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ESTIMATION OF BIT ERROR RATE IN OFDM SYSTEM OVER FADING CHANNELS

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ABSTRACT. In this paper, flat fading channel accompanied with Additive Gaussian White Noise (AWGN) based Orthogonal Frequency Division Multiplexing (OFDM) system model is developed. Matlab is used for the simulation. BER analysis is done initially in OFDM system considering the number of sub-carriers as 'N' using QPSK, BPSK, 8-PSK and 16-PSK. Analysis is carried out with the number of sub-carriers (N=64) with Ricean and Rayleigh fading. Due to multipath, phase errors occur in the channel which causes distorted result at the output of the receiver. RMS phase errors are calculated in OFDM system.

1. INTRODUCTION

A model of multi-carrier modulation is known as OFDM [1] where each and every sub carrier is chosen in such a way that they are orthogonal to the other sub carriers. It is fairly arbitrary for the number of sub channels transmitted with certain broad constraints, but practically, the subchannels tend to be extremely countless and close to each other. For example, in 802.11 wireless LAN the number of carriers is 48 while it is as high as 6000 subcarriers for Digital Video Broadcast (DVB) [2].At the edge of the assigned spectrum of the RECT pulse it is allowed that the rising and falling edges are softer (Raised cosine) in the IEEE Standards 802.11a.This helps the spectrum by without affecting the

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data transmissions. The advantages of the OFDM when compared to the single carrier system are observed as better multipath effect immunity ,relaxed timing acquisition constraints and simpler channel equalization [3]. But it is more prone to offset of local frequency and front-end non-linearity of radio [4]. Orthogonal frequencies are used in OFDM systems. Hence overlapping spectrum with Neighbouring frequencies can be used. This results in the systematic usage of Bandwidth. Therefore higher data rate can be provided for the same Band width by OFDM. The degradations experienced by many of these transmission systems are large attenuation ,noise, multipath, non- linearity ,time variation, interference, and finite transmit power.The recently gained popular technique is Multi-carrier modulation and it is a physical-layer technique which gained popularity because of its robustness in dealing with these damages. OFDM is a notable form of multi carrier modulation to produce waveform that are mutually orthogonal, which uses DSP algorithms such as Inverse Fast Fourier Transform (IFFT) for achieving it.

After many years of research OFDM is presently being widely used in high-speed digital communications .Due to the latest advancements in VLSI and DSP technologies, the barrier of OFDM applications will be eliminated here after [5].The requirement of arrays of sinusoidal generators and coherent demodulation in parallel data systems are eliminated by FFT algorithms and it shows the implementation technology value effective. In recent years, lot of interest has been gained by OFDM in diverse digital communication applications. In this work, OFDM system model is developed over flat fading channel with additive Gaussian white noise (AWGN) using Matlab 7.2. In OFDM system, Initially BER analysis is carried out with the number of subcarriers 'N' using QPSK ,BPSK, 16-PSK and 8-PSK.The Orthogonal frequency-division multiplexing (OFDM) has the advantage of averaging fades,when the symbol duration is longer than the length of the fades [6]. Because of this , the OFDM symbols are injured partly under fading. This can be obtained by adopting subcarriers at a larger number, and increasing the system complexity, but it resulted in a fair peak-to-average power ratio. By careful adjustment of the system parameters, the system performance can be improved by adjusting the number of subcarriers and the bit rate under certain fading.Analysis is carried out with the number of subcarriers (N=64) with Rayleigh and Ricean fading. Due to multipath , phase errors occur in the channel ,which causes distorted result at the output of the receiver.Therefore, a need is raised to calculate BER due to phase errors using BPSK which has less complexity than other modulation techniques in OFDM system. Section 2 presents an overview of Mathematical model of OFDM performance. Section 3 presents the performance of above model in terms of BER and SNR. Finally, section 4 provides concluding remarks.

2. Mathmatical Model of OFDM Performance

At the subcarriers, consider the input data sequence with a symbol rate, where N is the Number of Subcarriers. Considering cyclic prefix at the transmitter with sufficient Guard interval by padding zeros. The transmission modulation method is phase shift keying and Multi path fading is used and the channel noise considered is Gaussian noise. Each subcarrier is modulated using M-ary PSK, therefore, are selected from the sets of. Under time variant multipath fading channel, the demodulated signal in the frequency domain is given by [7–9]. Due to noise in the channel, the transmitted signal may develop a phase error and magnitude error at the received signal which leads a bit error at the output of the system. Initially magnitude error is considered in the channel and estimates its BER over Ricean and Rayleigh fading. Later the magnitude error is constant and random phase error is considered in the channel to estimates its BER. The received signal y in fading channel bears the form as follows.

$$y = hx + n. \tag{2.1}$$

In flat fading environment, where 'y' is the received symbol , 'h' is complex scaling factor for Rayleigh or Ricean multipath channel, 'x' is the transmitted symbol (taking values +1's and -1's) and 'n' is the Additive White Gaussian Noise (AWGN) Each transmitted symbol gets multiplied by a randomly varying complex number 'h' and the channel is randomly varying with time. Let 'h' be modelled as Rayleigh channel or Ricean channel in the system, the real and imaginary parts are taken as zero mean Gaussian distributed and unity variance. For Ricean channel, Rice factor is taken as k=3 to estimate the BER. For the computation of BER in AWGN, the probability of error is computed for transmission of either +1 or -1 for a given bit energy to noise ratio $\frac{E_b}{N_0}$ by integrating the tail of Gaussian probability density function. The bit error rate is given as follows.

$$P_b = \frac{1}{2} erfc\left(\sqrt{\frac{E_b}{N_0}}\right). \tag{2.2}$$

Nevertheless, in the existence of channel 'h', the productive bit energy to noise ratio is $|h^2|\frac{E_b}{N_0}$. Hence, for a given value of 'h', the bit error probability is given as follows.

$$P_{bjh} = \frac{1}{2} erfc\left(\sqrt{|h^2|\frac{E_b}{N_0}}\right) = \frac{1}{2} erfc\sqrt{k}, \tag{2.3}$$

where, $k = |h^2|\frac{E_b}{N_0}$. To realize the error probability of the over all random values of $|h^2|$, the conditional probability density function P_{bjh} is evaluated over the probability density function. Using chi-square random variable the probability density function of k [9] is as follows.

$$P_k = \frac{1}{\frac{E_b}{N_0}} e^{\frac{-k}{\frac{E_b}{N_0}}}, k \geq 0. \tag{2.4}$$

So the error probability is as follows.

$$P_b = \int_0^\infty \frac{1}{2} erfc(\sqrt{k}) P(k) dk. \tag{2.5}$$

2.1. **BER verses Channel Noise.** The transmitted signal has amplitude of one and fixed magnitude phasor, the phase is correlate with the data to be transmitted. For the transmitted signal the noise can be considered as the random vector added to it. The vector sum of the transmitted signal and the noise will be the received vector. The relative phase angle of the noise vector, and the magnitude of the noise are the contribution factors for the magnitude of the phase error. A constant magnitude vector equal to its RMS magnitude and a random phase angle is assumed to be the noise. The effect of noise on received phase angle is the received phase error given as θ. Assume that the transmitted signal is as follows.

$$x = 1 + Lcos\phi, \tag{2.6}$$

where 'L' is the noise vector length with phase angle ϕ then the signal received is as follows.

$$y = Lsin\phi \tag{2.7}$$

$$\theta = tan^{-1}\left(\frac{Lsin\phi}{1 + Lcos\phi}\right). \tag{2.8}$$

The noise level and the relative amplitude of the received signal are determined by the signal to noise ratio. Since the amplitude of signal is scaled to 1, the noise amplitude is as follows.

$$L = \frac{1}{SNR}. \tag{2.9}$$

The amplitude of the signals must be scaled correctly since the SNR is based on it. By substituting Eq. 2.9 in Eq. 2.8 then,

$$\theta = tan^{-1}\left(\left(\frac{1}{SNR}\right)\frac{sin\phi}{1 + \frac{1}{SNR}cos\phi}\right) \tag{2.10}$$

$$\theta = tan^{-1}\left(\frac{sin\phi}{SNR + cos\phi}\right). \tag{2.11}$$

Any phase angle can be taken by the noise signal. So it is required to obtain the RMS phase error and the average phase error (with an assumption that always positive value for the noise phase angle). By integrating ϕ over a half circle (i.e ϕ varies from 0 to π) the average phase angle can be found out.

$$Average\theta_{rms} = \frac{1}{\pi}\int_0^{\pi} tan^{-1}\left(\left(\frac{1}{SNR}\right)\frac{sin\phi}{1 + \frac{1}{SNR}cos\phi}\right)d\phi \tag{2.12}$$

$$\theta_{rms} = \frac{1}{\pi}\int_0^{\pi} tan^{-1}\left(\frac{sin\phi}{SNR + cos\phi}\right)d\phi. \tag{2.13}$$

2.2. **Allowable Maximum Phase Angle.** On the received word, before an error occurs, let us consider be the allowed maximum phase error on the received symbol. Table 1 shows the Modulation Technique and their Maximum allowed Phase error. Standard deviation 'SD' is calculated as follows.

$$SD = \frac{\phi_{max}}{\theta_{rms}}. \tag{2.14}$$

TABLE 1. Modulation technique & maximum allowed phase error

Modulation technique	Maximum phase error allowed in degrees
BPSK	90
QPSK	45
8-PSK	22.5
16-PSK	11.25

3. SIMULATION RESULTS

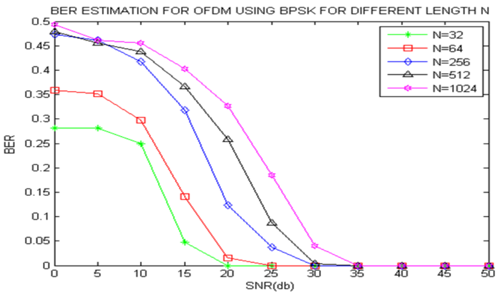
For high performance wireless telecommunications, OFDM appears to be a suitable Modulation technique under the current research. OFDM signals are carried out Rayleigh, Ricean faded signal with zero mean and unity variance to understand the effect of channel fading and obtain BER with different SNR using Matlab 7.2. Fig. 1(A) shows that number of sub carriers 'N' taken from 32 to 1024 using BPSK modulation with channel noise as white Gaussian in OFDM system. It is observed that for larger values of N, the bit error rate (BER) is high.

Fig. 1(B) shows that number of sub carriers 'N' taken from 32 to 1024 using QPSK modulation with channel noise as white Gaussian in OFDM system. The bit error rate (BER) is more for larger values of N.

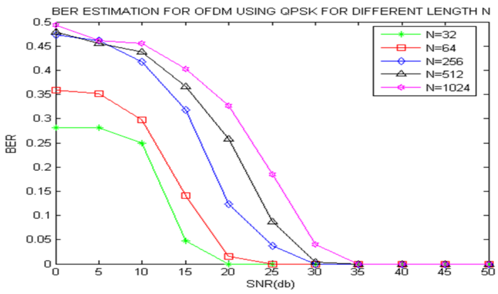
Fig. 1(C-D) shows that number of sub carriers 'N' taken from 32 to 1024 using 8-PSK and 16-PSK modulation with channel noise as AWGN in OFDM system. The bit error rate (BER) is more for larger values of N [10].

Fig. 2 shows that number of subcarriers 'N' 64 and modulation used is M-ary PSK with channel noise in OFDM system as AWGN. For BPSK the bit error rate (BER) is less.

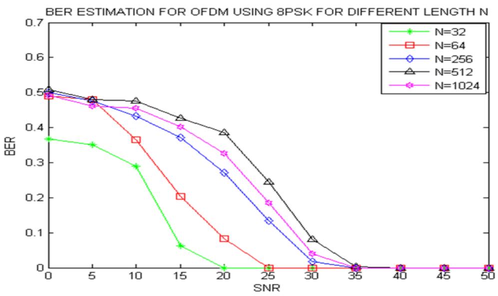
Fig. 3 shows the performance of OFDM in terms of BER verses SNR. As the number of sub carriers 'N' is taken from 32 to 1024 using BPSK modulation with channel noise is AWGN and multipath fading is Ricean with Rice factor K=3 . For different signal to noise ratios (SNR). the bit error rate (BER) is more for larger values of N. In Fig. 4, the number of subcarriers 'N' is taken as 64, using different M-ary PSK modulations for M=2,4,8,16 with channel noise is AWGN and multipath fading is Ricean with Rice factor K=3 in OFDM . for different signal to noise ratios(SNR). For different signal to noise ratios (SNR),the bit error rate (BER) is more for larger values of 'M'.



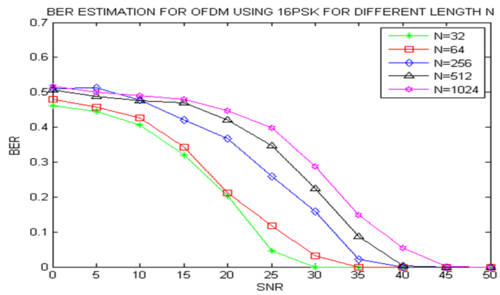
(A) BPSK modulation



(B) QPSK modulation



(C) 8-PSK modulation



(D) 16-PSK modulation

FIGURE 1. Bit error rate of OFDM for subcarriers ‘N’

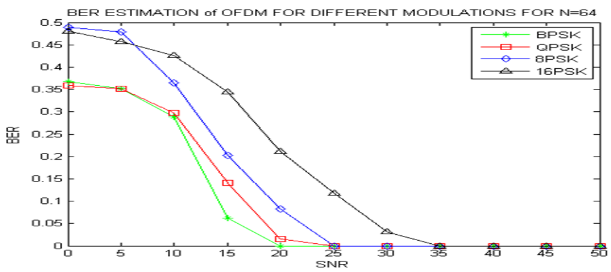


FIGURE 2. BER of OFDM using M-ary PSK modulation for N

In Fig. 5, the number of subcarriers ‘N’ is taken as 64, using different M-ary PSK modulations for M=2,4,8,16 with channel noise is AWGN and multipath fading is Rayleigh in OFDM . For different signal to noise ratios (SNR) the bit error rate (BER) is more for larger values of ‘M’.

Fig. 6 shows the performance of OFDM in terms of BER verses SNR. As the number of sub carriers ‘N’ is taken from 32 to 1024 using BPSK modulation with channel noise is AWGN and multipath fading is Rayleigh with zero mean and

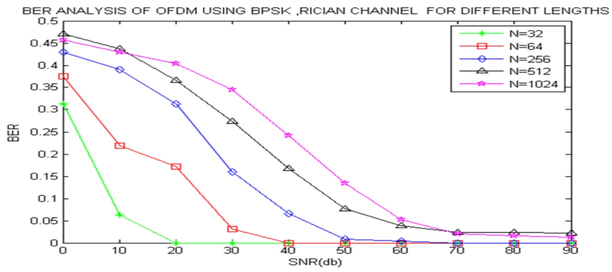


FIGURE 3. OFDM Bit error rate for N using Ricean channel

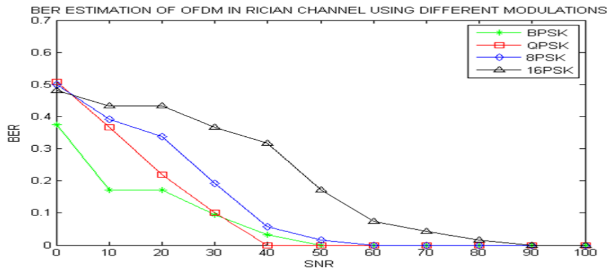


FIGURE 4. OFDM Bit error rate using M-ary PSK modulation for N=64

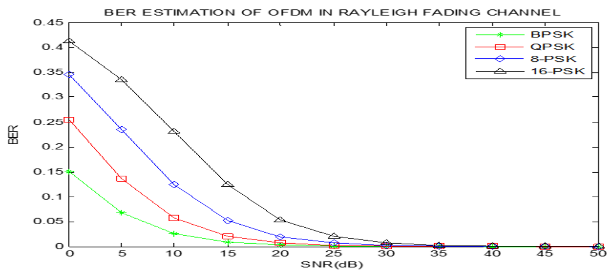


FIGURE 5. Bit error rate of OFDM using M-ary PSK for N=64

variance one. For different signal to noise ratios (SNR),the bit error rate (BER) is more for larger values of N.

The performance of OFDM system using phase shift keying is shown in Table 2. It shows the RMS phase error due to noise in the channel. As SNR increases phase error decreases. OFDM system and Bit Error Rate (BER) is calculated based on the signal to noise ratio (SNR) of the channel and BPSK phase modulation used .This is obtained by calculating the expected RMS phase error (θ_{rms}) at the receiver. Table 3 presents the expected Bit Error Rate for various noise levels.

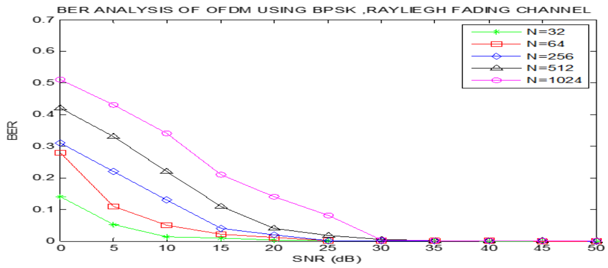


FIGURE 6. Bit error rate of OFDM for N using Rayleigh channel

TABLE 2. RMS phase error verses SNR (dB)

SNR (dB)	RMS Phase Error, θ_{rms} (degrees)
0	63.63
10	16.5
20	5.164
30	5.164
40	0.5158
50	0.1631

TABLE 3. RMS phase error verses SNR (dB)

SD	BER
0	1.000000
1.0	0.317311
2.0	0.045500
3.0	0.002700
4.0	34e-05
5.0	74e-07

4. CONCLUSION

In this chapter, the Bit error rate (BER) performance is evaluated in OFDM system using BPSK, QPSK, 8-PSK and 16-PSK modulations with number of sub-carriers N is taken as 32, 64, 256, 512 and 1024 and channel noise is AWGN. Graphical results shows that BER is less for BPSK with N=32. The same analysis is carried out for OFDM system using Ricean and Rayliegh fading distribution with zero mean and unity variance and channel is AWGN. From the results it is observed that for BPSK with N=64, the bit error rate is zero with SNR> 40 dB. It is also assumed a constant magnitude of noise and a random phase angle in the channel, the RMS phase errors are decreased by increasing the SNR in the system. The bit error rate is estimated using BPSK with different RMS phase errors occur in the channel.

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