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ON NANO GENERALIZED PRE *c*-CONTINUOUS FUNCTIONS IN NANO TOPOLOGICAL SPACES

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ABSTRACT. The purpose of this paper is to introduce a new class of continuous functions called Nano generalized pre c-continuous functions in Nano Topological Spaces and derive their characterizations in terms of nano generalized pre c-closure, nano generalized pre c-interior, nano generalized pre c-kernel and nano generalized pre c-surface.

1. INTRODUCTION

Continuous functions play a major role in general topology. Several authors have studied different types of generalization of continuous functions. Lellis Thivagar and Carmel Richard [1] introduced the notion of Nano Topology with respect to a subset X of a universe which is defined in terms of approximations and boundary region. They defined nano closed sets, nano interior and nano closure. They [2] also introduced nano continuous functions, nano open maps, nano closed maps and nano homeomorphisms in nano topological spaces. Padmavathi and Nithyakala [4] introduced nano generalized pre c-closed sets.

In this paper we have introduced a new class of continuous functions called nano generalized pre c continuous functions and established some of their representations in terms of nano interior, nano closure, nano kernel, nano generalized pre c-interior, nano generalized pre c-closure, nano generalized pre c-kernel and nano generalized pre c-surface of sets.

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2. PRELIMINARIES

Definition 2.1. *[1] Let U be a non empty finite set of objects called the universe and R be an equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be approximation space. Let $X \subseteq U$. Then*

(1) *The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \subseteq X\}$ where $R(x)$ denotes the equivalence class determined by $L_R(X)$.*
(2) *The upper approximation of X with respect to R is the set of all objects which can be possibly classified as X with respect to R and is denoted by $U_R(X)$. $U_R(X) = \bigcup_{x \in U} \{R(x) : R(x) \cap X \neq \emptyset\}$.*
(3) *The boundary region of X with respect to R is the set of all objects which can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.*

Proposition 2.1. *[1] If (U, R) is an approximation space and $X, Y \subseteq U$, then*

(1) $L_R(X) \subseteq X \subseteq U_R(X)$
(2) $L_R(\emptyset) = U_R(\emptyset) = \emptyset$
(3) $L_R(U) = U_R(U) = U$
(4) $U_R(X \bigcup Y) = U_R(X) \bigcup U_R(Y)$
(5) $U_R(X \bigcap Y) \subseteq U_R(X) \bigcap U_R(Y)$
(6) $L_R(X \bigcup Y) \supseteq L_R(X) \bigcup L_R(Y)$
(7) $L_R(X \bigcap Y) = L_R(X) \bigcap L_R(Y)$
(8) $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$
(9) $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$
(10) $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
(11) $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$.

Definition 2.2. *[1] Let U be the universe, R be an equivalence relation on U as $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms.*

- *U and $\emptyset \in \tau_R(X)$.*
- *The union of all the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.*

- *The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.*

Then $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as a nano topological space. The elements of $\tau_R(X)$ are called as nano open sets. The complement of the nano open sets are called nano closed sets.

Remark 2.1. *[1] If $\tau_R(X)$ is a nano topology on U with respect to X, then the set $B = \{U, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.*

Definition 2.3. *[1] If $(U, \tau_R(X))$ is a nano topological space with respect to X where $X \subseteq U$ and if $A \subseteq U$ then*

(1) *The nano interior of A is defined as the union of all nano open subsets contained in A and is denoted by $Nint(A)$.*
(2) *The nano closure of A is defined as the intersection of all nano closed sets containing A and is denoted by $Ncl(A)$.*

Definition 2.4. *[3] Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. The set $Nker(A) = \cap\{U : A \subseteq U, U \in \tau_R(X)\}$ is called the nano kernel of A and is denoted by $Nker(A)$.*

3. Nano Generalized pre c-Continuous Functions

In this section, we define nano generalized pre c-continuous function and study its characterization with Ngpc-int, Ngpc-cl, Ngpc-ker and Ngpc-surf of sets.

Definition 3.1. *Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two nano topological spaces. The function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is said to be Nano generalized pre c-continuous (briefly Ngpc- continuous) on U if the inverse image of every nano open set in V is a Ngpc-open set in U.*

Example 1. *Let $U = \{a, b, c, d\}$ with $\frac{U}{R} = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{b, d\}$. Then $\tau_R(X) = \{\emptyset, U, \{b\}, \{c, d\}, \{b, c, d\}\}$ is a nano topology on U. Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{z\}, \{y, w\}\}$ and $Y = \{x, y\}$. Then $\tau'_R(Y) = \{\emptyset, V,\ \{x\}, \{y, w\}, \{x, y, w\}\}$ is a nano topology on V.*

Then $\tau_R^C(X) = \{\emptyset, U, \{a\}, \{a,b\}, \{a,c,d\}\}$ *and* $\tau_{R\prime}^C(Y) = \{\emptyset, V, \{z\}, \{x,z\}, \{y,z,w\}\}$ *are the complements of* $\tau_R(X)$ *and* $\tau'_R(Y)$ *respectively.*

Define $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ *as* $f(a) = z, f(b) = y, f(c) = x, f(d) = w$*.Then* $f^{-1}(\{x\}) = \{c\}, f^{-1}(\{y,w\}) = \{b,d\}, f^{-1}(\{x,y,w\}) = \{b,c,d\}$*. Thus the inverse image of every nano open set in* V *is Ngpc-open in* U*.*

Definition 3.2. *The Nano generalized pre c-surface of* A *is defined as the union of all Ngpc-closed sets of* U *contained in* A *and it is denoted by Ngpc-surf*(A)*.*

Example 2. *Let* $U = \{a,b,c,d\}$ *with* $\frac{U}{R} = \{\{a\}, \{b\}, \{c,d\}\}$ *and* $X = \{b,d\}$*. Then* $\tau_R(X) = \{\emptyset, U, \{b\}, \{c,d\}, \{b,c,d\}\}$ *is a nano topology with respect to* X *and the complement* $\tau_R(X)^c = \{\emptyset, U, \{a\}, \{a,b\}, \{a,c,d\}\}$*.*

Then Ngpc-surf$(\{a\}) = \{a\}$*, Ngpc-surf*$(\{b\}) = \phi$ *and Ngpc-surf*$(\{a,c\}) = \{a,c\}$*.*

Theorem 3.1. *A function* $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ *is Ngpc-continuous iff the inverse image of every nano closed set in* V *is Ngpc-closed in* U*.*

Proof. Let f be Ngpc-continuous. Let A be a nano closed set in V. Then $V - A$ is nano open in V. Since f is Ngpc-continuous, $f^{-1}(V - A)$ is Ngpc-open in U. That is $U - f^{-1}(A)$ is Ngpc-open in U. Therefore $f^{-1}(A)$ is Ngpc closed in U. Thus the inverse image of every nano closed set in V is Ngpc-closed in U if f is Ngpc-continuous. Conversely, let the inverse image of every nano closed set in V is Ngpc-closed in U. Let B be a nano open set in V. Then $V - B$ is nano closed in V. Then $f^{-1}(V - B)$ is Ngpc-closed in U. That is $U - f^{-1}(B)$ is Ngpc-closed in U. Therefore $f^{-1}(B)$ is Ngpc open in U. Hence f is Ngpc-continuous. □

Theorem 3.2. *Let* $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ *be a function. Then the following statements are equivalent.*

(i) *f is Ngpc-continuous.*
(ii) *For every subset A of U,* $f(Ngpc - cl(A)) \subseteq Ncl(f(A))$*.*
(iii) *For every subset B of V, Ngpc-cl*$(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$*.*

Proof.

(i) $\Leftrightarrow$ (ii). Let f be Ngpc-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. $Ncl(f(A))$ is nano closed in V. Since f is Ngpc-continuous, $f^{-1}(Ncl(f(A)))$ is Ngpc-closed in U. Since $f(A) \subseteq Ncl(f(A))$, $A \subseteq f^{-1}(Ncl(f(A)))$. $f^{-1}(Ncl(f(A)))$ is a Ngpc-closed set containing A. But Ngpc-cl(A) is the smallest Ngpc-closed set containing A. Therefore Ngpc-cl$(A) \subseteq f^{-1}(Ncl(f(A)))$. That is $f(Ngpc - cl(A)) \subseteq Ncl(f(A))$.

Conversely let $f(Ngpc-cl(A)) \subseteq Ncl(f(A))$ for every subset A of U. Let G be a nano closed set in V. Since $f^{-1}(G) \subseteq U, f(Ngpc-cl(f^{-1}(G))) \subseteq Ncl(f(f^{-1}(G))) = Ncl(G)$. That is Ngpc-cl$(f^{-1}(G)) \subseteq f^{-1}(Ncl(G)) = f^{-1}(G)$ since G is nano closed. Hence Ngpc-cl$(f^{-1}(G)) \subseteq f^{-1}(G)$. But $f^{-1}(G) \subseteq Ngpc-cl(f^{-1}(G))$. Therefore $f^{-1}(G) = Ngpc-cl(f^{-1}(G))$. This implies $f^{-1}(G)$ is Ngpc-closed in U. Thus the inverse image of every nano closed set in V is Ngpc-closed in U. Hence f is Ngpc-continuous.

(ii) $\Leftrightarrow$ (iii). Assume (ii) holds. Let B be any subset of V. Then replacing A by $f^{-1}(B)$ in (ii) we have $f(Ngpc-cl(f^{-1}(B))) \subseteq Ncl(f(f^{-1}(B))) = Ncl(B)$. That is Ngpc-cl$(f^{-1}(B)) \subseteq f^{-1}(Ncl(B))$.

Conversely suppose (iii) holds. Let A be any subset of U. Then $f(A) \subseteq V$. Let $B = f(A)$. Then we have Ngpc-cl$(A) = Ngpc-cl(f^{-1}(B)) \subseteq f^{-1}(Ncl(B)) = f^{-1}(Ncl(f(A)))$. This implies Ngpc-cl$(A) \subseteq f^{-1}(Ncl(f(A)))$. Thus $f(Ngpc-cl(A)) \subseteq Ncl(f(A))$. □

Theorem 3.3. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Also let $A \subseteq U$ and $B \subseteq V$. Then*

(i) *f is Ngpc-continuous* $\Leftrightarrow Nintf(A) \subseteq f\,(Ngpc{-}int(A)) \Leftrightarrow f^{-1}(Nint\,(B)) \subseteq Ngpc-int\,(f^{-1}\,(B))$.

(ii) *f is Ngpc-continuous* $\Leftrightarrow A \subseteq Ngpc-int(f^{-1}Nker\,(f(A))) \Leftrightarrow f^{-1}(B) \subseteq Ngpc-int(f^{-1}(Nker(B)))$.

Proof. Proof is similar as theorem 3.2 □

Theorem 3.4. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be Ngpc-continuous. Then for every subset A of U we have*

(i) $f(Ngpc-surf\,(A)) \subseteq Ncl\,(f\,(A))$.

(ii) $Nintf(A) \subseteq f\,(Ngpc-ker(A))$.

(iii) $f(Ngpc-ker(A)) \subseteq Nker(f\,(A))$.

Proof. (i) Let f be Ngpc-continuous and $A \subseteq U$. Then $f(A) \subseteq V.Ncl\,(f(A))$ is nano closed in V. Since f is Ngpc-continuous, $f^{-1}(Ncl\,(f\,(A)))$ is Ngpc-closed in U. Therefore Ngpc-surf $(f^{-1}(Ncl\,(f\,(A)))) = f^{-1}(Ncl\,(f\,(A)))$. But we know that $f\,(A) \subseteq Ncl\,(f\,(A))\,, A \subseteq f^{-1}(Ncl\,(f\,(A)))$ which implies Ngpc-surf $A \subseteq Ngpc-surf(f^{-1}(Ncl\,(f\,(A))))$. Hence Ngpc-surf $(A) \subseteq f^{-1}(Ncl\,(f\,(A)))$. That is $f(Ngpc-surf(A)) \subseteq Ncl\,(f\,(A))$. Proof of (ii) and (iii) are similar. □

4. Nano Contra Generalized pre c-Continuous functions

In this section, we define nano contra generalized pre c-continuous function and study its characterization with Ngpc-int and Ngpc-cl of sets.

Definition 4.1. *Let $(U, \tau_R(X))$ and $(V, \tau'_R(Y))$ be two nano topological spaces. The function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is said to be Nano contra generalized pre c-continuous (briefly Ncgpc-continuous) on U if the inverse image of every nano open set in V is Ngpc-closed in U.*

Example 3. *In Example 1, Define $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ as $f(a) = w$, $f(b) = y$, $f(c) = x$, $f(d) = z$. Then $f^{-1}(\{x\}) = \{c\}, f^{-1}(\{y, w\}) = \{a, b\}, f^{-1}(\{x, y, w\}) = \{a, b, c\}$. The inverse image of every nano open set in V is Ngpc-closed in U.*

Theorem 4.1. *A function $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ is Ncgpc-continuous iff the inverse image of every nano closed set in V is Ngpc-open in U.*

Proof. Let f be Ncgpc-continuous. Let A be a nano closed set in V. Then $V - A$ is nano open in V. Since f is Ncgpc-continuous, $f^{-1}(V - A) = U - f^{-1}(A)$ is Ngpc-closed in U. Therefore $f^{-1}(A)$ is Ngpc-open in U. Thus the inverse image of every nano closed set in V is Ngpc-open in U. Conversely, assume that the inverse image of every nano closed set in V is Ngpc-open in U. Let B be a nano open set in V. Then $V - B$ is nano closed in V. By our assumption $f^{-1}(V - B) = U - f^{-1}(B)$ is Ngpc-open in U. Therefore $f^{-1}(B)$ is Ngpc-closed in U. That is the inverse image of every nano open set in V is a Ngpc-closed set in U. Hence f is Ncgpc-continuous. □

Theorem 4.2. *Let $f : (U, \tau_R(X)) \to (V, \tau'_R(Y))$ be a function. Then the following statements are equivalent.*

(i) *f is Ncgpc-continuous.*

(ii) *For every subset A of U, $f(Ngpc - cl(A)) \subseteq Nker(f(A))$.*

(iii) *For every subset B of V, $Ngpc - cl(f^{-1}(B)) \subseteq f^{-1}(Nker(B))$.*

Proof.

(i) ⇒ (ii). Let f be Ncgpc-continuous and $A \subseteq U$. Then $f(A) \subseteq V$. $Nker(f(A))$ is nano open in V. Since f is Ncgpc-continuous, $f^{-1}(Nker(f(A)))$ is Ngpc-closed in U. Since $f(A) \subseteq Nker(f(A)), A \subseteq f^{-1}(Nker(f(A)))$. $f^{-1}(Nker(f(A)))$

is a Ngpc-closed set containing A. But $Ngpc-cl(A)$ is the smallest Ngpc-closed set containing A. Therefore $Ngpc-cl(A) \subseteq f^{-1}(Nker(f(A)))$. That is $f(Ngpc-cl(A)) \subseteq Nker(f(A))$.

(ii) $\Rightarrow$ (iii). Assume (ii) holds. Let B be any subset of V. Then $f^{-1}(B) \subseteq U$. By our assumption $f(Ngpc-cl(f^{-1}(B))) \subseteq Nker(f(f^{-1}(B))) = Nker(B)$. That is Ngpc-$cl(f^{-1}(B)) \subseteq f^{-1}(Nker(B))$.

(iii) $\Rightarrow$ (i). Suppose (iii) holds. Let G be a nano open subset of V. Then by our assumption Ngpc-cl$(f^{-1}(G)) \subseteq f^{-1}(Nker(G)) = f^{-1}(G)$. But we know that $f^{-1}(G) \subseteq Ngpc-cl(f^{-1}(G))$. That is Ngpc-cl$(f^{-1}(G)) = f^{-1}(G)$ implies $f^{-1}(G)$ is Ngpc-closed in U. Therefore f is Ncgpc continuous. □

Theorem 4.3. *Let $f : U \to V$ be a function. Also let $A \subseteq U$ and $B \subseteq V$. Then we have*

(1) *f is Ncgpc-continuous $\Leftrightarrow$ Ngpc-cl(f^{-1}(Nint($f(A)$))) $\subseteq A$*
$\Leftrightarrow$ *Ngpc-cl(f^{-1}(Nint(B)) $\subseteq f^{-1}(B)$.*

(2) *f is Ncgpc-continuous $\Leftrightarrow A \subseteq$ Ngpc-int f^{-1}(Ncl f(A)))*
$\Leftrightarrow$ *$f^{-1}(B) \subseteq$ Ngpc-int (f^{-1}(Ncl(B))).*

Proof. Proof is similar as theorem 4.2. □

References

[1] M. Lellis Thivagar, Carmel Richard: *On nano forms of weakly open sets*, International Journal of Mathematics and Statistics Invention, **1**(2013), 31–37.

[2] M. Lellis Thivagar, Carmel Richard: *On Nano continuity*, Mathematical Theory and Modeling, **3**(7) (2013), 32–37.

[3] M. Lellis Thivagar: *On new class of Contra Continuity in Nano Topology*, Italian Journal of Pure and Applied Mathematics, (2017), 1–10.

[4] P. Padmavathi, R. Nithyakala: *A note on nano generalized pre c-closed sets*, International Journal of Advanced Science and Technology, **29**(3s) (2020), 194–201.

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