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BIMAGIC OF GRAPHS MERGING FROM PATH AND STAR

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ABSTRACT. The graphs one point union of a finite copies of P_n, $P_n * 2S_k$ of category-I, caterpillar graph of category of level-I, one point union of stars and switching each edge in a path are graceful with bi-magic labeling In this article, bi-magic labelings are obtained with suitable examples due to certain rules in the proofs of all results for one point union of paths of same size, finite copies of stars, two stars merging with pendent vertices of as path, caterpillar, roots of finite copies stars merging with all pendent vertices of a star, and switching each edge in a path.

1. INTRODUCTION

Choudum and Kishore [1999] found graceful labeling of the union of paths and cycles, Bhat-Nayak and Selvam [2003] got graceful labeling for n-cone $C_m \vee K_n$. Barrientos [2005] obtained the graceful labeling for unions of cycles and complete bipartite graphs. Guo [1994, 1995] investigated graceful labelings for bipartite graph $B(m,n)$ and $B(m,n,p)$. Liu [995] proved that the star graph with top sides is graceful. Seoud and Youssef [2000] showed that some classes of families in terms of disconnected from paths and cycles are graceful. Xu et.al. [2008] verified that the graphs $C_{13^{(t)}}$ are graceful where $t \equiv 0, 1 \pmod 4$.

The labeling of graphs found an increasingly useful in a variety of scientific applications. Gallian dynamic survey of graph labeling provided the focus on

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variety of graph labeling. The graph labeling is magic if it obey the condition $G(V, E)$ with p vertices and q edges is a bijection f from the set of vertices and edges to such that for every edge uv in E, $f(u) + f(uv) + f(v)$ is a constant k. Furthermore the graph is bi-magic if it possess two constants k_1 and k_2 with $f(u) + f(v) + f(uv) = k_1$ or k_2 [Harary, 1988].

In our previous work we reported the super magic and arithmetic labelings of the graphs, $P_3 \times P_n$ and $C_3 \times C_n$. Bi-magic labelings of the graphs, $W_n * S_k, W_n * 2S_k$and $W_n * nP_2$ were reported. In this article, the n copies of stars graph was graceful with magic labeling and the graphs one point union of a finite copies of $P_n, P_n * 2S_k$ of category-I, caterpillar graph of category of level-I, one point union of stars and switching each edge in a path were graceful with bi-magic labeling defined by two constants k_1 and k_2.

2. Basic definitions

Definition 2.1. *A graph G with p vertices and q edges is magic if there is a bijection $f : V \cup E \to \{1, 2 \ldots, p + q\}$ such that $f(u) + f(uv) + f(v) = k =$ constant for every edge uv in E.*

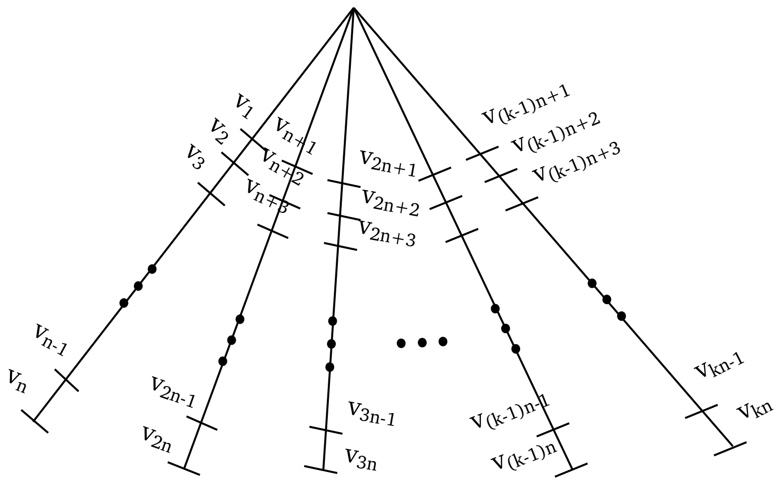
Definition 2.2. *A graph G with p vertices and q edges is edge bi-magic total if there exists a bijection $f : V \cup E \to \{1, 2, \ldots, p+q\}$ such that there are two constants k_1 and k_2 such that $f(u) + f(v) + f(uv) = k_1$ or k_2 for any edge $uv \in E$ [5, 6].*

Definition 2.3. *One point union of a finite copies of a path P_n is a graph whose vertex set is $\{V_1, V_2 \ldots V_{K_n}\}$ and edge set is $\{V_iV_{i+1}; i = 1$ to $k_n; i \neq 0(\mod n)\}$ $\cup \{V_0V_i; i = 1, 2, \ldots, n+1, (2n+1), \ldots (k-1)n+1\}$.*

3. Some bi-magic labelings of graphs

Theorem 3.1. *One point union of a finite copies of a path P_n is bi-magic.*

Proof. One of the arbitrary labelings for vertices is given below.



Define $f : V(G) \to \{1, 2, \ldots, p\}$ by $f(V_0) = 1$; and let for $i = 1$ to n,

$f(V_i) = 2 + k(i-1); f(V_{n+i}) = 3 + k(i-1)$;

$f(V_{2n+i}) = 4 + k(i-1); f(V_{(k-1)n+i}) = (k+1) + k(i-1)$.

Let $j = 1, 2, \ldots, k$. Define $f : E(G) \to \{p+1, p+2, \ldots, p+q\}$ by

$f(V_{(j-1)n+1}V_{(j-1)n+2}) = k(2n-1) - 2(j-1)$;

$f(V_{(j-1)n+2}V_{(j-1)n+3}) = k(2n-1) - 2(k-1) - 2(j-1); f(V_{(j-1)n+3}V_{(j-1)n+4}) = k(2n-1) - 4(k-1) - 2(j-1)$.

Case (1): $n = k$.

Suppose n is odd, k is even; $j = 1, 2, \ldots, k$.

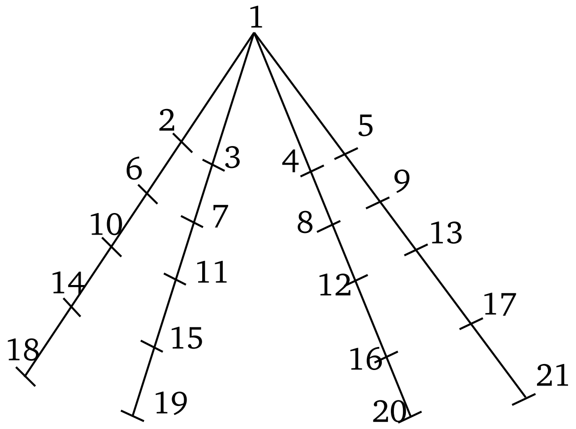
$f(V_{(j-1)n+(n/2+1)}V_{(j-1)n+(n/2+2)})$ [n is even]$= (2kn + 1 - k) - 2(j-1)$;

$f(V_{(j-1)n+(n+1)/2}V_{(j-1)n+[(n+1)/2]+j})$ [n is odd] $= (2kn + 1 - k) - 2(j-1)$;

Case (2). $n \neq k$.

Suppose n is odd and k is even. Magic constant $= n(2k+1) + 1$; Second magic constant $= n$. □

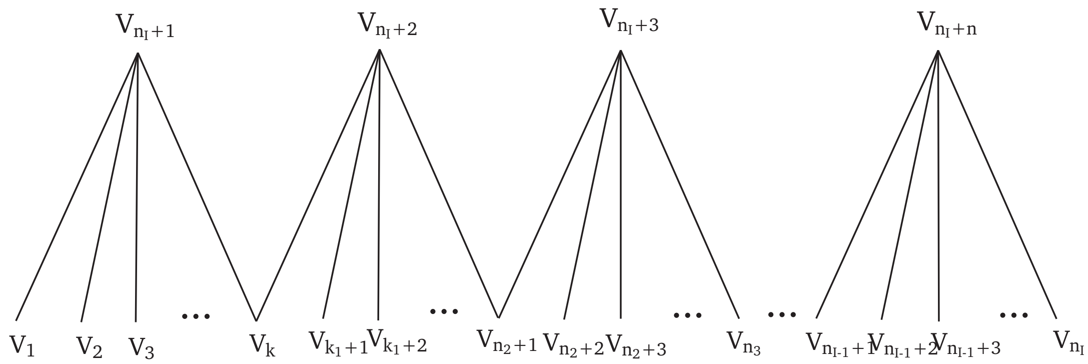
Example 1. *One point of four copies of P_6 is bimagic with magic constants* 44 *and* 61.



Definition 3.1. *n copies of stars is defined a finite of stars with different sizes whose vertex set is $\{V_1, V_2, \dots, V_{k_1}, V_{(k_1+1)}, \dots, V_{(n_2+1)}, V_{(n_2+2)}, \dots, V_{(n_3)}, \dots, V_{(n_{(l-1)}+1)}, \dots, V_{(n_l)}, V_{(n_l+1)}, \dots, V_{(n_l+n)}\}$ and edge set $= \{V_{n_l+1}V_i; i = 1$ to $k_l\} \cup \{V_{n_l+2}V_{k_1+i}; i = 1$ to $k_2\} \cup \{V_{n_l} + 3V_{n_2} + i; i = 1$ to $k_3\} \cup \cdots \cup \{V_{n_l+n}V_{n_{(l-1)}+i}; i = 1 \dots n_l\}$ where, $n_1 = k_1; n_2 = k_1 + k_2; n_3 = k_1 + k_2 + k_3; n_l = k_1 + k_2 + \cdots + k_l$.*

Theorem 3.2. *n copies $S_{k_1} * S_{k_2} * \cdots * S_{k_l}$ of stars are magic.*

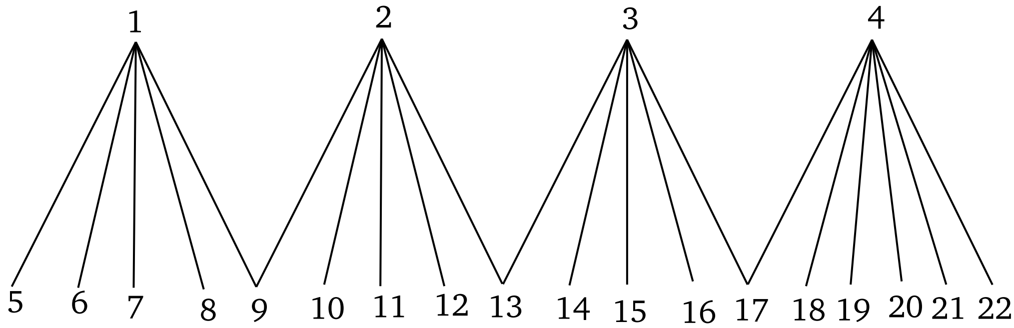
Proof. One of the arbitrary labelings for vertices of $S_{k_1} * S_{k_2} * \cdots * S_{k_l}$ is given below.



Define $f : V(G) \to \{1, 2, \dots, p\}$ by $f(V_{n_{l+i}}) = i; i = 1$ to n; $f(V_i) = n + i; i = 1 \dots n_l$, and define $f : E(G) \to \{p+1, p+2, \dots, p+q\}$ by $f(V_{n_l+1}V_i) = (2n + 2n - 1) + (i - 1); i = 1, 2, \dots, n_l$.

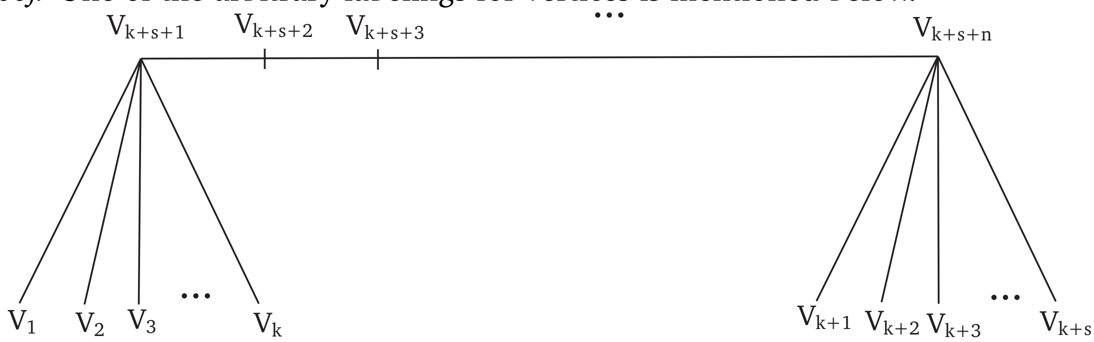
Magic constant $= (2n_l + 2n - 1) + n = 2n_l + 3n + 1$. □

Example 2. *$Sk_5 * Sk_5 * Sk_5 * Sk_6$ is magic with magic number 49.*



Theorem 3.3. *$P_n * 2S_k$ of category I is bi-magic.*

Proof. One of the arbitrary labelings for vertices is mentioned below.



Define $f : V(G) \to \{1, 2, \ldots, p\}$ by

$f(V_{k+s+i}) = i, i = 1$ to n; $f(V_i) = n + i, i = 1$ to k;

$f(V_{k+i}) = n + k + i, i = 1$ to s.

Also, define $f : E(G) \to \{p + 1, p + 2, \ldots, p + q\}$ by

$f(V_{k+s+n}V_{k+s+1-i}) = (k + s + n) + i; i = 1$ to s;

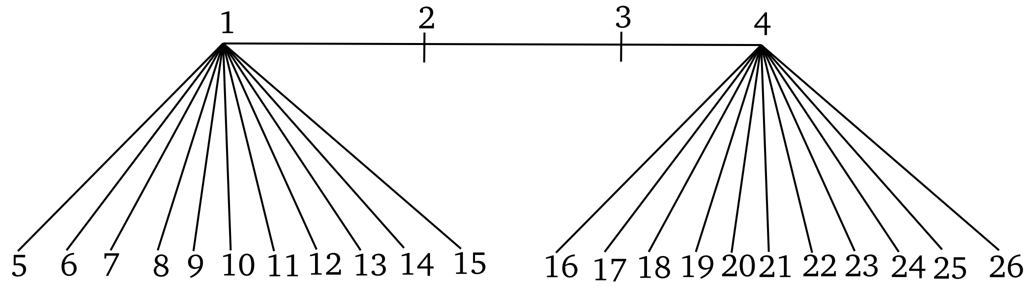
$f(V_{k+s+1}V_{k+1-i}) = k + 2s + n + i; i = 1$ to k.

$f(V_{k+s+i}) = 2(k + s + n) + 1 - 2i; i = 1$ to $n/2$ or $(n + 1)/2$.

$$f(V_{k+s+i}) = \begin{cases} 2(k+s+n) - 2 - 2[i - (n/2)], & i = (n/2) \text{ to } n \text{ if } n \text{ is even} \\ 2(k+s+n) - 2 - 2[i - (n+3/2)], & i = (n+3/2) \text{ to } n \text{ if } n \text{ is odd} \end{cases}$$

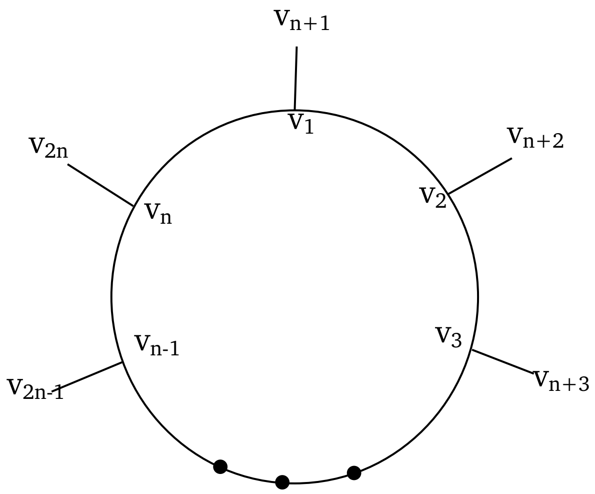
Magic constant = $2(k + s + n + 1)$; Second magic constant = $k + s$, □

Example 3. *$P_4 * 2S_{11}$ of category I is bimagic with magic numbers 54 and 57.*



Definition 3.2. *Caterpillar graph of category of level I is a graph whose vertex set is* $\{V_1, V_2 \ldots V_{2n}\}$ *and edge set* $= \{V_iV_{i+1}; i = 1 \text{ to } n-1\} \cup \{V_iV_{n+i}; i = 1, 2, \ldots n\}$.

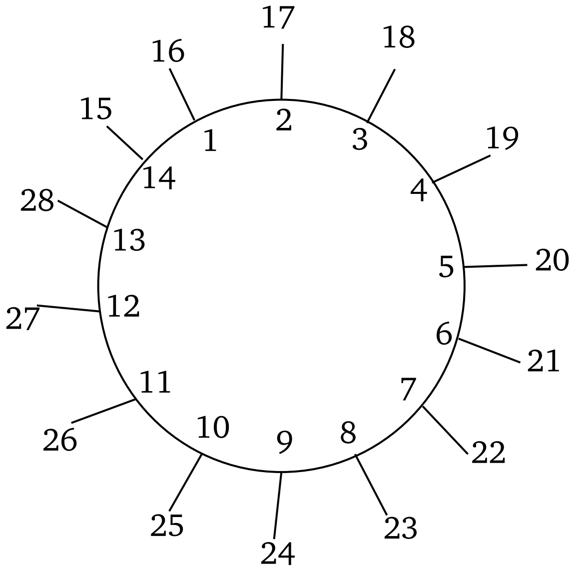
Theorem 3.4. *Caterpillar graph* C_n *with chords of category of level I is bimagic.*



Proof.
Define $f : V(G) \to \{1, 2 \ldots p\}$ by $f(V_i) = i, i = 1$ to n; $f(V_{n+i}) = n+i+1, i = 1$ to $(n-1)$; $f(V_{2n}) = n+1$, and define $f : E(G) \to \{p+1, p+2, \ldots, p+q\}$ by $f(V_iV_{i+1}) = 4n - 2i + 2; i = 1$ to $n-2$; $f(V_{n-1}V_n) = 4n-1$; $f(V_iV_{n+i}) = (4n-1) - 2i; i = 1$ to $(n-1)$; $f(V_nV_{2n}) = 2n+2$.

Magic constant $= 4n+3$; Second magic constant $= 5n$. □

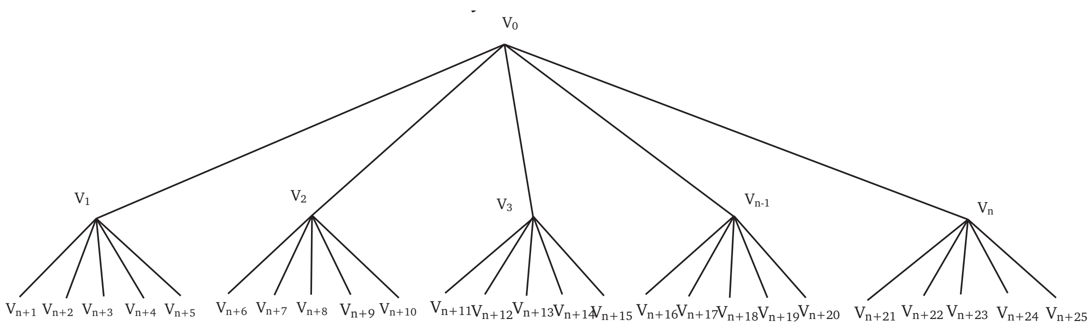
Example 4. *Caterpillar graph* C_{14} *with chords of category of level I is bi-magic with magic numbers* 59 *and* 70.



Definition 3.3. *Merging of stars with a star is a graph whose vertex set* $\{V_0, V_1 \dots V_n\}$ *and edge set is* $\{V_0V_i; i = 1 \dots n\} \cup \{V_jV_{n+i}; i = 1 \dots j; j = 1 \dots n\}$.

Theorem 3.5. *Merging of different size of stars* $Sk_1 * Sk_2 \cdots * Sk_l$ *with a star is bi-magic.*

Proof.



Define $f : V(G) \to \{1, 2 \dots p\}$ by

$f(V_0) = 6n + 1; f(V_i) = 1 + 6(i - 1), i = 1 \dots n;$

$f(V_{n+i}) = i + 1 + k, i = 5k + j, 0 \le j \le 4, i = n + 1$ to $5n$.

Define $f : E(G) \to \{p + 1, p + 2, \dots, p + q\}$ by

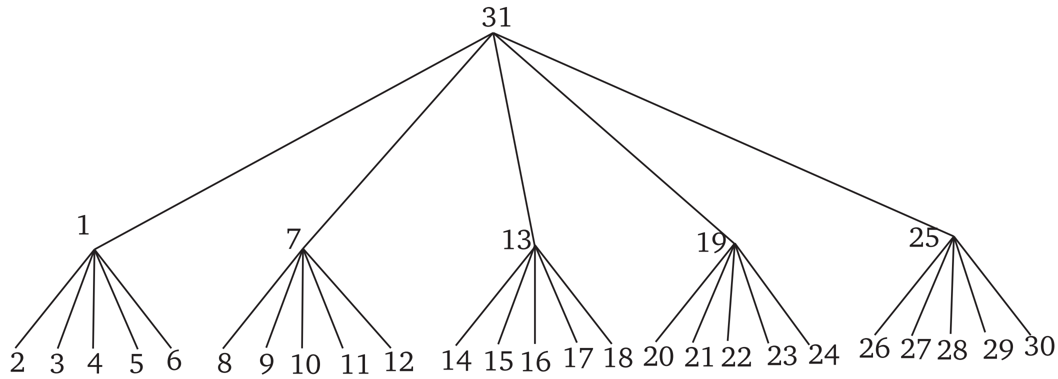
$f(V_0V_1) = 6n + 2; f(V_0V_{n+1-i}) = 6n + 2 + 6i, i = 1$ to $(n - 1)$.

$f(V_jV_{n+i}) = (12n + 2) + (j - 1)(-2n) + 2j - i, j = 1$ to $(n + 1)/2; i = 1$ to 5.

$$f(V_jV_{n+i}) = (11n+2) + (j-(n+1)/2)(-2n) + 2(j-(n+1)/2),$$
$j = (n+3)/2, (n+5)/2, \dots n; i = 1$ to 5.

Magic first constant $= 12n+4$; Second magic constant $= C_2 = 18n+4$. $\square$

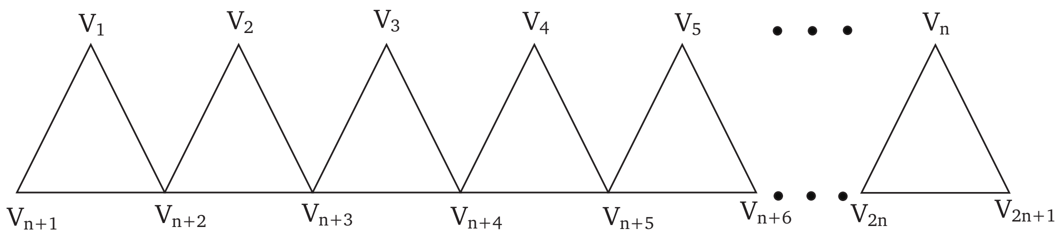
Example 5. *Merging of different size of stars $S_5 * S_5 * S_5 * S_5$ with a star S_5 is bimagic with magic numbers 59 and 70.*



Definition 3.4. *Switching each edge in a path is a graph whose vertex set is $\{V_1, V_2 \dots V_{2n+1}\}$ and edge set is Edge set $= \{V_iV_{n+i}; i = 1 \dots n\} \cup \{V_iV_{n+1+i}; i = 1 \dots n\} \cup \{V_{n+i}V_{n+1+i}; i = 1 \dots 2n\}$.*

Theorem 3.6. *Switching each edge in a path is bimagic.*

Proof. One of arbitrary labelings for vertices of given graph is as follows.



Define $f : V(G) \to \{1, 2, \dots, p\}$ by $f(V_i) = i, i = 1$ to $2n+1$, and define $f : E(G) \to \{p+1, p+2, \dots, p+q\}$ by $f(V_iV_{n+i}) = 6n+3-2i, i = 1$ to n; $f(V_iV_{n+1+i}) = 6n+2-2i, i = 1 \dots n$,

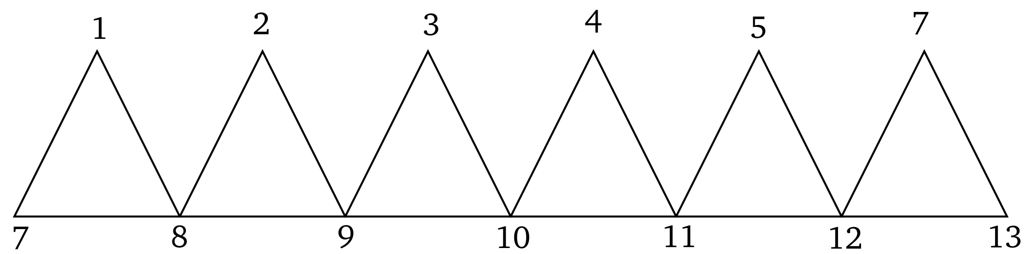
$$f(V_iV_{i+1}) = \begin{cases} 3n+2-2i, & i = 1 \text{ to } n/2 \text{ if } n \text{ is even} \\ 3n+2-2i, & i = 1 \text{ to } (n-1)/2 \text{ if } n \text{ is odd} \end{cases}$$

and

$$f(V_jV_{n+i}) = \begin{cases} 3n+3-2[i-(n+2)/2], & i=(n+2)/2 \text{ to } n \text{ if } n \text{ is even} \\ 3n+3-2[i-(n-1)/2], & i=(n+1)/2 \text{ to } n \text{ if } n \text{ is odd.} \end{cases}$$

Magic constant $= 6n+3$; Second magic constant $= 7n+3$. □

Example 6. *Switching each edge in a path P_7 is bimagic with magic numbers* 59 *and* 70.



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