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AN IMPROVED ASM METHOD FOR THE TRANSPORTATION PROBLEM

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ABSTRACT. Vogel's Approximation Method (VAM) is a method, which has been the most efficient solution procedure for more than sixty years, for obtaining an Initial Basic Feasible Solution (IBFS) for the transportation problems (TPs) as it provides a very good IBFS. Abdul Quddoos et al. developed a new method called ASM method (July 2012) and Revised Version of ASM method (June 2016) for obtaining best IBFS for TPs with minimum effort of mathematical calculations. While solving various TPs by the ASM method, we faced with the difficulty of identifying and selecting an appropriate zero-entry cell for allocation from a reduced cost matrix (RCM) when tie occurs at the situation where the total sum of all the row elements and column elements in the considered zero-entry cell from the RCM have the same magnitude. In that case we have found very simple tie breaking techniques to resolve this and proposed a new algorithm, called improved ASM method (IASM), which produces better IBFS than the best IBFS produced by the ASM method.

1. INTRODUCTION

Transportation problems have been broadly studied in Operations Research and Computer Science. They play a vital role in logistics and supply-chain management for reducing the distribution cost and improving the service. In 1941 Hitchcock [4] developed the basic transportation problem along with the constructive method of solution. In 1951, Dantzig [3] formulated the transportation

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problem as linear programming problem and also provided the solution method. During 1960s, quite few methods such as North West Corner (NWC) Method, Least Cost Method (LCM) and Vogel's Approximation Method (VAM) [5] have been established for finding the IBFS of TPs. In the recent years several methods have been projected by several researchers to find the optimal solution for TPs. directly. But no method is attaining optimal solution directly to all TPs. Among them, in July 2012, Abdul Quddoos et al. [2] proposed a new method, named ASM method, based on making allocations to zero-entry cell of reduced cost matrix, for finding an optimal solution directly for a wide range of TPs. In October 2012, Mohammad Kamrul Hasan [8] proposed that direct methods (including the ASM method) for finding optimal solution of a TP do not reflect optimal solution continuously. Murugesan [10] confessed and recognized the statement of Mohammad Kamrul Hasan by testing the ASM method for various benchmark problems. Meanwhile by doing further research, Abdul Quddoos et al. [1] encountered a few problems in which ASM method does not directly provide optimal solution to each and every problem, but provides a best IBFS, which is very close to optimal solution. One basic problem encountered was the unbalanced TP (UTP) in which an IBFS, not optimal but very close to optimal, was obtained. To overcome this problem, in July 2016, Abdul Quddoos et al. [1] presented a Revised Version of the ASM method, which provides optimal solution directly for most of the problems, and if not, it provides best IBFS. Hereafter, throughout this paper 'ASM Method' we mean the Revised Version of the ASM Method. Murugesan and Esakkiammal [10] established Abdul Quddoos et al.'s claim by testing 50 benchmark instances. Again by our further research we have observed that Kirca and Satir (1990) [6] first introduced the concept of Total Opportunity Cost Matrix (TOCM) and applied the Least Cost Method with some tie-breaking policies on the TOCM to determine the feasible solution of the TP. Mathirajan and Meenakshi (2004) [7] extended TOCM of Kirca and Satir by using VAM procedure on the TOCM (called the VAM-TOC, also same as the TOCM-VAM). According to the authors, this approach yielded the optimal solution and about 80 percentage of the time it yielded a solution very close to the optimal (0.5 percentage loss of optimality). Murugesan and Esakkiammal (2020) [10] introduced a new approach called TOCM-ASM which applies the ASM method on the TOCM of the given TP. To verify the performance of this approach, 50 classical benchmark instances from the literature have been tested. Simulation results validate that the proposed TOCM-ASM approach has produced optimal solution directly to 40 TPs. Though the ASM method produces optimal solution to a good number of TPs, Murugesan and Esakkiammal (2020) [9] have identified some challenging TPs for which the ASM method produces only near optimal solution. While solving various TPs by the ASM method, we faced with the difficulty of identifying and selecting an appropriate zero-entry cell for allocation from an RCM when tie occurs at different situations. Therefore, we have found and applied very simple tie breaking techniques, due to Mathirajan and Meenakshi (2004) [7] to resolve this and proposed a new algorithm, called IASM method to find best IBFS of TPs. As the proposed method produces better IBFS than that of by the ASM method, the name Improved ASM (IASM) is designed. The paper is organized as follows: In Section 2, step-by-step algorithm of the existing ASM method is presented. Section 3 describes the difficulty in selecting the zero-entry cell by the ASM method. In Section 4, the proposed algorithm for the IASM method is presented. In Section 5, two benchmark problems from unbalanced type and one from balanced type have been illustrated. Section 6 demonstrates the comparison of the results and discussion of the proposed IASM method. The observed advantages of the proposed IASM method over the existing ASM method are given in Section 7. Lastly, in Section 8 conclusions are drawn.

2. EXISTING ALGORITHM: THE ASM METHOD

A set of non-negative allocations is said to be an IBFS to a TP, if it satisfies each row as well as each column limitations. The IBFS is also known as a starting solution or initial solution of a TP. From 1958 itself, number of methods have been proposed in the literatures to find IBFS of a TP. The generated IBFS may or may not be optimal directly. But it is established that the recently developed ASM method provides best IBFS to a TP in the sense that which is either optimal directly or very close to the optimal solution. In the case of near optimal solution, it can be improved to reach an optimal solution in fewer numbers of iterations. In this section, we present the existing algorithm of the ASM method by Abdul Quddoos et al. [1]. Please refer the paper [1] or [9] for the algorithm.

3. DIFFICULTY IN SELECTING THE ZERO-ENTRY CELL BY THE ASM METHOD

While solving a TP by applying the existing ASM method, the operations of row minimum subtraction (RMS) and column minimum subtraction (CMS) are carried out successively to obtain a reduced cost matrix (RCM). Each and every row and column of an RCM will have at least one zero element (or cost or entry). The cells having zero elements are called zero-entry cells. The main concept of the ASM method is to identify an appropriate zero-entry cell in an RCM and allocating the maximum possible amount in that cell. Actually the allocations depend on the nature of the position of the zero-entry cells in an RCM. The second action of Step 6 in the ASM method states that, 'if tie occurs for some zeros in Step 5, choose the cell of that zero for breaking tie such that the sum of all the elements in the row and column is maximum. Supply maximum possible amount to that cell.' If tie occurs in this 'sum', by the ASM algorithm, one can select any zero-entry cell among the zero-entry cells having the same 'sum' magnitude. This situation motivated us to propose the IASM method. In the proposed IASM method, we have suggested very simple tie breaking techniques to break the tie in the same 'sum' magnitude situation as well as in the subsequent other tie occurring situations. This tie breaking technique ensures better IBFS than the best IBFS produced by the ASM method. In that way, one can easily solve any given TP using the IASM method. All the possible tie arising situations are resolved easily and are explained with three numerical examples in Section 5.

4. ALGORITHM FOR THE PROPOSED IASM METHOD

In this section, algorithm for the proposed IASM method for determining best IBFS of TPs has been proposed. The following are the sequence of steps involved in it:

(1) Checking the Balanced Condition. Construct a transportation table, if the given TP is in statement form. Check whether the problem is balanced or not. If the problem is balanced, go to Step 3; otherwise, go to Step 2.

(2) Conversion to Balanced TP. If the problem is not balanced, then anyone of the following two cases may arise:

a) If total supply exceeds total demand, introduce an additional dummy column to the transportation table to absorb the excess supply. The unit transportation cost for the cells in this dummy column is set to 'M', where $M > 0$ is a very large but finite positive quantity. Go to Step 3.

(OR)

b) If total demand exceeds total supply, introduce an additional dummy row to the transportation table to satisfy the excess demand. The unit transportation cost for the cells in this dummy row is set to 'M', where $M>0$ is a very large but finite positive quantity. Go to Step 4.

(3) Constructing the Reduced Cost Matrix (RCM).

a) Perform the Row Minimum Subtraction (RMS) Operation. Subtract the minimum cost from each of the costs of every row of the balanced TP. This will result in a resultant matrix.

b) Perform the Column Minimum Subtraction (CMS) Operation. Subtract the minimum cost from each of the costs of every column of the resultant matrix obtained in Step 3(a). Go to Step 5.

(4) Constructing the Reduced Cost Matrix (RCM).

a) Perform the Column Minimum Subtraction (CMS) Operation. Subtract the minimum cost from each of the costs of every column of the balanced TP. This will result in a resultant matrix.

b) Perform the Row Minimum Subtraction (RMS) Operation. Subtract the minimum cost from each of the costs of every row of the resultant matrix obtained in Step 4(a). Go to Step 5. /* The resultant matrix obtained in Step 3(b) or Step 4(b) is known as the reduced cost matrix (RCM). It is noted that there will be at least one zero entry in each row and in each column of an RCM. The cells having only zero entries in an RCM are called zero-entry cells.

(5) Making an Allocation by applying the Tie Breaking Techniques.

(i) List all the zero-entry cells (row-wise) from the obtained RCM.

(ii) For each such cell, count the total number of zeros (excluding the selected one) in its row and column. Now choose a zero-entry cell for which the number of zeros counted is the minimum and allocate the maximum possible allocation value to that cell.

(iii) If tie occurs in case of (ii), then make the allocation to that cell for which the total sum of all the elements in the corresponding row and column is the maximum.

(iv) Again, if tie occurs in case of (iii), then make the allocation to that cell for which maximum allocation value can be made.

(v) Yet again, if tie occurs in case of (iv), then make the allocation to that cell for which the sum of demand and supply in the original transportation table is maximum.

(vi) Over again, if tie occurs in case of (v) then make the allocation to that cell for which the i value (row number) is less in case of occurrence of tie in the same column [or the j value (column number) is less in case of occurrence of tie in the same row].

(vii) All over again, if tie occurs in case of (vi) then select the cell at random for allocation.

(6) Reducing the RCM. After performing Step 5, delete the row or column for further calculation where the supply from a given source is exhausted or the demand for a given destination is satisfied. [If we delete both the row and column where the supply from a given source is exhausted as well as the demand for a given destination is satisfied, then this will generate a degenerate solution. To get a non-degenerate solution, delete either the corresponding row only or the column only (but not both), and adjust the supply (demand) as zero, if column (row) is deleted]. Adjust the supply (demand), if column (row) is deleted.

(7) Checking the Reducibility of the Resultant Matrix. Check whether the resultant matrix obtained in Step 6 possesses at least one zero in each row and in each column. If so, go to Step 5 for making the next allocation; otherwise, go to Step 3 for constructing a further RCM.

(8) Repeat Steps 3 to 7 until and unless all the demands are satisfied and all the supplies are exhausted.

(9) Writing the allocation values. Write the allocations one by one row-wise.

(10) Computing the Total Transportation Cost. Finally, calculate the total transportation cost, which is the sum of the product of unit transportation cost (from the original TP) and the corresponding allocation value.

TABLE 1. The given TP

Sources	D1	D2	D3	Supply
S1	15	22	17	20
S2	11	17	16	25
S3	20	25	21	40
Demand	35	45	30	40

TABLE 2. IBFS generated due to the ASM method

Sources	D1	D2	D3	Supply
S1	15	22	$\angle 20$17	20
S2	$\angle 25$11	17	16	25
S3	$\angle 10$ 20	$\angle 20$25	$\angle 10$21	40
S4	M	$\angle 25$ M	M	25
Demand	35	45	30	40

5. NUMERICAL EXAMPLES

Suitable illustrative solution makes the readers to understand the proposed algorithm completely. Bearing in mind three examples are provided.

5.1. **EXAMPLE-1.** Consider the following cost minimizing TP with three sources and three destinations shown in Table 1:

5.1.1. *IBFS GENERATED BY THE ASM METHOD.*. The IBFS generated due to the ASM method is shown in Table 2.

Computing the Total Transportation Cost (TTC). $Z = (20\times 17)+(25\times 11)+(10\times 20)+(20\times 25)+(10\times 21) = 340+275+200+500+210 = 1525.$

Optimality Checking: By checking the condition for optimality by the MODI method, it is found that the generated IBFS by the ASM method is not an optimal one. The ASM-MODI process produces the optimal solution with Z = 1515 in a single iteration. The optimal solution is shown in Table 3.

Computing the TTC. $Z = (10\times 15)+(10\times 17)+(25\times 11)+(20\times 25)+(20\times 21) = 150+170+275+500+420 = 1515.$

Important Observation: It is observed that the existing ASM method has not produced the optimal solution directly for the given UTP. By applying the MODI

TABLE 3. Optimal solution derived from the IBFS of the ASM method

Sources	D1	D2	D3	Supply
S1	$\angle 10$15	22	$\angle 10$17	20
S2	$\angle 25$11	17	16	25
S3	20	$\angle 20$25	$\angle 20$21	40
S4	M	$\angle 25$ M	M	25
Demand	35	45	30	40

TABLE 4. IBFS (Optimal Solution) generated due to the proposed IASM method

Sources	D1	D2	D3	Supply
S1	$\angle 20$15	22	17	20
S2	$\angle 15$11	$\angle 10$ 17	16	25
S3	20	$\angle 10$25	$\angle 30$21	40
S4	M	$\angle 25$ M	M	25
Demand	35	45	30	40

method, the obtained IBFS has been improved towards optimality in a single iteration.

5.1.2. *SOLUTION BY THE PROPOSED IASM METHOD.*. The IBFS generated due to the proposed IASM method is shown in Table 4.

Computing the TTC. $Z = (20\times15)+(15\times11)+(10\times17)+(10\times25)+(30\times21) = 300 + 165 + 170 + 250 + 630 = 1515.$

Optimality Checking: By checking the condition for optimality by MODI method, it is found that the generated IBFS by the proposed IASM is an optimal one.

Important Observations:

1. It is observed that the proposed IASM method has produced the optimal solution directly whereas, the existing ASM method has produced only best IBFS to the given UTP.
2. Also, it is noted that the optimal solution generated directly by the proposed IASM method is different from that of produced by the existing ASM method via the MODI improvement method.
3. Further, the IBFS (optimal solution with Z = 1515) generated by the proposed

TABLE 5. The given TP- Destinations

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	9	12	9	6	9	10	5
S2	7	3	7	7	5	5	6
S3	6 4	9	11	3	11	2	5
S4	6 8	11	2	2	10	2	6
Demand	4	4	6	2	4	2	6

IASM method is better than the best IBFS (with Z = 1525) generated by the existing ASM method.

5.2. **EXAMPLE-2.** Consider the following cost minimization type UTP with four sources and six destinations, as given in Table 5.

5.2.1. *SOLUTION BY THE ASM METHOD..* The given UTP is solved by the existing ASM method, which produces an IBFS with the total transportation cost of Z = 77. By checking the condition for optimality by MODI method, it is found that the generated solution by the ASM method is not an optimal one. The ASM-MODI process produces the optimal solution with Z = 71 in two iterations.

5.2.2. *SOLUTION BY THE PROPOSED IASM METHOD..* The given UTP is solved by the proposed IASM method, which also produces an IBFS with the same total transportation cost of Z = 77. However, the IASM-MODI process produces the optimal solution with Z = 71 in a single iteration.

Important Observations:

1. It is observed that the IBFSs generated by the existing ASM method and the proposed IASM method are different, but with the same total transportation cost of Z = 77.
2. Also, it is noted that the optimal solution derived from the IBFS of the ASM method and that of derived from the IBFS of the proposed IASM method are different.
3. The IBFS generated by the ASM method takes two iterations to reach the optimal solution, whereas the IBFS generated by the proposed IASM method takes a single iteration only to reach the optimal solution.

TABLE 6. The given TP- Destinations

Sources	D1	D2	D3	D4	D5	D6	Supply
S1	5	3	7	3	8	5	30
S2	5	6	11	5	7	12	40
S3	2	7	3	4	8	2	20
S4	9	7	10	5	10	9	40
S5	5	3	7	3	7	5	30
Demand	10	40	40	20	10	40	10

5.3. **EXAMPLE 3.** Consider the following cost minimizing BTP with five sources and six destinations as shown in Table 6.

5.3.1. *SOLUTION BY THE EXISTING ASM METHOD.*. The given BTP is solved by the existing ASM method, which produces an IBFS with the total transportation cost of Z = 880. By checking the condition for optimality by MODI method, it is found that the generated solution by the ASM method is not an optimal one. The ASM-MODI process produces the optimal solution with Z = 860 in two iterations.

5.3.2. *SOLUTION BY THE PROPOSED IASM METHOD.*. The given BTP is a special TP which faces all the possible tie breaking techniques provided in the proposed IASM method. In particular, during the 1st, 7th, 9th and 10th allocations, the tie breaking techniques (ii) to (vii) provided in the proposed IASM method are applied and during the 2nd allocation the tie breaking techniques (ii) to (vi) are applied. Also, the tie breaking situation during the 7th allocation produces two different IBFSs (IBFS 1 and IBFS 2) with the same total transportation cost of Z = 880. By checking the condition for optimality by the MODI method, it is found that the generated two IBFSs by the IASM method are not optimal one. For the two IBFSs the IASM-MODI process produces the optimal solution with Z = 860 in two iterations.

Important Observations:

1. It is observed that the IBFSs generated by the existing ASM method and the proposed IASM method are different, but with the same total transportation cost of Z = 880.

2. Also, it is noted that the optimal solution derived from the IBFS of the ASM

TABLE 7. Comparison of the ASM and IASM methods for the Illustration-1

Problem Type	Problem Size	Method Applied	TTC by the IBFS	Minimum TTC by the OS	No. of iterations required to reach optimality
UTP	3 x 3	ASM	1525	1515	1
UTP	3 x 3	IASM	1515	1515	0

TABLE 8. Comparison of the ASM and IASM methods for the Illustration-2

Problem Type	Problem Size	Method Applied	TTC by the IBFS	Minimum TTC by the OS	No. of iterations required to reach optimality
UTP	4 x 6	ASM	77	71	2
UTP	4 x 6	IASM	77	71	1

TABLE 9. Comparison of the ASM and IASM methods for the Illustration-3

Problem Type	Problem Size	Method Applied	TTC by the IBFS	Minimum TTC by the OS	No. of iterations required to reach optimality
BTP	5 x 6	ASM	880	860	2
BTP	5 x 6	IASM	880	860	2

method and that of derived from the IBFS-2 of the proposed IASM method are different.

3. The IBFSs generated by the ASM method as well as the proposed IASM method takes two iterations to reach the optimal solution with Z = 860.

6. RESULTS AND DISCUSSION

To measure the effectiveness of the proposed IASM method, three benchmark problems have been tested and the results are compared with the results of the existing ASM method. The comparison of results by the two methods due to the three examples is shown in Table 7, Table 8 and Table 9 respectively.

Table 7 shows that the proposed IASM method produces the optimal solution (with the minimum total transportation cost of 1515) directly to the given TP shown in Example-1 whereas, the ASM method produces only best IBFS (with the total transportation cost of 1525). Also, it shows that in a single iteration the best IBFS by the ASM method can be improved to the optimal solution with the minimum total transportation cost of 1515.This means that the proposed IASM method produces better IBFS (here optimal solution) than the best IBFS produced by the ASM method. Table 8 shows that both the proposed IASM method as well as the ASM method generate best IBFSs with the same total transportation cost of 77 to the given TP shown in Example-2. Also, it shows that in two iterations the best IBFS by the ASM method can be improved to the optimal solution, whereas in a single iteration the best IBFS by the IASM method can be improved to the optimal solution with the minimum total transportation cost of 71. This means that the proposed IASM method produces better IBFS than the best IBFS produced by the ASM method. Table 9 shows that both the proposed IASM method as well as the ASM method produces best IBFSs with the same total transportation cost of 880 to the given TP shown in Illustration-3. Also, it shows that in two iterations the best IBFS by the ASM method as well as by the proposed IASM method can be improved to the optimal solution with the minimum total transportation cost of 860.

7. MAIN ADVANTAGES OF THE PROPOSED IASM METHOD OVER THE ASM METHOD

1. By the algorithm of the ASM method, one can choose any zero-entry cell for allocation when tie occurs among the zero-entry cells of a reduced cost matrix. But, in such a situation the algorithm of the proposed IASM method identifies and chooses the exact zero-entry cell for allocation using the tie breaking techniques provided. This may result in to produce different IBFSs by the ASM and IASM methods, but with the same total transportation cost.
2. In case of the different IBFSs produced by the ASM and the IASM methods, the solution produced by the proposed IASM method takes less number of iterations than that of by the ASM method to reach the optimal solution. The resulting optimal solutions by the two methods may also be different.
3. The proposed IASM method produces better IBFS than the best IBFS produced by the ASM method.

8. CONCLUSION

In this paper, while solving a TP by the existing ASM method, we have found the difficulty of identifying an appropriate zero-entry cell for allocation in an RCM in case of tie occurs among certain zero-entry cells. In the proposed IASM method, we have suggested very simple tie breaking techniques to break the tie in all possible tie occurring situations. In that way, one can easily solve any given TP using the IASM method. Each tie arising circumstances and resolving them exactly have been explained with suitable numerical examples. Also, it is established that the proposed IASM method produces better IBFS than the best IBFS produced by the existing ASM method.

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