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HEAT AND MASS TRANSFER SLIP FLOW OF MHD CASSON LIQUID PAST A VERTICALLY ROTATING CONE WITH CONVECTIVE CONDITIONS

KOLLI VIJAYA[1], G. VENKATA RAMANA REDDY, AND Y. HARI KRISHNA

ABSTRACT. This article has been developed for studying the heat alongside mass transport attribute of MHD Casson liquid flow past a vertically circulating cone with convective conditions. Thermal motion in the liquid flow is anticipated with viscous dissipation while concentrations of liquid particles are affected by diffusion-thermo alongside thermo-diffusion numbers. The basic model equations are transformed in to total differential equations by employing the suitable similarity terms. The R-K approach with shooting approach was used to provide solution to the simplified model equations. A nice conformity of the present outcomes has been noticed by checking them with the previously available literature.

1. INTRODUCTION

A substance that deforms regularly owning to an imposed shear stress is a fluid. Fluids represent the state of matters consist of liquids, gases and plasma. The examination of heat alongside mass transport at the condition of double diffusion has pulled attention among the scholars owning to its applications in chemical, industrial, as well as geophysical engineering and so on. Due to change in buoyancy forces the effect of diffusion came into existence in fluids. The buoyancy forces cause variations in temperature and density. As a result

[1]*corresponding author*

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diffusion in the midst of heavier as well as lighter liquid molecules arise. Energy diffusion is originated owning to the gradient in molecules constituent, while species diffusion is generated owning to slope temperature. Hence, the analysts have explained these diffusions in technology as thermo diffusion alongside diffusion thermo effects. Madhu and Kishan [1] computationally inspected heat alongside mass transport of MHD non-Newtonian model on nano liquids towards a stretching surface. Das [2] has represented the unsteady hydromagnetic Casson liquid flow with heat alongside mass transport. Eldabe [3] reported about the heat along with mass transport of MHD non-Newtonian liquid flow in a penetrable channel. Hamdy [4] made the effect of interchanging the cyclone cone length. Stefan [5] has investigated convex cone for stochastic PDEs. Pal [6] studied the fluid characters in his article using the geometry of nonlinear stretching sheet affected by diffusion-thermo, thermo-diffusion on MHD nanofluids. Sravanthi [7] in her article represented the attributes of Soret alongside Dufour on the exponentially stretching sheet. Ramzan [8] has declared the attribute of Soret alongside Dufour on viscoelastic nanofluid using three dimensional flows. By using the method of homotopy, Omowaye [9] represented the flow characteristics of MHD liquid in a penetrable channel with viscosity subject to temperature. Ganesan and Loganadhan examined the radiation alongside mass transport impacts on incompressible motion of viscous liquid over a locomotive vertical cylinder.

2. Flow analysis

Take three-dimensional unsteadiness, MHD and axi-symmetric motion of incompressible viscous Casson fluid over a cone with convective condition and time dependent circulation. The time dependent circulatory velocity of conies Ω . Let u, v and w be the velocity flow towards x, y and z, where x, y, z are coordinate in tangential, perpendicular as well as azimuthal direction to the cone. The attribute of radiation on the flow is owning to thermal condition of the working fluid. The consideration of thermal radiation in this paper shows that the cone at the wall is not cooled. Gravitational force is acting towards cone's coordinates towards downward position. Wall temperature alongside concentration change with tangential coordinate x, ambient temperature and ambient concentration are surmised to be constant. The geometry model is given in Fig. 1.

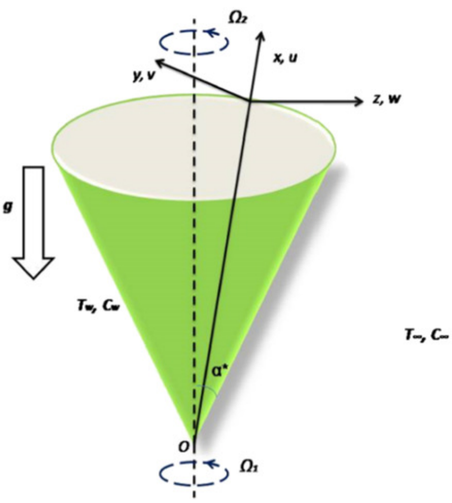

FIGURE 1. The geometric configuration over cone

By taking Boussinesq approximation along with above assumption, the governing system can be written as:

$$\frac{\partial (xu)}{\partial x}+\frac{\partial (xv)}{\partial z}=0 \tag{2.1}$$

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+w\frac{\partial u}{\partial z}-\frac{v^2}{x}=v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial z^2}+g\beta\left(T-T_\infty\right)\cos\alpha^*-\frac{\sigma B_0^2}{\rho}u \tag{2.2}$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+w\frac{\partial v}{\partial z}+\frac{uv}{x}=v\left(1+\frac{1}{\beta}\right)\frac{\partial^2 v}{\partial z^2}-\frac{\sigma B_0^2}{\rho}v \tag{2.3}$$

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+w\frac{\partial T}{\partial z}=\alpha\frac{\partial^2 T}{\partial z^2}+\left(\frac{Dk}{C_z c_p}\right)\frac{\partial^2 C}{\partial z^2}+\frac{v}{c_p}\left[\left(\frac{\partial u}{\partial z}\right)^2+\left(\frac{\partial v}{\partial z}\right)^2\right]-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y} \tag{2.4}$$

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+w\frac{\partial C}{\partial z}=D\frac{\partial^2 C}{\partial z^2}+\left(\frac{Dk}{T_m}\right)\frac{\partial^2 T}{\partial z^2}-K_1\left(C-C_\infty\right), \tag{2.5}$$

where kinematic viscosity $= v$, magnetic induction $= B_o$, gravity $= g$, temperature expansion coefficient $= \beta_1$, density of the fluid $= \rho$, temperature $= T$, ambient temperature $= T_\infty$, half-vertical angle $= \alpha^*$, thermal conductivity $= \alpha$, Casson fluid parameter $= \beta$, mass diffusivity $= D$, specific heat $= c_p$, thermal diffusion ratio $= D_k$, concentration susceptibility $= S_s$, mean fluid temperature $= T_m$, chemical reaction rate $= K_1$, fluid concentration $= C$ and ambient concentration $= C_\infty$, respectively.

3. Results and Discussion

This section is focused for computing numerical approach of the system. The impacts of various important terms such as slip parameter (γ), Casson fluid parameter, magnetic field parameter, mixed convection(λ), Dufour number (Df), Prandtl number (Pr), chemical reaction (Ch), Schmidt number (Sc), Eckert number (Ec), Soret number (Sr), unsteadiness parameter(s), thermal radiation parameter (R), and concentration biot number on velocity field, energy field, concentration profile, skin friction factor, heat as well as mass transport rate are explained through graphs The following values are taken for important parameters in calculation.

$$M = \beta = Df = Ec = Bi_1 = Bi_2 = s = Ch = \lambda = 0.5,$$

$$\mathrm{Pr} = Sc = 10, Sr = 0.33, \gamma = 0.1, \mathrm{R} = 1.0$$

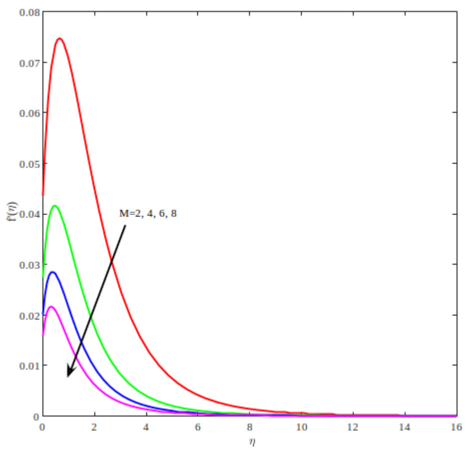

FIGURE 2. Influence of M on tangential velocity

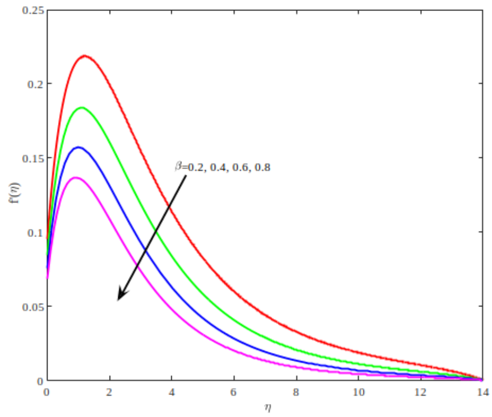

FIGURE 3. Influence of β on tangential velocity

In Fig. 2 we observe the decrease in tangential velocity alongside momentum layer thickness with the boosting values of magnetic. This detected that the hike in M helps to thinning the boundary layer. The velocity plot exponentially reduce to zero at shorter distance from the sheet for higher values of M. Physically, the magnetic term is derived from the magnetism imposed on the flow from the rotating cone. However, the imposed magnetism produces the drag force called Lorentz force. This force suppresses the movement of an electrically conducting liquid such as Casson considered in this study. Hence, an incremental value of the magnetic term hike the strength of the Lorentz force and hereby pull down the velocity as well as momentum layer thickness. Fig. 3 indicates the decrease in tangential velocity for growing values of non-Newtonian liquid parameter. Non-Newtonian fluid parameter has a discouraging impact on velocity profiles. By improving the values of non-Newtonian fluid parameter, tangential velocity distribution is reduced. This may happen due to the fact that the thickness of the momentum boundary layer reduces with raising values of β. In the physical geometry shown in Fig. 1, the motion of the Casson fluid is subjected to the magnetic field. The magnetism is responsible for dragging the flow of an electrically conducting liquid. It originates Lorentz force which drags the flow of the Casson fluid by decreasing the momentum layer. Our experiment proven that large value of β changes the present model to Newtonian model. This means that as $\beta \to \infty$, the behavior of β within the flow region obeys the law of viscosity.

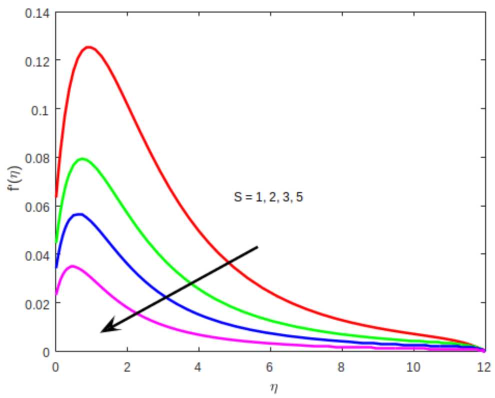

FIGURE 4. Influence of S on tangential velocity

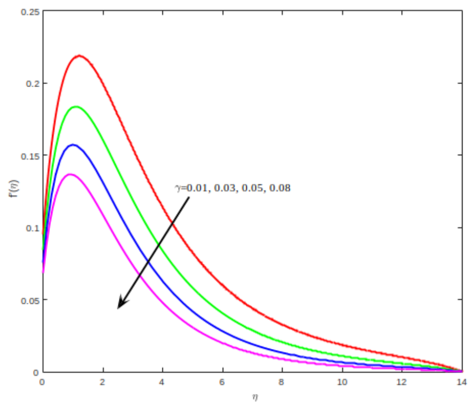

FIGURE 5. Influence of γ on tangential velocity.

From Fig. 4 we observe the decrease in tangential velocity with hike in unsteady term. This may happen owning to the momentum boundary layer is decreased with improving values of unsteady term. The unsteadiness flow means the fluid properties is with respect to time. An incremental value of the unsteadiness parameter declines the tangential velocity in the boundary layer. When $s < 0$ a steady state flow is observed but the moment $S > 0$ the state of the flow becomes unsteady. The momentum boundary layer thickness reduces with the increase in the slip parameter as illustrated in Fig.5. Closely at the wall, the slip constraints is clearly seen and velocity does not reach zero. Experimentally, the boundary slip condition shows that the fluid velocity is not equals to zero. Hence, an incremental value of the slip parameter declines the velocity profile.

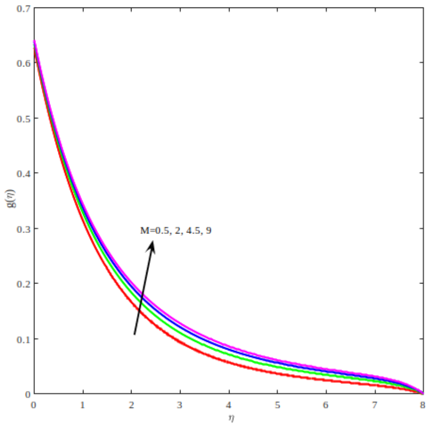

FIGURE 6. Influence of M on circumferential velocity

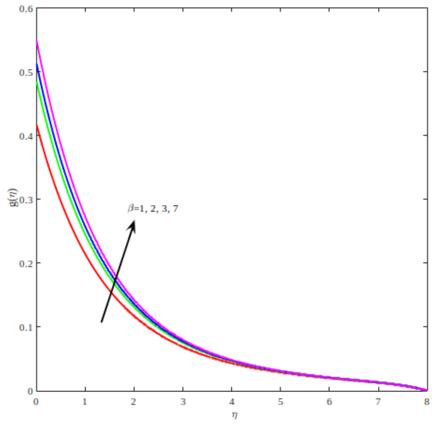

FIGURE 7. Influence of β on circumferential velocity

From Fig. 6 it is traced out that the circumferential velocity is intensified with magnetic field. It shows enhancement in $\theta(\eta)$ with the improved values of M. The imposed magnetic field has great impact on the circumferential velocity. This is because of the originated Lorentz force by the magnetic field which lead to declination in the momentum layer thickness of the circumferential velocity. The reason for this trend is that M assists to increase the momentum layer thickness. In Fig. 7 we observed the increase in circumferential velocity with hike in non- Newtonian liquid parameter. Physically, the plastic dynamic viscosity in the Casson liquid causes a resistance to the liquid flow in the boundary layer. On the other hand, the imposed magnetic field slows down Casson liquid motion due to the Lorentz force. The Lorentz force drags the fluid velocity as well as momentum boundary layer thickness of the circumferential velocity.

From Fig. 8 we observed that the circumferential velocity is decreasing with unsteady parameter. Slip velocity parameter has the tendency to decrease circumferential velocity profile with its growing values. This tendency is shown in Fig. 9 This attitude is explained owning to slip phenomenon the dimensionless

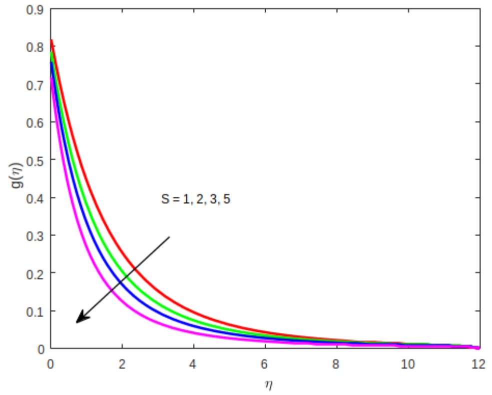

FIGURE 8. Influence of S on circumferential velocity

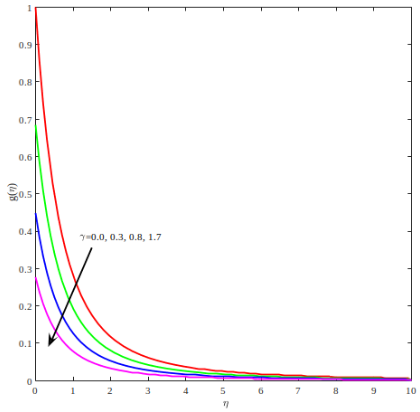

FIGURE 9. Influence of γ on circumferential velocity

velocity at the wall is not equivalent to stretchable velocity. Thus the dragging of stretchable wall cannot be completely transformed to the fluid flow and hence cause degeneration in liquid flow along azimuthal direction.

CONCLUSION

In this study numerical examination has been carried out on heat alongside mass transport of MHD Casson liquid over a vertically rotating cone. Discussion about the impact of different constitute parameters on tangential velocity $f'(\eta)$, circumferential velocity $g(\eta)$, temperature distribution $\theta(\eta)$, concentration $\phi(\eta)$.

(1) Slip parameter, mixed convection parameter, Dufour number, non-Newtonian fluid parameter, thermal biot number cause increase in Tangential velocity; The reverse behaviour is true in case of unsteady parameter, magnetic parameter.
(2) Circumferential velocity is boosted with raising values of magnetic parameter, non-Newtonian fluid parameter. The reverse attitude is detected in case of slip parameter, mixed convection parameter, unsteady parameter.

(3) Temperature profile is increased with improving values of Schmidt number, Eckert number, chemical reaction parameter, Dufour number, thermal biot number, concentration biot number, non-Newtonian liquid parameter. But it depreciates with growing values of Prandtl number, slip parameter, mixed convection parameter, unsteady parameter.

(4) Soret, thermal biot, concentration biot number, magnetic parameter have the tendency to intensify concentration profile. Reduction in concentration profile is observed with rising values of chemical reaction parameter, mixed convection parameter, unsteady parameter, Schmidt number, non-Newtonian liquid parameter.

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DEPARTMENT OF MATHEMATICS,
KONERU LAKSHMAIAH EDUCATION FOUNDATION,
VADDESWARAM, INDIA-522 502
Email address: vijji.maths@gmail.com

DEPARTMENT OF MATHEMATICS,
KONERU LAKSHMAIAH EDUCATION FOUNDATION,
VADDESWARAM, INDIA-522 502

DEPARTMENT OF MATHEMATICS,
ANURAG ENGINEERING COLLEGE,
ANANTHAGIRI, KODAD, INDIA-508 206