

SERIES DECOMPOSITION METHOD FOR ASYMMETRIC NONLINEAR OSCILLATIONS

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ABSTRACT. STSDM is considered to be an efficient method to solve a large number of problems in Engineering and Sciences. It gives divergent series solution which is similar to DTM. In order to get the periodic solution, the phenomenon of modified DTM is applied by inserting LT. Pade's Approximant and ILT. By observing a Standard Duffing Equation of Motion with symmetric oscillations, many researchers studied the competence of STSDM. This research article explores on the applicability of STSDM in getting the asymmetric oscillations of a Simple Helmholtz Equation of Motion which is nonlinear and substantiates its inconsistency nature in differentiating the asymmetrical oscillation behavior and the non-periodic nature of the models

1. INTRODUCTION

A ODE of the Duffing oscillator which is nonlinear and 2nd order is

$$\ddot{x} + \epsilon x + g(x) = G_0(t)$$
$$\ddot{x} = \frac{d^2 x}{dt^2}.$$

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$$\epsilon = \text{dampingFactor}.$$

$$G_0(t) = \text{ForcingFunctiondependingont}.$$

Restoring force function is $g(x)$ and is given by

$$(1.1) \quad g(x) = ax + bx^2 + cx^3 + d$$

When b is non-zero and/or d is nonzero (1.1) stands for asymmetric oscillations. If b and d are zeros (1.1) describes symmetric oscillations. In the case of positive values of c , is considered for the hardening type of the system while the -ve values of c , treat (1.1) as softening type of the system. STSDM is considered to be an efficient method to solve a large number of physical problems in Engineering and Sciences. ST reduces the difficulty in the integration of highly integral functions which are nonlinear. For the series expansion of the solution the rate of convergence is always high. The terms in DEq which are nonlinear are decomposed by the AP expression. Akinola et al. applied the STSDM to investigate the behavior of oscillations by merging the ST, Series Expansion and AP Expressions. This research article explores on the applicability of STSDM in getting the solution for asymmetric oscillations of a Simple Nonlinear Helmholtz Equation of Motion and concludes its incapability in differentiating the non-periodicity behavior and the asymmetric oscillation nature.

For further references see [1-27].

2. ANALYSIS

A Simple Nonlinear Helmholtz Equation of Motion is given by

$$(2.1) \quad \ddot{x} + ay + by^2 = 0$$

$$(2.2) \quad att = 0, x = B \text{ and } \dot{x} = 0$$

owing to non-odd restoring force function $g(x) = ax + bx^2$ asymmetric oscillations are received. The PHASE DIAGRAM for (2.1) and (2.2) are created by

$$(2.3) \quad (\ddot{x})^2 = (B - x)[a(B + x) + \frac{2}{3}b(B^2 + Bx + x^2)]$$

The singular points of (2.1) in plane from the zeros of are: origin and $(-\frac{a}{b}, 0)$. Interpreting the differential coefficient of with respect to x as $g'(x)$, one can get $g'(0) = a > 0$ and $g'(-\frac{a}{b}) = -a > 0$. So, origin is the centre and $(-\frac{a}{b}, 0)$

is a saddle point. The separatrix from (2.3) passes through the points $(-\frac{a}{b}, 0)$ and $(-\frac{a}{2b}, 0)$. Periodicity occurs only when the amplitude B lies in the interval $(-\frac{a}{2b}, \frac{a}{2b})$. Applying ST to (2.1) and (2.2), one can observe

$$Sx(t) = B - v^2 S[ax(t) + bx^2(t)]$$

Applying the IST to (2.3), one can see

$$(2.4) \quad x(t) = B - S^{-1}[v^2(ax(t) + bx^2(t))]$$

Considering the series solution $x(t) = \sum_{m=0}^{\infty} x_m(t)$ and putting in (2.4) as

$$(2.5) \quad \sum_{m=0}^{\infty} x_m(t) = B - S^{-1}[v^2 S(\sum_{m=0}^{\infty} B_m)].$$

The AP functions in (2.5) are

$$(2.6) \quad A_n = \frac{1}{m!} \frac{d^m}{d\theta^m} [\sum_{k=0}^{\infty} \theta^k ax_k + b(\sum_{k=0}^{\infty} \theta^k x_k)^2] |_{\theta=0} = ax_m + b \sum_{k=0}^{\infty} x_k x_{m-k}.$$

Inserting (2.6) in (2.5) and making a comparison imply

$$\begin{aligned} x_0(t) &= B \\ x_{m+1}(t) &= -S^{-1}[v^2 S(A_m)] m \geq 0 \\ x_1(t) &= -(aB + bB^2) \frac{t^2}{2} \\ x_2(t) &= (a + 2bB)(aB + bB^2) \frac{t^4}{24} \\ x_3(t) &= -[(a + 2bB)^2 + 6b(aB + bB^2)](aB + bB^2) \frac{t^6}{720}. \end{aligned}$$

For B=1, and the series solution of equations (2.1) and (2.2) are obtained is

$$x(t) = x_0(t) + x_1(t) + x_2(t) + x_3(t) + \dots$$

$$(2.7) \quad x(t) = 1 - 1.1 \frac{t^2}{2} + 1.32 \frac{t^4}{24} - 2.31 \frac{t^6}{720}.$$

The series solution (2.7) is incapable to present the periodicity. Inserting LT, PA and the ILT as in the MDTM [19], the solution of the problem is got as

$$(2.8) \quad x(t) = (\text{COSINE}(1.03944t))(0.0996529) + (\text{COSINE}(2.5922t))(0.003471)$$

Fig.1 depicts the comparison between (2.7) and (2.8) . (2.7) is diverging, while, (2.7) depicts behaviour of oscillation. In the same way, the STSDM solution for

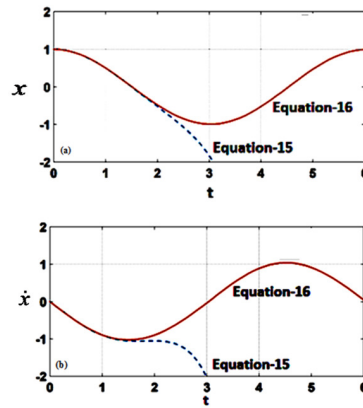


FIGURE 1. Comparison between (2.7) and (2.8)

$B=5$, and $a=1, b=0.1$ as

(2.9)

$$x(t) = (\text{COSINE}(1.186306t))(4.943389) + (\text{COSINE}(3.097204t))(0.056611)$$

The solution for , and is got as

$$(2.10) \quad x(t) = (\text{COSINE}(1.2205t))(5.9250) + (\text{COSINE}(3.2109t))(0.07496)$$

The periodic solution for and in (3) is thought as when the range of amplitude is in the interval $(-10, 5)$. If B does not fall in $(-10, 5)$ the solution is non-periodic. The +ve and -ve amplitudes of oscillations from (5) for $B=1$ are -1.0717 and 1, which gives the asymmetry of the PHASE DIAGRAM wrt x-axis whereas it is symmetric wrt x-axis. Fig.2 depicts the PHASE DIAGRAM created by (5) for the amplitude (B) values of 1, 5 and 6. The PHASE DIAGRAM is a closed boundary for . For the PHASE DIAGRAM represents a separatrix. The PHASE DIAGRAMS do not represent closed boundary for while (2.8) to (2.10) depicts the closed boundaries with almost symmetric oscillation behaviour (see Fig.3).

3. CONCLUSION

Helmholtz Equation of Motion's asymmetric oscillations have been examined initially by phase diagrams. Though STSDM is claimed to be an efficient method

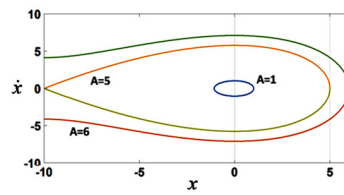


FIGURE 2. Phase diagrams from(2.3) for the amplitudes(B)

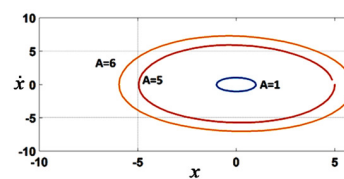


FIGURE 3. Phase diagrams from (2.8) to(2.10) for the amplitudes(B)

for oscillatory systems which are non-linear, it fails to recognise the asymmetric oscillations and the non-periodic nature of a Simple Helmholtz Equation of Motion. Though STSDM seems to be simple, the computations are tedious due to change of initial conditions with repetition of the procedure. Large discrepancy is observed in negative amplitudes of the asymmetrical oscillations. This simple Helmholtz equation serves as a benchmark to the newly developed mathematical/numerical techniques.

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8076 J. P. PRAVEEN, B. N. RAO, B. MAHABOOB, C. RAJIAH, Y. HARNATH, AND C. NARAYANA

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