ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.10, 7733–7739
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.10.4 Spec. Issue on ACMAMP-2020

FUZZY TOTALLY SOMEWHAT COMPLETELY IRRESOLUTE MAPPINGS

S. MUTHUKUMARAN, B. ANANDH, AND A.SWAMINATHAN[1]

ABSTRACT. This article is to study the concept of fuzzy totally somewhat completely irresolute and fuzzy totally somewhat completely continuous mappings. Further, some interesting properties of those mappings are given.

1. INTRODUCTION

After the introduction of fuzzy sets by L. A. Zadeh in his classical paper, C. L. Chang [3] first introduced the concept of fuzzy topology in 1968. The notions of fuzzy totally continuous functions and fuzzy totally semicontinuous functions were investigated by Anjan Mukherjee in [1]. G.Thangaraj and G.Balasubramanian introduced the concepts of somewhat fuzzy continuous and somewhat fuzzy semicontinuous functions . The notions of fuzzy totally somewhat continuous mapping and fuzzy totally somewhat semicontinuous mapping were investigated by A.Vadivel and A. Swaminathan.

In this paper we introduce the ideas of fuzzy totally somewhat completely irresolute and fuzzy totally somewhat completely continuous mappings and since some examples and relationships between these new classes with other classes of fuzzy functions are obtained. Finally, in section 3, some preservation of these mappings are discussed.

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 54A40, 03E72.

Key words and phrases. Fuzzy totally somewhat completely irresolute, Fuzzy totally somewhat completely continuous mapping, Fuzzy strongly compact, Fuzzy almost regular, Fuzzy almost normal, Fuzzy r-closed.

The known definitions which are used in this paper are seen in [1, 2, 7, 9].

2. Fuzzy totally somewhat completely irresolute mappings

In this section, we introduce fuzzy totally somewhat completely irresolute and fuzzy totally somewhat completely continuous mappings and we characterize these mappings.

Definition 2.1. *A mapping $f : X \to Y$ is called fuzzy totally somewhat completely irresolute if there exists a fuzzy regular clopen set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy semiopen set $\nu \neq 0_Y$ on Y.*

Definition 2.2. *A mapping $f : X \to Y$ is called fuzzy totally somewhat completely continuous if there exists a fuzzy regular clopen set $\mu \neq 0_X$ on X such that $\mu \leq f^{-1}(\nu) \neq 0_X$ for any fuzzy open set $\nu \neq 0_Y$ on Y.*

From the above definitions the following reverse implications are false:

(i) Every fuzzy totally completely irresolute is a fuzzy totally somewhat completely irresolute.

(ii) Every fuzzy totally somewhat completely irresolute is a fuzzy totally somewhat completely continuous.

(iii) Every fuzzy totally completely continuous is a fuzzy totally somewhat completely continuous.

Example 1. *Let $K_1(x)$ and $K_2(x)$ be fuzzy sets on $I = [0, 1]$ defined as follows:*

$$K_1(x) = \begin{cases} \frac{1}{4}, & 0 \leq x \leq \frac{1}{2}, \\ \frac{1-x}{2}, & \frac{1}{2} \leq x \leq 1. \end{cases}, \qquad K_2(x) = \begin{cases} \frac{1}{2}, & 0 \leq x \leq \frac{1}{2}, \\ 1-x, & \frac{1}{2} \leq x \leq 1. \end{cases}$$

$$K_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x-1, & \frac{1}{2} \leq x \leq 1 \end{cases}, \qquad K_4(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{4} \\ -4x+2, & \frac{1}{4} \leq x \leq \frac{1}{2} \\ 0, & \frac{1}{2} \leq x \leq 1 \end{cases}.$$

Let $\mathcal{J}_1 = \{0, K_1, K_1^c, K_2, 1\}$ be a fuzzy topology on I. Let $f : (I, \mathcal{J}_1) \to (I, \mathcal{J}_1)$ defined by $f(x) = x$ for each $x \in I$. It is observed that, for the fuzzy regular clopen set K_1 on $(I, \mathcal{J}_1)$, $K_1 \leq f^{-1}(K_1) = K_1$, $K_1 \leq f^{-1}(K_1^c) = K_1^c$ and $K_1 \leq f^{-1}(K_2) = K_2$. Therefore, f is fuzzy totally somewhat completely irresolute mapping. But *for a fuzzy semiopen set K_2 on $(I, \mathcal{J}_2)$, $f^{-1}(K_2) = K_2$ which is not fuzzy regular clopen set on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally completely irresolute mapping.*

Example 2. *Consider the fuzzy topologies $\mathcal{J}_3 = \{0, K_3, K_3^c, K_3 \vee K_3^c, K_3 \wedge K_3^c, 1\}$ and $\mathcal{J}_4 = \{0, K_3, K_4, K_3 \vee K_4, 1\}$ a mapping $f : (I, \mathcal{J}_3) \to (I, \mathcal{J}_4)$ defined by $f(x) = \frac{x}{2}$ for each $x \in I$. It is observed that, for the fuzzy open sets K_3, K_4 and $K_3 \vee K_4$ on $(I, \mathcal{J}_4)$, $f^{-1}(K_3) = 0$ and $K_3^c \leq f^{-1}(K_3 \vee K_4) = K_3^c = f^{-1}(K_4)$. Therefore, f is fuzzy totally somewhat completely continuous mapping. But for a fuzzy semiopen set K_5 on $(I, \mathcal{J}_4)$,*

$$f^{-1}(K_5) = K_5 f(\frac{x}{2}) = M(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3}(2x-1), & \frac{1}{2} \leq x \leq 1 \end{cases},$$

for each $x \in I$. Hence there is no non-zero fuzzy regular clopen set smaller than $f^{-1}(K_5) = K_5 f(\frac{x}{2}) = M(x)$ on $(I, \mathcal{J}_3)$. Therefore f is not fuzzy totally somewhat completely irresolute mapping.

Example 3. *As described in Example 3.1, consider the fuzzy topology $\mathcal{J}_3 = \{0, K_1, K_2, 1\}$ and a mapping $f : (I, \mathcal{J}_1) \to (I, \mathcal{J}_3)$ defined by $f(x) = x$ for each $x \in I$. It is observed that, for the fuzzy open sets K_1 and K_2 on $(I, \mathcal{J}_3)$, $K_1 \leq f^{-1}(K_1) = K_1$ and $K_1 \leq f^{-1}(K_2) = K_2$. Therefore, f is fuzzy totally somewhat completely continuous mapping. But for a fuzzy open set K_2 on $(I, \mathcal{J}_3)$, $f^{-1}(K_2) = K_2$ which is fuzzy regular open but not regular closed on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally completely continuous mapping.*

Theorem 2.1. *Let X_1 be product related to X_2 and let Y_1 be product related to Y_2. If $f_1 : X_1 \to Y_1$ and $f_2 : X_2 \to Y_2$ is fuzzy totally somewhat completely irresolute mappings, then the product $f_1 \times f_2 : X_1 \times X_2 \to Y_1 \times Y_2$ is also fuzzy totally somewhat completely irresolute mappings.*

Proof. Let $\lambda = \vee_{i,j}(\mu_i \times \nu_j)$ be a fuzzy semiopen set on $Y_1 \times Y_2$ where $\mu_i \neq 0_{Y_1}$ and $\nu_j \neq 0_{Y_2}$ are fuzzy semiopen sets on Y_1 and Y_2 respectively. Then $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j))$. Since f_1 is fuzzy totally somewhat completely irresolute, there exists a fuzzy regular clopen set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\mu_i) \neq 0_{X_1}$. And, since f_2 is fuzzy totally somewhat completely irresolute, there exists a fuzzy regular clopen set $\eta_j \neq 0_{X_2}$ such that $\eta_j \leq f_2^{-1}(\nu_j) \neq 0_{X_2}$. Now $\delta_i \times \eta_j \leq f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j) = (f_1 \times f_2)^{-1}(\mu_i \times \nu_j)$ and $\delta_i \times \eta_j \neq 0_{X_1} \times 0_{X_2}$ is a fuzzy regular clopen set on $X_1 \times X_2$. Hence $\bigvee_{i,j}(\delta_i \times \eta_j) \neq 0_{X_1} \times 0_{X_2}$ is a fuzzy regular clopen set on $X_1 \times X_2$ such that $\bigvee_{i,j}(\delta_i \times \eta_j) \leq \bigvee_{i,j}(f_1^{-1}(\mu_i) \times f_2^{-1}(\nu_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\mu_i \times \nu_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is fuzzy totally somewhat completely irresolute. □

Definition 2.3. *A fuzzy topological space $(X, \mathcal{F})$ is said to be fuzzy super connected [4] if it has no regular open subsets.*

Proposition 2.1. *If f is fuzzy totally somewhat fuzzy irresolute mapping from a fuzzy super connected space X into any fuzzy topological space Y, then Y is indiscrete fuzzy topological space.*

Proof. If possible suppose Y is not indiscrete. Then Y has a proper ($\neq 0$ and $\neq 1$) fuzzy open set λ (say). Then by hypothesis on $f, f^{-1}(\lambda)$ is a proper fuzzy regular clopen set of X, which is a contradiction to the assumption that X is fuzzy super connected. Hence the proposition. □

Definition 2.4. *Let $(X, \mathcal{F})$ be any fuzzy topological space. X is called fuzzy semi T_0 [5](fuzzy almost T_0 [8]) if and only if for any pair of distinct fuzzy points x_t and y_s, there is a fuzzy semiopen(regular open) set λ such that $x_t \in \lambda,\ y_s \notin \lambda$ or $x_t \notin \lambda$, $y_s \in \lambda$.*

Definition 2.5. *Let $(X, \mathcal{F})$ be any fuzzy topological space. (X, T) is called fuzzy semi T_2 [5] if for any pair of distinct fuzzy points x_t and y_s there exist fuzzy semi open sets λ and μ such that $x_t \in \lambda,\ y_s \in \mu$ and $sCl\lambda \leq 1 - sCl\mu$.*

Definition 2.6. *Let $(X, \mathcal{F})$ be any fuzzy topological space. (X, T) is called fuzzy almost T_2 [8] if for any pair of distinct fuzzy points x_t and y_s there exist fuzzy regular open sets λ and μ such that $x_t \in \lambda,\ y_s \in \mu$ and $rCl\lambda \leq 1 - rCl\mu$.*

Proposition 2.2. *Let $f : (X, \mathcal{F}) \to (Y, \mathcal{H})$ be an injective fuzzy totally somewhat completely irresolute mapping. If Y is fuzzy semi T_0, then X is fuzzy almost T_2.*

Proof. Let x_t and y_s be any two distinct fuzzy points of X. Then $f(x_t) \neq f(y_s)$. That is $(f(x))_t \neq (f(y))_s$. Since Y is fuzzy semi T_0, there exists a fuzzy semiopen set say $\lambda \neq 0$ in Y such that $f(x_t) \in \lambda$ and $f(y_s) \notin \lambda$. This means $x_t \in f^{-1}(\lambda)$ and $y_s \notin f^{-1}(\lambda)$. Since f is fuzzy totally somewhat completely irresolute, there exists fuzzy regular open set $\mu \neq 0$ in X such that $\mu \leq f^{-1}(\lambda)$ is fuzzy regular open set of X.

Also $x_t \in f^{-1}(\lambda)$ and $y_s \in 1 - f^{-1}(\lambda)$. Now put $\mu = 1 - f^{-1}(\lambda)$. Then $f^{-1}(\lambda) = rCl(\lambda)$ and $rCl(1 - f^{-1}(\lambda)) = rCl\mu = 1 - f^{-1}(\lambda)$ and $rCl(\lambda) = f^{-1}(\lambda) = 1 - rCl(\mu) \leq 1 - rCl(\mu)$. Hence the Proposition. □

Proposition 2.3. *Let $(X, \mathcal{F})$ be any fuzzy super connected space. Then every fuzzy totally somewhat completely irresolute mapping from a space X onto any fuzzy semi T_0-space Y is constant.*

Proof. Given that $(X, \mathcal{F})$ is fuzzy super connected. Suppose $f : X \to Y$ be any fuzzy totally somewhat completely irresolute mapping and we assume that Y is a fuzzy semi T_0 space. Then by Proposition 2.1, Y should be an indiscrete space. But an indiscrete fuzzy topological space containing two or more points cannot be fuzzy semi T_0. Therefore, Y must be singleton and this proves that f must be a constant function. □

Definition 2.7. *A fuzzy space X is called fuzzy almost regular [8] if for each fuzzy regular open set λ and each fuzzy point $x_t \notin \lambda$ there exist disjoint fuzzy open sets σ and δ such that $\lambda \leq \sigma$ and $x_t \in \delta$.*

Definition 2.8. *A fuzzy space X is called fuzzy almost normal [8] if for every pair of disjoint fuzzy sets sets λ_1 and λ_2 in X, where λ_1 is fuzzy open and λ_2 is fuzzy regular open, there exist disjoint fuzzy open sets σ and δ such that $\lambda_1 \leq \sigma$ and $\lambda_2 \leq \delta$.*

Theorem 2.2. *If $f : (X, \mathcal{F}) \to (Y, \mathcal{H})$ be an injective fuzzy totally somewhat completely continuous mapping and X is a fuzzy almost regular space, then Y is fuzzy regular.*

Proof. Let λ be a fuzzy open set on Y and a fuzzy point $y_\beta \notin \lambda$. Take $y_\beta = f(x_\alpha)$. Since f is fuzzy totally somewhat completely continuous such that $f^{-1}(\lambda)$ is a fuzzy regular open set of X. Take $\mu = f^{-1}(\lambda)$. We have $x_\alpha \notin \mu$. Since X is fuzzy almost regular, there exist disjoint fuzzy open sets η and δ in X such that $\mu \leq \eta$ and $x_\alpha \in \delta$. We obtain that $\lambda = f(\mu) \leq f(\eta)$ and $y_\beta = f(x_\alpha) \in \delta$ such that $f(\eta)$ and $f(\delta)$ are disjoint fuzzy semiopen sets of Y. This shows that Y is fuzzy regular. □

Theorem 2.3. *If $f : (X, \mathcal{F}) \to (Y, \mathcal{H})$ be an injective fuzzy totally somewhat completely continuous and X is a fuzzy almost normal space, then Y is fuzzy normal.*

Proof. Let λ_1 and λ_2 be disjoint fuzzy open set in Y. Since f is fuzzy totally somewhat completely continuous, $f^{-1}(\lambda_1)$ and $f^{-1}(\lambda_2)$ are fuzzy regular open sets in X. Let $\alpha = f^{-1}(\lambda_1)$ and $\beta = f^{-1}(\lambda_2)$. We have $\alpha \wedge \beta = 0$. Since X is fuzzy almost normal, there exist disjoint fuzzy open sets γ and δ such that $\alpha \leq \gamma$ and $\beta \leq \delta$. We obtain that $\lambda_1 = f(\alpha) \leq f(\gamma)$ and $\lambda_2 = f(\beta) \leq f(\delta)$ such that $f(\gamma)$ and $f(\delta)$ are disjoint fuzzy open sets. Thus, Y is fuzzy semi normal. □

3. Some preservation results

In this section by means of fuzzy totally somewhat completely irresolute and fuzzy totally somewhat completely continuous mapping preservation of some fuzzy topological structures are discussed.

Definition 3.1. *A fuzzy topological space $(X, \mathcal{F})$ is called*

(i) *fuzzy semi-compact [6] if every fuzzy semiopen cover has a finite subcover.*
(ii) *fuzzy r-closed [9] if every fuzzy regular clopen cover has a finite subcover.*

Theorem 3.1. *Every surjective fuzzy totally somewhat completely irresolute image of a fuzzy r-closed space is fuzzy semi-compact.*

Proof. Let $f : X \to Y$ be a fuzzy totally somewhat completely irresolute mapping of a fuzzy r-closed space $(X, \mathcal{F}_1)$ onto a fuzzy space $(Y, \mathcal{F}_2)$. Let $\{\beta_a : a \in A\}$ be any fuzzy semiopen cover of Y. Since f is fuzzy totally somewhat completely irresolute, $\{f^{-1}(\beta_a) : a \in A\}$ is a fuzzy regular clopen cover of X. Since X is a fuzzy r-closed space, then there exists a finite subfamily $\{f^{-1}(\beta_{a_i}) : i = 1, \ldots, n\}$ of $\{f^{-1}(\beta)\}$ which covers X. It implies that $\{\beta_{a_i} : i = 1, \ldots, n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers Y. Hence $f(X) = Y$ is fuzzy semi-compact. □

Theorem 3.2. *Every surjective fuzzy totally somewhat completely continuous image of a fuzzy r-closed space is fuzzy compact.*

Proof. Let $f : X \to Y$ be a fuzzy totally somewhat completely continuous mapping of a fuzzy r-closed space $(X, \mathcal{F}_1)$ onto a fuzzy space $(Y, \mathcal{F}_2)$. Let $\{\beta_a : a \in A\}$ be any fuzzy open cover of Y. Since f is fuzzy totally somewhat completely continuous, $\{f^{-1}(\beta_a) : a \in A\}$ is a fuzzy regular clopen cover of X. Since X is a fuzzy r-closed space, then there exists a finite subfamily $\{f^{-1}(\beta_{a_i}) : i = 1, \ldots, n\}$ of $\{f^{-1}(\beta)\}$ which covers X. It implies that $\{\beta_{a_i} : i = 1, \ldots, n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers Y. Hence $f(X) = Y$ is fuzzy compact. □

REFERENCES

[1] A. MUKUHERJEE: *Fuzzy totally continuous and totally semi-continuous functions*, Fuzzy Sets and Systems **107**(2) (1999) 227-230.

[2] K.K. AZAD: *On fuzzy semi-continuity, fuzzy almost continuity and fuzzy weakly continuity*, J. Math. Anal. Appl., **82**(1) (1981), 14-32.

[3] C.L. CHANG: *Fuzzy topological spaces*, J. Math. Anal. Appl., **24**(1) (1968), 182-190.

[4] U.V. FATTEH, D.S. BASSAN: *Fuzzy connectedness and its stronger forms*, J. Math. Anal. Appl., **111** (1985), 449-464.

[5] B. GHOSH: *Semi-continuous and semi-closed mappings and semi connrctednrss in fuzzy setting*, Fuzzy Sets and Systems, **35** (1990) 345-355.

[6] S. MALAKAR: *On fuzzy semi-irresolute and strongly irresolute functions*, Fuzzy Sets and Systems, **45** (1992), 239-244.

[7] S. MUTHUKUMARAN, B. ANANDH, A. SWAMINATHAN: *Fuzzy totally somewhat irresolute mappings* (submitted).

[8] M.K. SINGAL, N.RAJVANSHI: *Regular open sets in fuzzy topological spaces*, Fuzzy sets and systems, **50** (1992), 343-353.

[9] A. SWAMINATHAN, M. SANKARI: *Somewhat Fuzzy Completely Pre-irresolute and Somewhat Fuzzy Completely Continuous Mappings*, The Journal of Fuzzy Mathematics, **27**(3) (2019), 687-696.

H.H. RAJAH'S COLLEGE (AUTONOMOUS)
PUDUKKOTAI-622 001,INDIA.
Email address: muthumaths28@gmail.com

DEPARTMENT OF MATHEMATICS
H.H. RAJAH'S COLLEGE(AUTONOMOUS)
PUDUKKOTAI-622 001, INDIA.
Email address: drbalaanandh@gmail.com

DEPARTMENT OF MATHEMATICS
GOVERNMENT ARTS COLLEGE (AUTONOMOUS)
KUMBAKONAM-612 002, INDIA.
Email address: asnathanway@gmail.com