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SIGNED UNIDOMINATION OF SOME STANDARD GRAPHS

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ABSTRACT. Let $G(V, E)$ a graph. A function $f : V \to \{-1, 1\}$ is called a signed unidominating function (SUDF) of G if $\sum_{u\in N[v]} f(u) = 1$,if $f(v) = -1$ and ≥ 1 if $f(v) = 1$ for each $v \in V$. The weight of a signed unidominating function f is defined as $(f(V) = \sum_{v\in V} f(v))$ and signed unidomination number is the minimum weight of a SUDF of G and it is denoted by $\gamma_{su}(G)$. In this paper signed unidomination number of some standard graphs are discussed.

1. INTRODUCTION

Domination theory is one in the developing area of Graph Theory and expanded collections of results in this area are extensively studied by Hayes at el.[1,2] in 1998. The concept of signed dominating function of a graph was introduced by Dunbar et al [3] and there is a variety of potential applications for this domination. By assigning the values -1 or +1 to the vertices of a graph we can model such things as networks of positive and negative electrical charges, networks of positive and negative spins of electrons, and networks of people or organizations in which global decisions must be made. Let G (V ,E) a graph. A function $f : V \to \{-1, 1\}$ is called a signed dominating function (SDF) of G if $f(N[v]) = \sum_{u\in N[v]} f(u) \geq 1$ for each $v \in V$. The weight of a signed dominating function G is defined as $W(f) = \sum_{v\in V} f(v)$. The signed domination number of

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G is the minimum weight of a SDF of G and it is denoted by $\gamma_s(G)$.

The Unidominating function was defined and studied by Anantha Lakshmi [4] in 2015 and results on Unidomination number and upper unidomination of some standard graphs are discussed. Let $G(V, E)$ a graph. A function $f : V \to \{0, 1\}$ is called a unidominating function (UDF) of G if $\sum_{u \in N[v]} f(u) = 1$ if $f(v) = 0$ and ≥ 1 if $f(v)= 1$, for each $v \in V$. The unidomination number is defined as $\min\{f(V) : f$ is unidominating function of $G\}$, Where $f(V) = \sum_{u \in V} f(u)$ is the weight of the unidominating function and it is denoted by $\gamma_u(G)$.

In this paper signed unidominating functions and signed unidomination number of a some standard graphs are discussed.

2. Signed Unidominating Functions and Signed Unidomination Number of a Graph

Let $G(V, E)$ be a graph. A function $f : V \to \{-1, 1\}$ called a signed unidominating function (SUDF) of G if

$$\text{for each } v \in V, \sum_{u \in N[v]} f(u) = 1, \text{ if } f(v) = -1$$

$$\text{and } \sum_{u \in N[v]} f(u) \geq 1, \text{ if } f(v) = 1.$$

The signed unidomination number of a graph $G(V, E)$ is the minimum weight of a signed unidominating function of G where the weight of f is $f(V) = \sum_{u \in V} f(u)$. In this section signed unidominating functions of some standard graphs are discussed and the results on signed unidomination number of path, cycle and complete graph are obtained.

Theorem 2.1. *The Signed Unidomination number of a path P_n is*

$$\gamma_{su}(P_n) = \begin{cases} \dfrac{n+6}{3}, & \text{if } n \equiv 0 \text{ (mod 3)} \\ \dfrac{n+8}{3}, & \text{if } n \equiv 1 \text{ (mod 3)} \\ \dfrac{n+4}{3}, & \text{if } n \equiv 2 \text{ (mod 3)} \end{cases}$$

Proof. Consider a path P_n with vertex set V $=\{v_1, v_2, \ldots, v_n\}$. Define a function $f : V \to \{-1, 1\}$ by

$$f(v_i) = \begin{cases} 1, \text{ if } i \equiv 1, 2 \text{ (mod 3)} \\ -1, \text{ if } i \equiv 0 \text{ (mod 3)} \end{cases}.$$

There are three possible cases arise in the process of proving f is a Signed Unidominating function of P_n.

Case 1: Let $n \equiv 0$ (mod 3).

Sub case 1: For vertex $v_i \in P_n$ with i $\equiv 0$ (mod 3).

If $i \neq n$, then $f(v_i)$ =-1 and the closed nbd of v_i, contains 3 vertices of P_n. So $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1 - 1 + 1 = 1$. If $i = n$, then $f(v_i) = 1$ and the closed nbd of v_i contains 2 vertices of P_n. So $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) = 1 + 1 = 2$.

Sub case 2: For vertex $v_i \in P_n$ with $i \equiv 1$ (mod 3) , we have $f(v_i) = 1$. If $i \neq 1$ then the closed nbd of v_i contains 3 vertices of P_n and $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1 - 1 + 1 = 1$. and if $i = 1$, $\sum\limits_{u \in N[v_i]} f(u) = f(v_i) + f(v_{i+1}) = 2$.

Sub case 3: For vertex $v_i \in P_n$ with $i \equiv 2$ (mod 3), we have $f(v_i) = 1$. For $i \neq n-1$, $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1 + 1 - 1 = 1$ and for $i = n-1$, $\sum\limits_{u \in N[v_{n-1}]} f(u) = f(v_{n-2}) + f(v_{n-1}) + f(v_n) = 3$.

From the above all cases, f is a signed unidominating function of P_n with $n \equiv 0$ (mod 3). Therefore

$$\begin{aligned} f(V) = &\sum_{u \in V} f(u) = \sum_{i=1}^{n-3} f(v_i) + f(v_{n-2}) + f(v_{n-1}) + f(v_n) \\ = &\frac{(1+1-1) + (1+1-1) + \cdots + (1+1-1)}{(n-3) - terms} + 1 + 1 + 1 = \frac{n+6}{3}. \end{aligned}$$

By the definition of signed unidomination number

$$\gamma_{su}(P_n) \leq \frac{n+6}{3}. \tag{2.1}$$

We know that degree of each vertex of path is 1 or 2. If f is a signed unidominating function of P_n then we can see that amongst three consecutive vertices in P_n at least one vertex must have functional value -1 and at most two vertices can have functional value 1. Therefore the sum of the functional values of three consecutive vertices is greater than or equal to 1. That means

$$\sum_{i=1}^{3} f(v_i) \geq 1, \quad \sum_{i=4}^{6} f(v_i) \geq 1, \quad \ldots, \quad \sum_{i=n-2}^{n} f(v_i) \geq 1.$$

Therefore

$$\begin{aligned} f(V) &= \sum_{i=1}^{3} f(v_i) + \sum_{i=4}^{6} f(v_i) + \cdots + \sum_{i=n-5}^{n-3} f(v_i) + [f(v_{n-2}) + f(v_{n-1}) + f(v_n)] \\ &\geq \frac{n+6}{3} \end{aligned}$$

This is true for any signed unidominating function of a path P_n. Therefore $\min\{f(V) : f$ is signed unidominating function of $P_n\} \geq \frac{n+6}{3}$. Thus

$$\gamma_{su}(P_n) \geq \frac{n+6}{3} \tag{2.2}$$

From (2.1) and (2.2) we have, $\gamma_{su}(P_n) = \frac{n+6}{3}$, if $n \equiv 0$ (mod 3).

Case 2: Let $n \equiv 1$ (mod 3). Define a function $f : V \to \{-1, 1\}$ by

$$f(v_i) = \begin{cases} 1, & \text{if } i \equiv 1, 2 \text{ (mod 3)} \\ -1, & \text{if } i \equiv 0 \text{ (mod 3)}, i \neq n-1 \text{ and } f(v_{n-1}) = 1 \end{cases}.$$

Sub case 1: For vertex $v_i \in P_n$ with $i \equiv 0$ (mod 3).
If $i = n-1$, $f(v_i) = -1$ and $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1$.
If $i = n$-1 , $f(v_i) = 1$ and $\sum\limits_{u \in N[v_{n-1}]} f(u) = f(v_{n-2}) + f(v_{n-1}) + f(v_n) = 3$.

Sub case 2: For vertex $v_i \in P_n$ with $i \equiv 1$ (mod 3), we have $f(v_i) = 1$.
Then $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1$, if $i \neq 1$ and $i \neq n$. Also $\sum\limits_{u \in N[v_1]} f(u) = f(v_1) + f(v_2) = 2$ and $\sum\limits_{u \in N[v_n]} f(u) = f(v_{n-1}) + f(v_n) = 2$.

Sub case 3: For vertex $v_i \in P_n$ with $i \equiv 2$ (mod 3), we have $f(v_i) = 1$.
Then $\sum\limits_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1$, if $i \neq n-2$. Also, $\sum\limits_{u \in N[v_i]} f(u) = 3$, if $i = n-2$.

From the above three cases we observed that f is a signed unidominating function of P_n. Now

$$\begin{aligned} f(V) =& \sum_{u\in V} f(u) = \sum_{i=1}^{n-4} f(v_i) + f(v_{n-4}) + f(v_{n-2}) + f(v_{n-1}) + f(v_n) \\ =& \frac{(1+1-1)+(1+1-1)+\cdots+(1+1-1)}{(n-4)-terms} + 1+1+1+1 \\ =& \frac{n-4}{3}(1) + 4 = \frac{n+8}{3}. \end{aligned}$$

Since three vertices are taken as one group whose functional values are $1, 1, -1$ and the sum is 1 and there are $\frac{n-4}{3}$ such groups. By the definition of signed unidomination number

$$\gamma_{su}(P_n) \leq \frac{n+8}{3}. \tag{2.3}$$

If f is any signed unidominating function of P_n then the sum of the functional values of any three consecutive vertices of P_n is greater than or equal to 1.

Therefore,

$$\begin{aligned} f(V) =& \sum_{i=1}^{3} f(v_i) + \sum_{i=4}^{6} f(v_i) + \dots \\ & \dots + \sum_{i=n-6}^{n-4} f(v_i) + [f(v_{n-3}) + f(v_{n-2}) + f(v_{n-1}) + f(v_n)] \\ \geq& \frac{n-4}{3}(1) + 4 \geq \frac{n+8}{3}. \end{aligned} \tag{2.4}$$

From (2.3) and (2.4), it is clear that $\gamma_{su}(P_n) = \frac{n+8}{3}$.

Case 3: Let $n \equiv 2$ (mod 3).
Define a function $f : V \to \{-1, 1\}$ by

$$f(v_i) = \begin{cases} 1, if i \equiv 1, 2 \text{ (mod 3)} \\ -1, if i \equiv 0 \text{ (mod 3)} \end{cases}.$$

As shown in Case 1, for vertex $v_i \in P_n$ with $i \equiv 0$ (mod 3),

$$\sum_{u\in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1,$$

and for vertex $v_i \in P_n$ with $i \equiv 1 \pmod 3$. and $i \neq 1$,

$$\sum_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1,$$

and $\sum\limits_{u \in N[v_i]} f(u) = 2$, if $i \equiv 1$ (mod 3) and $i = 1$, for vertex $v_i \in P_n$ with $i \equiv 2$ (mod 3) and $i \neq n$,

$$\sum_{u \in N[v_i]} f(u) = f(v_{i-1}) + f(v_i) + f(v_{i+1}) = 1,$$

and $\sum\limits_{u \in N[v_i]} f(u) = 2$, if $i \equiv 2$ (mod 3) and $i = n$.

From the above all possibilities, we have seen that f is a signed unidominating function of P_n. Now,

$$\begin{aligned} f(V) =& \sum_{u \in V} f(u) = \sum_{i=1}^{n-2} f(v_i) + f(v_{n-1}) + f(v_n) \\ =& \frac{(1+1-1)+(1+1-1)+\cdots+(1+1-1)}{(n-2)-terms} + 1 + 1 \\ =& \frac{n+4}{3}. \end{aligned}$$

By the definition of signed unidomination number, we have

$$\gamma_{su}(P_n) \leq \frac{n+4}{3}. \tag{2.5}$$

But the sum of the functional values of three consecutive vertices is greater than or equal to 1. Therefore,

$$f(V) = \sum_{i=1}^{3} f(v_i) + \cdots + \sum_{i=n-4}^{n-2} f(v_i) + f(v_{n-1}) + f(v_n). \geq \frac{n+4}{3} \tag{2.6}$$

From (2.5) and (2.6), it is clear that $\gamma_{su}(P_n) = \frac{n+4}{3}$. □

Theorem 2.2. *The Signed Unidomination number of a Cycle C_n is*

$$\gamma_{su}(C_n) = \begin{cases} \frac{n}{3}, & \textit{if } n \equiv 0 \textit{ (mod 3)} \\ \frac{n+2}{3}, & \textit{if } n \equiv 1 \textit{ (mod 3)} \\ \frac{n+4}{3}, & \textit{if } n \equiv 2 \textit{ (mod 3)}. \end{cases}$$

Proof. Let C_n be a cycle with n vertices with vertex set $V = \{v_1, v_2, v_3 \dots v_n\}$. From Theorem (2.1), the function $f : V \to \{-1, 1\}$ defined by

$$f(v_i) = \begin{cases} 1, \text{ if } i \equiv 1, 2 \text{ (mod 3)} \\ -1, \text{ if } i \equiv 0 \text{ (mod 3)} \end{cases},$$

$\forall v_i \in V$ is also a signed unidominating function of C_n.

To find the signed unidomination number of C_n, three possible cases arise.

Case 1: Let $n \equiv 0$ (mod 3). As shown in Case 1 of Theorem(2.1),

$$f(V) = \sum_{u \in V} f(u) = \frac{n}{3}.$$

By the definition of signed unidomination number

$$\gamma_{su}(C_n) \leq \frac{n}{3}. \tag{2.7}$$

If f is signed unidominating function of a cycle C_n then

$$f(V) = \sum_{u \in V} f(u) = \sum_{i=1}^{3} f(v_i) + \sum_{i=4}^{6} f(v_i) + \cdots + \sum_{i=n-2}^{n} f(v_i) \geq \frac{n}{3}.$$

Therefore $\min\{f(V) : f$ is singed unidominating function$\} \geq \frac{n}{3}$. Thus

$$\gamma_{su}(C_n) \geq \frac{n}{3}. \tag{2.8}$$

Therefore from (2.7) and (2.8), $\gamma_{su}(C_n) = \frac{n}{3}$.

Case 2: Let $n \equiv 1$ (mod 3).
As shown in Case 2 of Theorem(2.1),

$$f(V) = \sum_{u \in V} f(u) = \frac{n+2}{3}.$$

By the definition of signed unidomination number

$$\gamma_{su}(C_n) \leq \frac{n+2}{3}. \tag{2.9}$$

If f is signed unidominating function of a cycle C_n with minimum weight, the possibility of assigning functional values are $(1, 1, -1), (1, 1, -1), \dots, (1, 1, -1)$

and 1. Then

$$f(V) = \sum_{u \in V} f(u) = \sum_{i=1}^{3} f(v_i) + \sum_{i=4}^{6} f(v_i) + \cdots + \sum_{i=n-3}^{n-1} f(v_i) + f(v_n) \tag{2.10}$$
$$\geq \frac{n-1}{3} + 1 = \frac{n+2}{3}.$$

Thus

$$\gamma_{su}(C_n) \geq \frac{n+2}{3}. \tag{2.11}$$

Therefore from (2.9) and (2.11), $\gamma_{su}(C_n) = \frac{n+2}{3}$.

Case 3: Let $n \equiv 2$ (mod 3). From As shown in Case 3 of Theorem(2.1), f is a signed unidominating function of C_n. Then $f(V) = \sum_{u \in V} f(u) \geq \frac{n-2}{3} + 2 = \frac{n+4}{3}$.

By the definition of signed unidomination number we have

$$\gamma_{su}(C_n) \leq \frac{n+4}{3}. \tag{2.12}$$

To get the minimum weight, the possibility of assigning functional values is $(1, 1, -1), \ldots, (1, 1, -1), 1, 1$. Then $f(V) = \sum_{u \in v} f(u) \geq \frac{n+4}{3}$.

This is true for any signed unidominating function. Therefore

$$\gamma_{su}(C_n) \geq \frac{n+4}{3}. \tag{2.13}$$

From (2.12) and (2.13), $\gamma_{su}(C_n) = \frac{n+4}{3}$. □

Theorem 2.3. *The Signed Unidomination number of a complete graph*

$$\gamma_{su}(K_n) = \begin{cases} 1, & \text{if } n \equiv 1 \text{ (mod 2)} \\ n, & \text{if } n \equiv 0 \text{ (mod 2)} \end{cases}.$$

Proof. Let K_n be a complete graph of n vertices with vertex set $V = \{v_1, v_2, v_3, \ldots, v_n\}$. To find the signed unidomination number of K_n, the following cases arise.

Case 1: Let $n \equiv 1$ (mod 2).

Define a function $f : V \to \{-1, 1\}$ by $f(v_i) = \begin{cases} 1, & \text{if } i \text{ is odd} \\ -1, & \text{if } i \text{ is even} \end{cases}$.

Now we check the condition for Signed unidominating function at every vertex v_i of K_n and for $K_n, d(v_i) = n - 1$.

By the definition of function, there are $\frac{n+1}{2}$ vertices are of functional value 1 and $\frac{n-1}{2}$ vertices are of -1. Then for every vertex of K_n,

$$\sum_{u\in V} f(u) = \frac{n+1}{2}(1) + \frac{n-1}{2}(-1) = 1.$$

for $v \in V, \gamma_{su}(K_n) \geq 1$. By the definition signed unidomination number, $\gamma_{su}(K_n)$ ≤ 1. Therefore $\gamma_{su}(K_n) = 1$.

Case 2: Let $n \equiv 0$ (mod 2).
Defined a function $f : V \to \{-1, 1\}$ by $f(v) = 1 \forall v \in V$. For $v \in V$, $\sum\limits_{u\in V}$, $f(u) = n > 1$. Therefore f is a signed unidominating function of K_n. Now $f(V) = n$ and by the definition of signed unidominating function $\gamma_{su}(K_n) \leq n$. By the definition of the function, each vertex is of functional value 1. If any vertex $v_i \in K_n$, $f(v_i) = -1$ it fails to the condition of signed unidominating function since $\sum\limits_{u\in N[v_i]} f(u) = n - 2 \neq 1$.

Therefore $\gamma_{su}(K_n) \geq n$. Thus $\gamma_{su}(K_n) = n$. □

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