and cycles. Fukuchi [2000] identified a recursive theorem for super edge-magic labelings of trees. Fukuchi, and Oshima [2008] contributed super-edge-magic labeling of trees with large diameter. Ivanco and Semanicova [2007] constructed super-magic labelings for non-regular graphs.

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2. Basic Definitions

Definition 2.1. *A graph G with p vertices and q edges is magic if there is a bijection $f : V \cup E \to 1, 2 \ldots, p + q$ such that $f(u) + f(uv) + f(v) = k =$ constant for every edge uv in E.*

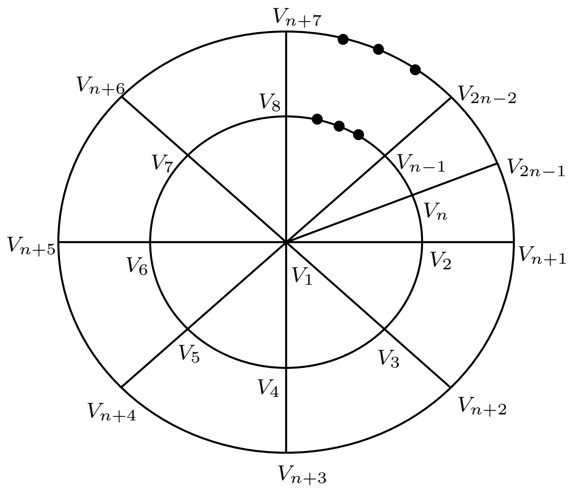
Definition 2.2. *A graph G with p vertices and q edges is edge bi-magic total if there exists a bijection $f : V \cup E \to 1, 2, \ldots, p + q$ such that there are two constants k_1 and k_2 such that $f(u) + f(v) + f(uv) = k_1$ or k_2 for any edge $uv \in E$[5, 6].*

Definition 2.3. *The graph wheel is defined as $W_n = K_1 + C_n$. Extension $W_n * nP_2$ of a wheel is defined by attaching one chord at each vertex of the circuit in the wheel W_n. Double wheel is obtained from extension of a wheel by joining all pendent vertices in term of a circuit. Thus double wheel is a graph whose vertex set is $\{V_1, V_2, \ldots, V_{2n-1}\}$, and edge set is $\{V_1V_i; i = 2to n\} \cup \{V_iV_{i+1}; i = 2to(n-1)\} \cup \{V_nV_2\} \cup \{V_iV_{n+i-1}; i = 2to n\} \cup \{V_iV_{i+1}; i = n+1, n+2, \ldots, (2n-2)\} \cup \{V_{2n-1}V_{n+1}\}$.*

3. Some classes of bi-magic graphs

Theorem 3.1. *Double wheel $2W_n$ is bi-magic graph if n is even.*

Proof. One of the arbitrary labelings for vertices is given as follows.

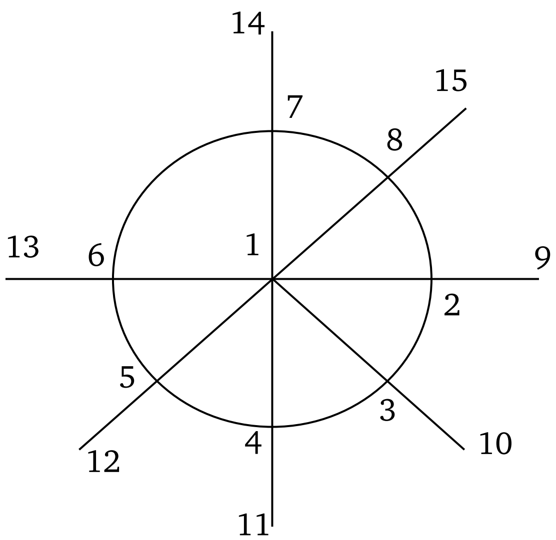


Define $f : V(G) \to \{1, 2, \ldots, p\}$ by $f(V_i) = i$, $i = 1$ to $(2n-1)$ and define $f : E(G) \to \{p+1, p+2, \ldots, p+q\}$ by $f(V_1V_i) = p + q - (i-2)$, $i = 2, 3 \ldots n$
$= 6n - 1 - i + 2, i = 2$ to n $\quad = 6n - i + 1, i = 1$ to n

$f(V_2V_n) = p + q - n + 1 = (6n - 1) - n + 1 = 5n.$
$f(V_2V_{n+1}) = p + q - n = 6n - 1 - n = 5n - 1.$
$f(V_iV_{i+1}) = (p + q - n + 1) - 2(i - 1), i = 2, 3, \ldots, (n - 1)$
$= 6n - 1 - n + 1 - 2i + 2, i = 2 \text{ to } (n - 1)$
$= 5n - 2i + 2, i = 2 \text{ to } (n - 1).$
$f(V_iV_{n+i-1}) = (p + q - n) - 2(i - 2), i = 3, 4 \ldots n$
$= 6n - 1 - n - 2i + 4, i = 3, 4 \ldots n$
$= 5n - 2i + 3, i = 3, 4 \ldots n$
$f(V_iV_{i+1}) = (p + q - 3n + 1) - 2(i - n - 1), i = n + 1, n + 2 \ldots (3n - 2)/2$
$= 6n - 1 - 3n + 1 - 2i + 2n + 2, i = n + 1, n + 2, \ldots, (3n - 2)/2$
$= 5n - 2i + 2, i = n + 1, n + 2, \ldots, (3n - 2)/2.$
$f(V_iV_{i+1}) = (p+q-3n+2)-2(i-[(3n+2)/2]-1),\ i = [(3n-2)/2]+1 \ldots (2n-2)$
$= 6n - 1 - 3n + 2 - 2(i - [(3n + 2)/2] - 1), i = [(3n - 2)/2] + 1 \ldots (2n - 2)$
$= 3n + 1 - 2(i - [(3n + 2)/2] - 1),\ i = [(3n - 2)/2] + 1 \ldots (2n - 2)$
$= 3n + 1 - 2i + 3n + 2 + 2,\ i = [(3n - 2)/2] + 1 \ldots (2n - 2)$
$= 6n - 2i + 5,\ i = [(3n - 2)/2] + 1 \ldots (2n - 2)$
$f(V_{2n-1}V_{2n}) = (p + q - 3n + 3) = 6n - 1 - 3n + 3 = 3n + 2.$

Magic constant $= C_1 = p+q+3 = 6n-1+3 = 6n+2$. Second magic constant $= C_2 = [C_1 - n] = [6n + 2 - n] = 5n + 2$. □

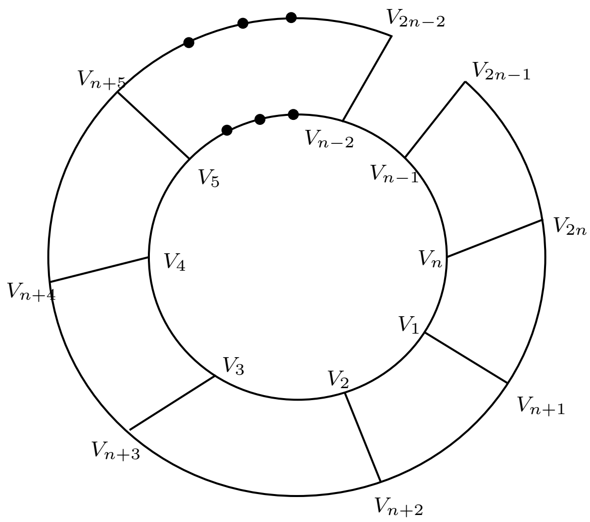
Example 1. *Double wheel $2W_7$ is bi-magic with magic numbers as 32 and 39.*



Definition 3.1. *Removal one edge form $2 * C_n$ is a graph whose vertex set is $\{V_1, V_2 \dots V_{2n}\}$ and edge set is $\{V_iV_{i+1}; i = 2 \dots (n-1)\} \cup \{V_nV_1\} \cup \{V_{n+i}V_{n+i+1}; i = 1, 2 \dots 2n-1\} \cup \{V_{2n-2}V_{2n-1}\} \cup \{V_iV_{n+i}; i = 1, 2 \dots n\}$.*

Theorem 3.2. *Removal one edge form $2 * C_n$ is bi-magic.*

Proof. One of the arbitrary labelings for vertices is mentioned below.



Define $f : V(G) \to 1, 2 \dots p$ by

$f(V_i) = i, i = 1 \dots n$

$f(V_{n+i)} = n + i + 1$, $i = 1$ to $(n-2)$

$f(V_{2n-1)} = 2n; f(V_{2n}) = n + 1$.

Define $f : E(G) \to p + 1, p + 2, \dots, p + q$ by

$f(V_{2n-1}V_{2n}) = 2n + 1; f(V_{2n}V_1) = 2n + 3;$

$f(V_{n+i}V_{n+i+1}) = (3n-1) - 2(i-1); i = 1$ to $[(n-1)/2]$

$= 3n - 2i + 1; i = 1$ to $[(n-1)/2]$.

$f(V_{n+i}V_{n+i+1}) = (5n-2) - 2(i-1); i = [(n+1)/2] \dots (2n-2)$

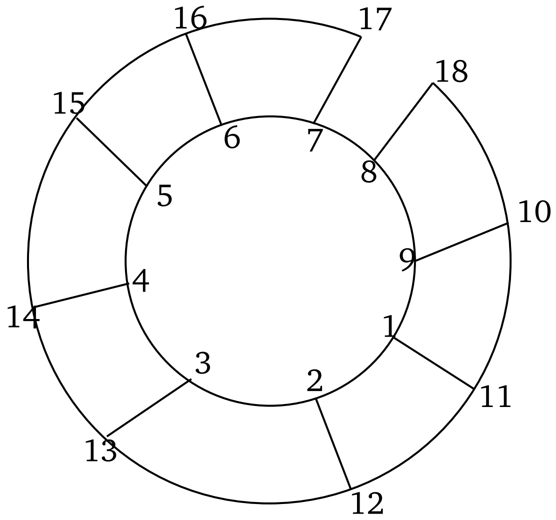
$= 5n - 2i; i = [(n+1)/2]$ to $(2n-2)$.

$f(V_iV_{i+1}) = 5n - 1; i = 1$ to $(n-1); f(V_nV_1) = 4n + 1;$

$f(V_iV_{n+i}) = (4n-1) - 2(i-1); i = 1to(n-1); f(V_nV_{2n}) = 3n + 1.$

Magic constant $= C_1 = p + q + 3 = 6n - 1 + 3 = 6n + 2$. Second magic constant $= C_2 = [C_1 - n] = [6n + 2 - n] = 5n + 2$. □

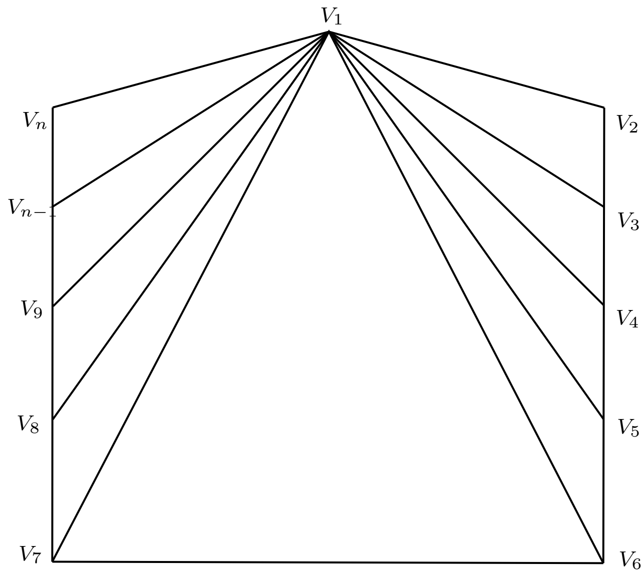
Example 2. $2 * C_{10}$ *is bi-magic with two magic numbers* 47 *and* 72.



Definition 3.2. *$C(n, n-3)$ is a graph whose vertex set is $\{V_1, V_2, \ldots, V_n\}$ and edge set is $\{V_iV_{i+1}; i = 1$ to $(n-1)\} \cup \{V_nV_1\} \cup \{V_1V_i; i = 3$ to$(n-1)\}$.*

Theorem 3.3. *$C(n, n-3)$ is bi-magic graph.*

Proof. One of the arbitrary labelings for the vertices of C(n, n-3) is denoted as follows:



Define $f : V(G) \to 1, 2 \ldots p$ by $f(V_i) = i, i = 1$ to n, and define $f : E(G) \to \{p+1, p+2 \ldots p+q\}$ by:
$f(V_1V_i) = (3n-3) - (i-2), i = 1$ to n

$= 3n - i - 1, i = 1$ to n

$f(V_iV_{i+1}) = (2n-2) - 2(i-2), i = 2$ to $(n/2)$ if n is even

$= 2n - 2i + 2, i = 2$ to $(n/2)$ if n is even

$= (2n-3) - 2(i-2), i = 2$ to $[(n-1)/2]$ if n is odd

$= 2n - 2i + 1, i = 2$ to $[(n-1)/2]$ if n is odd

$f(V_iV_{i+1}) = (2n-3) - 2[i - (n+2)/2], i = (n+2)/2$ to $(n-1)$ if n is even

$= 2n - 3 - 2i + n + 2, i = (n+2)/2$ to $(n-1)$ if n is even

$= 3n - 2i - 1, i = (n+2)/2$ to $(n-1)$ if n is even

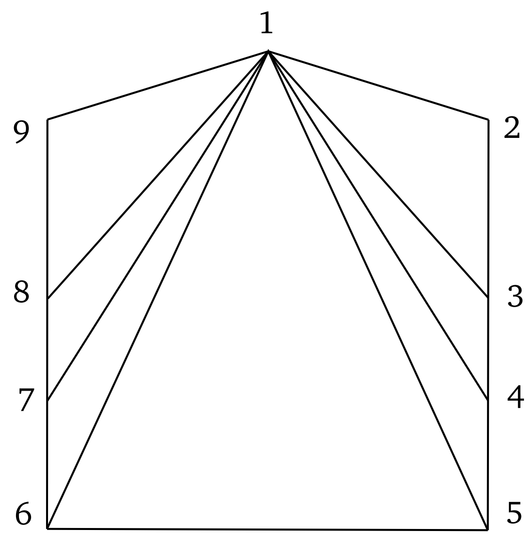
$f(V_iV_{i+1}) = (2n-2) - 2[i - (n+1)/2], i = (n+1)/2$ to $(n-1)$ if n is odd

$= 2n - 2 - 2i + n + 1, i = (n+1)/2$ to $(n-1)$ if n is odd

$= 3n - 2i - 1, i = (n+1)/2$ to $(n-1)$ if n is odd.

Magic constant = $C_1 = 3n$; Second magic constant = $C_2 = 2n+3$ if n is even $= 2n+2$ if n is odd. □

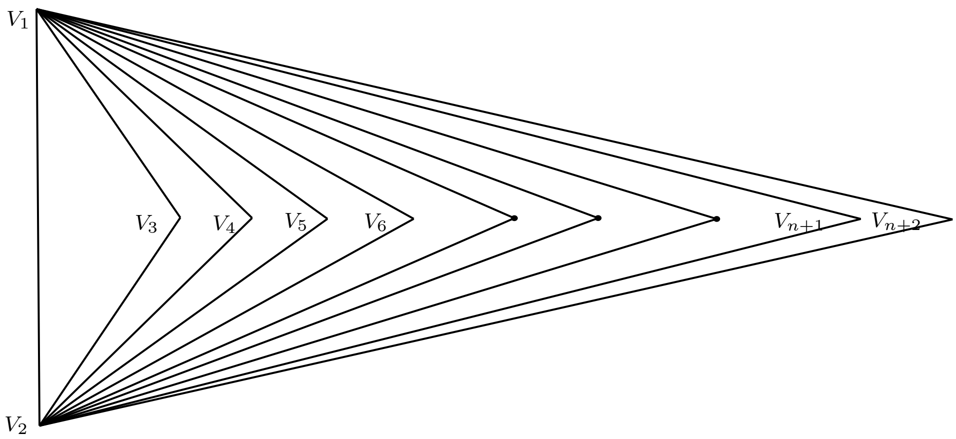
Example 3. *The shell graph $C(9,6)$ is bi-magic with magic numbers 29 and 20.*



Definition 3.3. *The graph $n*C_3$ is n copies of circuit of length 3 whose vertex set is $\{V_1, V_2, \ldots, V_{n+1}, V_{n+2}\}$ and so edge set is $\{V_1V_2\} \cup \{V_1V_i, i = 3 \ldots n+2\} \cup \{V_2V_i, i = 3 \ldots n+2\}$.*

Theorem 3.4. *n copies $(n * C_3)$ of triangles having a common edge is bimagic graph.*

Proof. One of arbitrary labelings for the vertices of $n * C_3$ is noted below.



Define $f : V(G) \to \{1, 2 \ldots p\}$ by $f(V_{n+2}) = 1; f(V_i) = i + 1, i = 1 \ldots n + 1$.
Define $f : E(G) \to \{p + 1, p + 2, \ldots, p + q\}$ by

$f(V_1V_{n+2}) = 3n + 3;$

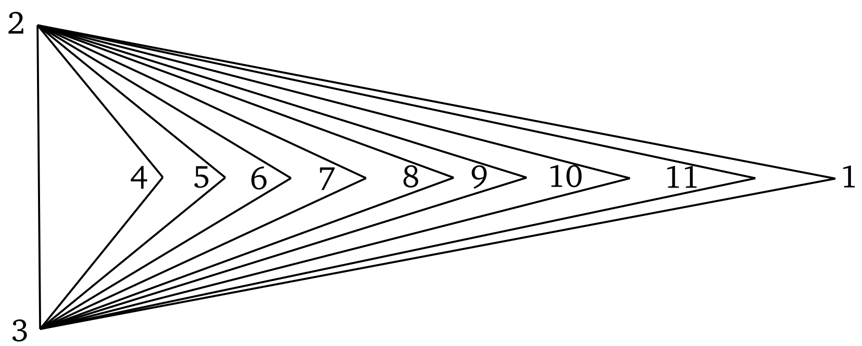
$f(V_2V_{n+2}) = 3n + 2; f(V_1V_2) = 3n + 1; f(V_1V_3) = 3n; f(V_2V_3) = 3n - 1;$

$f(V_1V_i) = 2n - (i - 4); i = 4, 5, \ldots, (n + 1).$

$f(V_1V_i) = (3n - 1) - (i - 3) = 3n - i + 2; i = 4, 5, \ldots, (n + 3).$

Magic constant = $C_1 = 3n + 6$; Second magic constant = $C_2 = 2n + 7$. □

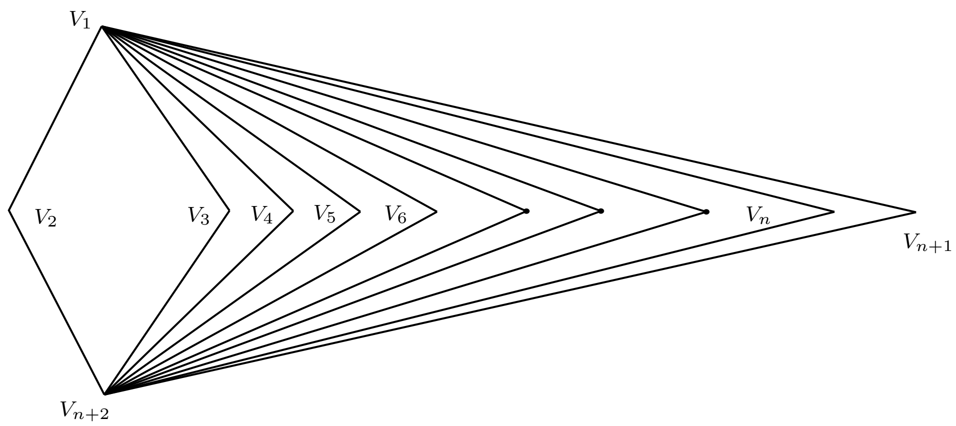
Example 4. *$9 * C_3$ is bi-magic with two magic numbers 33 and 25.*



Definition 3.4. *$n{*}C_4$ is n copies of squares having two common edges whose vertex set is a $\{V_1, V_2, \ldots, V_{n+1}, V_{n+2}\}$ and edge set is $\{V_1V_i; i = 2 \ldots (n+1)\} \cup \{V_{n+2}V_i; i = 2 \ldots (n + 1)\}$.*

Theorem 3.5. *n copies $(n * C_4)$ of squares having two common edges is bi-magic graph.*

Proof. One of the arbitrary labelings for the vertices of $n * C_4$ is given as follows.



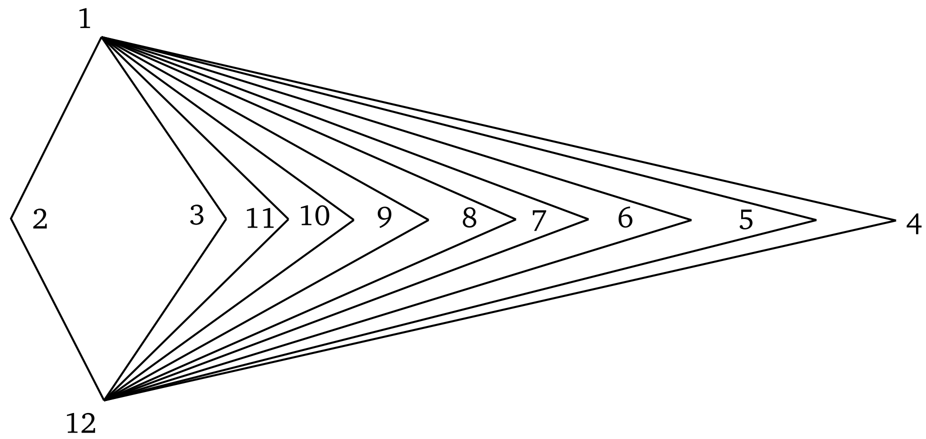
Define $f : V(G) \to \{1, 2 \ldots p\}$ by $f(V_i) = i, i = 1 \ldots n+2$, and define $f : E(G) \to p+1, p+2, \ldots, p+q$ by

$f(V_1V_i) = 3n + 2 - (i - 2); i = 2 \ldots (n + 1);$

$f(V_{n+1}V_{n+2}) = 2n + 2; f(V_{n+2}V_i) = (2n + 1) - (i - 2); i = 2 \ldots n.$

Magic constant $= C_1 = 3n + 5$; Second magic constant $= C_2 = 4n + 5$. □

Example 5. *The graph* $9 * C_4$ *is bi-magic with magic numbers* 35 *and* 45.



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