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MODELING, ANALYSIS AND SIMULATION OF MODIFIED SIX-PHASE STATOR WINDING INDUCTION MOTOR

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ABSTRACT. In this manuscript, an extended state space model based on stator currents and rotor cur-rents for dual-stator winding induction motor using reference frame theory is presented. The modified model is compared against existing six-phase stator winding induction motor model. By using Runge–Kutta method mathematical concept is used for solving non-linear ordinary differential six-phase stator winding induction motor model. Computer simulations are carried out for a 3.7 kW four-pole six-phase stator winding induction motor using MATLAB scientific environment. In realistic applications, current sensors are readily available on the six-phase stator winding induction motor. The six-phase stator winding induction motor model presented in this paper can be used directly without any further mathematical computation is the significant advantage of the proposed model.

1. INTRODUCTION

Induction machines are widely used electrical machines in industrial applications. Usually, three-phase induction motors are used in industries as readily available power supply is three-phase voltage supply systems. Nowadays, with the advancement in power inverter topologies encouraged a new range of multi-phase induction motors in industrial applications. Fault tolerant capability in case of phase damage is the critical advantage of the multi-phase machines.

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These machines are simple, robust and reliable in industrial drive applications [1-5]. In the recent years a great deal of interest on double-stator induction machines attracted many researchers. Researchers proposed DSWIM models in abc machine variables and $qd0$ reference frame theory. Most of the research in the literature is related to drive applications [6]. Recently, complex vector based DSWIM is presented in [7,8].

In this paper, a modified 11^{th} order six-phase stator induction motor model based on stator-currents, rotor-currents, rotor-speed and load-torque is proposed. The paper is organized as follows: Introduction in Section 1, followed by existing mathematical models of six-phase stator induction motor model in abc and $qd0$ reference frames in section 2. Modified state-space six-phase stator winding induction motor models using synchronously rotating reference frame are proposed in section 3. Simulation results and observations are presented in section 4. Finally, conclusions are placed in section 5.

2. Mathematical Model of Six-phase Stator Winding Induction Motor Model

In this section, closed-form representations of SPSWIM in abc machines are presented in this section.

2.1. Six-phase stator winding induction motor model based on stator-fluxes and rotor-fluxes in *abc*-machine variables.

$$\begin{aligned}(\dot{\lambda}^{s_1}_{abc})^T &= (V^{s_1}_{abc})^T - (R^{s_1}_{abc})^T \times (i^{s_1}_{abc})^T\\ (\dot{\lambda}^{s_2}_{abc})^T &= (V^{s_2}_{abc})^T - (R^{s_2}_{abc})^T \times (i^{s_2}_{abc})^T\\ (\dot{\lambda}^{r}_{abc})^T &= -(R^{r}_{abc})^T \times (i^{r}_{abc})^T, \dot{T}_l = 0\end{aligned} \tag{2.1}$$

$$\dot{\omega}_r = \frac{1}{J}\left(n_p\left(\begin{pmatrix}(i^{s_1}_{abc})^T\\(i^{s_2}_{abc})^T\end{pmatrix}\right)\frac{d}{d\Theta}\left[\begin{pmatrix}(L^{sr_1}_{abc})^T\\(L^{sr_2}_{abc})^T\end{pmatrix}\right](i^{r}_{abc})^T - T_l - B_l n_p \omega_r\right) \tag{2.2}$$

where,

$$\begin{pmatrix}\lambda^{s_1}_{abc}\\ \lambda^{s_2}_{abc}\\ \lambda^{r}_{abc}\end{pmatrix} = \begin{pmatrix}L^{s_1}_{abc} & L^{sr_1}_{abc} & L^{sr_1}_{abc}\\ L^{sr_2}_{abc} & L^{s_2}_{abc} & L^{sr_2}_{abc}\\ L^{rs_1}_{abc} & L^{rs_2}_{abc} & L^{rr}_{abc}\end{pmatrix}\begin{pmatrix}i^{s_1}_{abc}\\ i^{s_2}_{abc}\\ i^{r}_{abc}\end{pmatrix} \tag{2.3}$$

2.2. **Six-phase stator winding induction motor model based on stator-currents and rotor-currents in abc-machine variables.**

$$\begin{pmatrix} i^{s_1}_{abc} \\ i^{s_2}_{abc} \\ i^{r}_{abc} \end{pmatrix} = \begin{pmatrix} L^{s_1}_{abc} & L^{sr_1}_{abc} & L^{sr_1}_{abc} \\ L^{sr_2}_{abc} & L^{s_2}_{abc} & L^{sr_2}_{abc} \\ L^{rs_1}_{abc} & L^{rs_2}_{abc} & L^{rr}_{abc} \end{pmatrix}^{-1} \begin{pmatrix} V^{s_1}_{abc} \\ V^{s_2}_{abc} \\ 0 \end{pmatrix} - \begin{pmatrix} L^{s_1}_{abc} & L^{sr_1}_{abc} & L^{sr_1}_{abc} \\ L^{sr_2}_{abc} & L^{s_2}_{abc} & L^{sr_2}_{abc} \\ L^{rs_1}_{abc} & L^{rs_2}_{abc} & L^{rr}_{abc} \end{pmatrix}^{-1} \begin{pmatrix} 0 & R^{s_1}_{abc}\dot{L}^{sr_1}_{abc} & R^{s_2}_{abc}\dot{L}^{sr_2}_{abc} \\ R^{s_1}_{abc}\dot{L}^{sr_1}_{abc} & 0 & R^{s_2}_{abc}\dot{L}^{sr_2}_{abc} \\ R^{r}_{abc}\dot{L}^{rs_1}_{abc} & R^{r}_{abc}\dot{L}^{rs_2}_{abc} & 0 \end{pmatrix} \begin{pmatrix} i^{s_1}_{abc} \\ i^{s_2}_{abc} \\ i^{r}_{abc} \end{pmatrix} \tag{2.4}$$

$$\begin{aligned} \dot{\omega}_r &= \frac{1}{J}\left(n_p \left(\begin{pmatrix} (i^{s_1}_{abc})^T \\ (i^{s_2}_{abc})^T \end{pmatrix} \right) \frac{d}{d\Theta} \left[\begin{pmatrix} (L^{sr_1}_{abc})^T \\ (L^{sr_2}_{abc})^T \end{pmatrix} \right] (i^{r}_{abc})^T - T_l - B_l n_p \omega_r \right) \\ \dot{T}_l &= 0 \end{aligned} \tag{2.5}$$

2.3. **Six-phase stator winding induction motor model based on stator-fluxes and rotor-fluxes in synchronously rotating reference frame.**

$$\begin{aligned} \dot{\lambda}^{s_1}_q &= V^{s_1}_q - \omega_e \lambda^{s_1}_d - r_{s1} i^{s_1}_q \\ \dot{\lambda}^{s_1}_d &= V^{s_1}_d + \omega_e \lambda^{s_1}_q - r_{s1} i^{s_1}_d \end{aligned} \tag{2.6}$$

$$\begin{aligned} \dot{\lambda}^{s_2}_q &= V^{s_2}_q - \omega_e \lambda^{s_2}_d - r_{s2} i^{s_2}_q \\ \dot{\lambda}^{s_2}_d &= V^{s_2}_d + \omega_e \lambda^{s_2}_q - r_{s2} i^{s_2}_d \end{aligned} \tag{2.7}$$

$$\begin{aligned} \dot{\lambda}^{r}_q &= -(\omega_e - n_p\omega_r)\lambda^{r}_d - r_r i^{r}_q \\ \dot{\lambda}^{r}_d &= (\omega_e - n_p\omega_r)\lambda^{r}_q - r_r i^{r}_d \end{aligned} \tag{2.8}$$

$$\begin{aligned} \dot{\omega}_r &= \frac{1}{J}\left(\frac{3}{2} n_p \frac{L_m}{L_{ss1} + L_{ss2}} ((\lambda^{s_1}_q - \lambda^{s_2}_q) i^r_d - (\lambda^{s_1}_d - \lambda^{s_2}_d) i^r_q) - T_l - B_l n_p \omega_r \right) \\ \dot{T}_l &= 0 \end{aligned} \tag{2.9}$$

In SPSWIM model based on reference frame theory is presented above.

3. Modified Six-phase Stator Induction Motor Model

3.1. **Modified six-phase stator winding induction motor model based on stator-fluxes and rotor-fluxes in synchronously rotating reference frame.** Using reference frame theory, complete and accurate state–space models for six-phase stator winding induction motor are formulated in this section. Based on the reference frame theory transformations, and the SPSWIM model available in (2.6-2.9) is modified SPSWIM model based fluxes is presented in (3.2).

$$\begin{aligned}
T_{em} &= \frac{3}{2} n_p L_m \frac{(\lambda_q^{s_1} - \lambda_q^{s_2})\lambda_d^r - (\lambda_d^{s_1} - \lambda_d^{s_2})\lambda_q^r}{(L_{ss1} + L_{ss2})L_r + L_m^2} \\
T_{em} &= \frac{3}{2} n_p ((i_q^{s_1} - i_q^{s_2})\lambda_d^s - (i_d^{s_1} - i_d^{s_2})\lambda_q^s) \\
T_{em} &= \frac{3}{2} n_p L_m \frac{(\lambda_q^{s_1} - \lambda_q^{s_2}) i_d^r - (\lambda_d^{s_1} - \lambda_d^{s_2}) i_q^r}{(L_{ss1} + L_{ss2})} \\
T_{em} &= \frac{3}{2} n_p L_m ((i_q^{s_1} - i_q^{s_2}) i_d^r - (i_d^{s_1} - i_d^{s_2}) i_q^r) \\
T_{em} &= \frac{3}{2} n_p \frac{L_m}{L_r} ((i_q^{s_1} - i_q^{s_2})\lambda_d^r - (i_d^{s_1} - i_d^{s_2})\lambda_q^r) \\
T_{em} &= \frac{3}{2} n_p (\lambda_q^r i_d^r - \lambda_d^r i_q^r)
\end{aligned} \tag{3.1}$$

Electro-magnetic torque used in (2.6-2.9), (3.2) can used any one of the forms presented in (3.1).

$$\begin{aligned}
\dot{\lambda_q^{s1}} &= v_q^{s1} - \omega_e \lambda_d^{s1} - \frac{r_{s1}}{\Delta}(\alpha\lambda_q^{s1} - \gamma\lambda_q^{s2} - \beta\lambda_q^r) \\
\dot{\lambda_q^{s1}} &= v_q^{s1} - \omega_e \lambda_d^{s1} - \frac{r_{s1}}{\Delta}(\alpha\lambda_q^{s1} - \gamma\lambda_q^{s2} - \beta\lambda_q^r) \\
\dot{\lambda_0^{s1}} &= v_0^{s1} - \frac{r_{s1}}{L_{ls1}}\lambda_0^{s1} \\
\dot{\lambda_q^{s2}} &= v_q^{s2} - \omega_e \lambda_d^{s2} - \frac{r_{s2}}{\Delta}(\eta\lambda_q^{s1} - \gamma\lambda_q^{s2} - \rho\lambda_q^r) \\
\dot{\lambda_d^{s2}} &= v_d^{s2} - \omega_e \lambda_q^{s2} - \frac{r_{s2}}{\Delta}(\eta\lambda_d^{s1} - \gamma\lambda_d^{s2} - \rho\lambda_d^r) \\
\dot{\lambda_0^{s2}} &= v_0^{s1} - \frac{r_{s2}}{L_{ls2}}\lambda_0^{s2}
\end{aligned} \tag{3.2}$$

$$\begin{aligned}
\dot{\lambda_q^r} &= -(\omega_e - n_p\omega_r)\lambda_d^r - \frac{r_r}{\Delta}(\zeta\lambda_q^r - \beta\lambda_q^{s1} - \rho\lambda_q^{s2}) \\
\dot{\lambda_d^r} &= -(\omega_e - n_p\omega_r)\lambda_q^r - \frac{r_r}{\Delta}(\zeta\lambda_d^r - \beta\lambda_d^{s1} - \rho\lambda_d^{s2}) \\
\dot{\lambda_0^r} &= -\frac{r_r}{L_{lr}}\lambda_0^r \\
\dot{\omega_r} &= \frac{1}{J}(T_{em} - T_l - B_l n_p \omega_r) \\
\dot{T_l} &= 0
\end{aligned} \tag{3.3}$$

3.2. **Modified six-phase stator winding induction motor model based on stator currents and rotor currents in synchronously rotating reference frame.** The modified six-phase stator winding induction motor model based on stator currents, rotor currents, rotor speed and load torque is presented in this subsection.

(3.4)

$$
\begin{aligned}
i_q^{s1} =& \frac{\alpha}{\Delta}(V_q^{s1} - \omega_e(L_{ss1}i_d^{s1} + L_m i_d^{s1} + L_m i_d^r) - r_{s1}i_q^{s1}) - \frac{\gamma}{\Delta}(V_q^{s2} - \omega_e(L_{ss2}i_d^{s2} \\
&+ L_m i_d^{s2} + L_m i_d^r) - r_{s2}i_q^{s2}) - \frac{\beta}{\Delta}(-\omega_e - n_p\omega_r)(L_m i_d^{s1} + L_m i_d^{s2} + L_r i_d^r) - r_r i_q^r \\
i_d^{s1} =& \frac{\alpha}{\Delta}(V_d^{s1} - \omega_e(L_{ss1}i_q^{s1} + L_m i_q^{s1} + L_m i_q^r) - r_{s1}i_d^{s1}) - \frac{\gamma}{\Delta}(V_d^{s2} - \omega_e(L_{ss2}i_q^{s2} \\
&+ L_m i_q^{s2} + L_m i_q^r) - r_{s2}i_d^{s2}) - \frac{\beta}{\Delta}(-\omega_e - n_p\omega_r)(L_m i_q^{s1} + L_m i_q^{s2} + L_r i_q^r) - r_r i_d^r \\
i_0^{s1} =& v_0^{s1} - r_{s1}i_0^{s1} \\
i_q^{s2} =& \frac{\gamma}{\Delta}(V_q^{s1} - \omega_e(L_{ss1}i_d^{s1} + L_m i_d^{s1} + L_m i_d^r) - r_{s1}i_q^{s1}) - \frac{\eta}{\Delta}(V_q^{s2} - \omega_e(L_{ss2}i_d^{s2} \\
&+ L_m i_d^{s2} + L_m i_d^r) - r_{s2}i_q^{s2}) - \frac{\rho}{\Delta}(-\omega_e - n_p\omega_r)(L_m i_d^{s1} + L_m i_d^{s2} + L_r i_d^r) - r_r i_q^r \\
i_d^{s2} =& \frac{\gamma}{\Delta}(V_d^{s1} - \omega_e(L_{ss1}i_q^{s1} + L_m i_q^{s1} + L_m i_q^r) - r_{s1}i_d^{s1}) - \frac{\eta}{\Delta}(V_d^{s2} - \omega_e(L_{ss2}i_q^{s2} \\
&+ L_m i_q^{s2} + L_m i_q^r) - r_{s2}i_d^{s2}) - \frac{\rho}{\Delta}(-\omega_e - n_p\omega_r)(L_m i_q^{s1} + L_m i_q^{s2} + L_r i_q^r) - r_r i_d^r \\
i_0^{s2} =& v_0^{s2} - r_{s2}i_0^{s2}
\end{aligned}
$$

(3.5)

$$
\begin{aligned}
i_q^r =& \frac{\beta}{\Delta}(V_q^{s1} - \omega_e(L_{ss1}i_d^{s1} + L_m i_d^{s1} + L_m i_d^r) - r_{s1}i_q^{s1}) - \frac{\rho}{\Delta}(V_q^{s2} - \omega_e(L_{ss2}i_d^{s2} \\
&+ L_m i_d^{s2} + L_m i_d^r) - r_{s2}i_q^{s2}) - \frac{\zeta}{\Delta}(-\omega_e - n_p\omega_r)(L_m i_d^{s1} + L_m i_d^{s2} + L_r i_d^r) - r_r i_q^r \\
i_d^r =& \frac{\beta}{\Delta}(V_d^{s1} - \omega_e(L_{ss1}i_q^{s1} + L_m i_q^{s1} + L_m i_q^r) - r_{s1}i_d^{s1}) - \frac{\rho}{\Delta}(V_d^{s2} - \omega_e(L_{ss2}i_q^{s2} \\
&+ L_m i_q^{s2} + L_m i_q^r) - r_{s2}i_d^{s2}) - \frac{\zeta}{\Delta}(-\omega_e - n_p\omega_r)(L_m i_q^{s1} + L_m i_q^{s2} + L_r i_q^r) - r_r i_d^r \\
i_0^r =& v_0^r - r_r i_0^r \\
\dot{\omega}_r =& \frac{1}{J}(\frac{3}{2}n_p L_m((i_q^{s_1} - i_q^{s_2})i_d^r - (i_d^{s_1} - i_d^{s_2})i_q^r)) \\
\dot{T}_l =& 0
\end{aligned}
$$

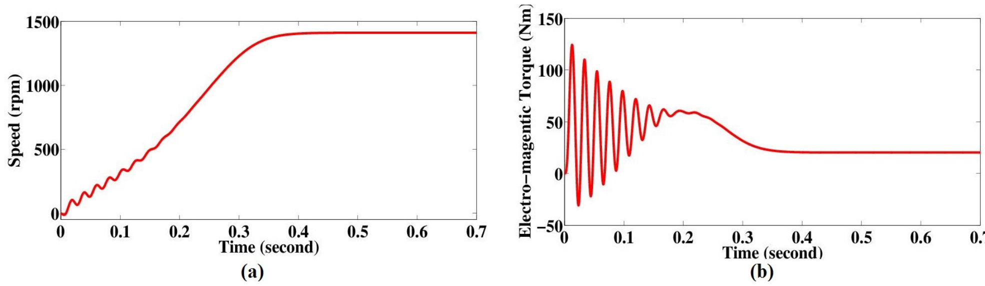

FIGURE 1. (a) Rotor speed in rpm, (b) Electromagnetic torque in N.m

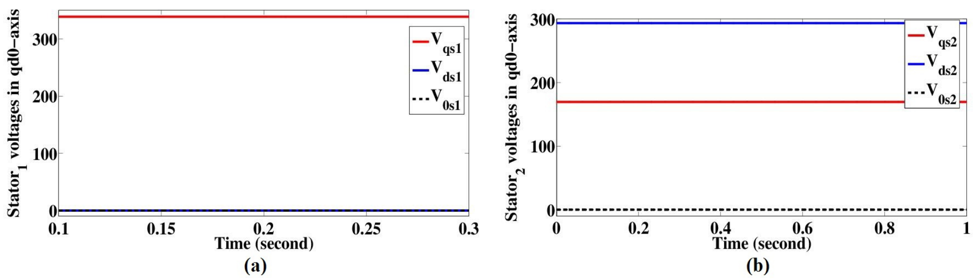

FIGURE 2. (a) Stator-1 line voltages in synchronous $qd0$-axis, (b) Stator-2 line voltages in synchronous $qd0$-axis

4. Simulation Results and Discussions

In this section, the proposed 11^{th} order SPSWIM models presented in section 3 are programmed using MATLAB software. The proposed models are verified with existing model presented in this section. The simulation results are presented in this section.

4.1. **SPSWIM model based on stator-currents and rotor-currents.**

4.1.1. *Case I: Stationary reference frame.* In the sub-section 4.1.1, SPSWIM model presented in (2.6-2.9) is supplied from balanced supply. For all the states of the SPSWIM model presented in (2.6-2.9) in stationary reference frame are presented in Fig. 1.

4.2. **SPSWIM model based on stator-currents and rotor-currents.**

4.2.1. *Case II: Synchronously rotating reference frame.* For all the states of the SPSWIM model presented in (3.4-3.5) synchronously rotating reference frame are presented in Fig. 2 to Fig. 4. The other simulation results are omitted

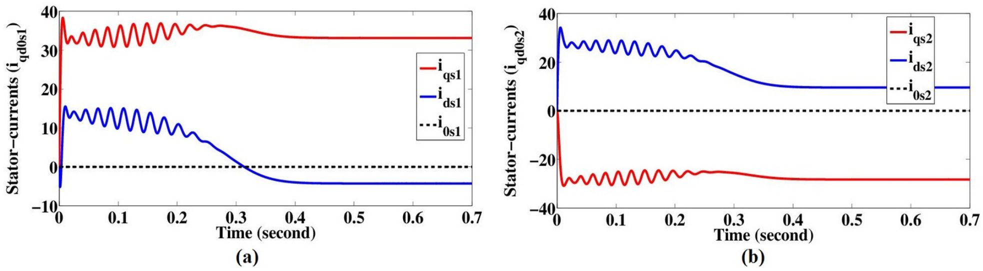

FIGURE 3. (a) Stator-currents for stator-1 in synchronous $qd0$-axis, (b) Stator-currents for stator-2 in synchronous $qd0$-axis

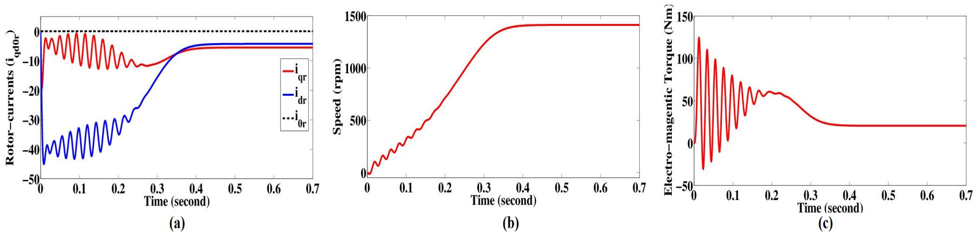

FIGURE 4. (a) Rotor-currents for rotor in synchronous $qd0$-axis, (b) Rotor speed in rpm, (c) Electromagnetic torque in N.m

due to conciseness. Table 1 and Table 2 shows the electrical specification of the machine and simulation running time.

5. CONCLUSIONS

In this paper, a modified extended state-space model based on stator currents and rotor currents in synchronously rotating reference frame is proposed. Usually, current sensors are placed in the SPSWIM drives. SPSWIM model based on fluxes model requires either flux sensors or additional mathematical computations to measure stator-currents. A modified 11^{th} order SPSWIM model based on stator-currents rotor currents, rotor-speed and load torque can directly measure six-phase stator-currents without any additional computational burden. The simulation results for the proposed model are compared with existing SPSWIM model. The simulation results are in very good agreement with each other. Non-invasiveness is critical advantage of the proposed SPSWIM model. At present, three-phase induction machines are used in electric vehicles. Due to additional

TABLE 1. Six-phase Stator Winding Induction Motor Parameters

Parameter	Symbol	Value	Unit
Rated power	P_{rated}	3.7	kW
Line voltage for Stator-1	V_l^1	415	V
Line voltage for Stator-2	V_l^2	415	V
Rated speed	N_{rpm}	1410	rpm
Load torque	T_l	20	N.m
Rated frequency	f_s	50	Hz
Number of pole-pairs	n_p	2	-
Stator winding resistance for Stator 1 and 2	r_{s1}=r_{s2}	2.283	Ω
Rotor winding resistance (Stator referred)	r_r	2.133	Ω
Stator leakage inductance	$L_{ls1} = L_{ls2}$	11.11	mH
Rotor leakage inductance (Stator referred)	L_{lr}	11.11	mH
Magnetizing inductance	$L_{ms1} = L_{ms2}$	146.7	mH
Stator self inductance	$L_{ss1} = L_{ss2}$	23.11	mH
Rotor inductance (Stator referred)	L_r	23.11	mH
Mutual-inductance	L_m	22.01	mH
Rotor inertia	J	0.06	$kg.m^2$
Friction coefficient	B_l	0.001	N.m/(rad/s)

TABLE 2. Time taken for simulations

State variables		Stationary	Synchronous
Stator fluxes and rotor fluxes	Time (Seconds)	190.39	196.98
Stator currents and rotor currents	Time (Seconds)	99.98	108.16

advantages of six-phase induction machines, three-phase induction machines may be replaced with multi-phase machines in future. The proposed model in this manuscript can be used for model-based fault detection of SPSWIMs.

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