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JORDAN GENERALIZED DERIVATIONS IN GAMMA NEARRINGS

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ABSTRACT. This research article explores on Jordan generalized derivations in Gamma nearrings. The main objective of this talk is to prove one most important result in Gamma nearings namely: let N be a two torsion free Γ-nearring. If N has two elements p and q so that for any $\alpha \in \Gamma, r\gamma y\gamma[p,q]_\alpha = 0$ or $[p,q]_\alpha\gamma y\gamma r = 0$ implies $r = 0$ for all $y \in N, \gamma \in \Gamma$ then every Jordan generalized (σ,τ) derivation on N is a generalized (σ,τ) derivation. In order to present an innovative proof to this result four prerequisites in terms of lemmas are proposed. Moreover elegant proofs for these three lemmas have been depicted. In this discourse we crave to present some fundamental characteristic properties of Gamma nearrings and an evolution in the conjecture of Gamma nearrings. The innovatory proofs of three lemmas proposed here ensure a way for young researchers.

1. INTRODUCTION

Hvala [1] is the first researcher to introduce the concept of generalized derivations in rings and derived a number of significant results in classical rings. Thammam El-Sayiad [2] and Daif investigated the generalized derivations in semiprime rings. Aydin [3] investigated the phenomenon of generalized derivations of prime rings.Ceven and Ozturk [4] studied the concept of Jordan generalized derivations in Gamma rings. Ali and Choudary [5] proposed some

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ideas concerning with generalized derivations of semiprime rings consider a Γ-nearring N with the (σ,τ) Jordan generalized derivation and if p,q,r $\in$ N a two torsion free Γ-nearring, then

(i) $\delta_a(p,q)\alpha[p,q]_\alpha. = 0(when[p,q]_\alpha = p\alpha q - q\alpha p)$
(ii) $\delta_\alpha(p,q)\beta y\beta[p,q]_\alpha = 0$ for any $y \in N$.

By using these, one can prove that a two torsion free Γ-nearring has two elements p and q so that for any $\alpha \in \Gamma$ we have $r\gamma y\gamma[p,q]_\alpha = 0$ or $[p,q]_\alpha\gamma y\gamma r = 0$ implies r=0 for all $y \in N, \gamma \in \Gamma$ then every Jordan generalized (σ,τ) derivation on N is a generalized (σ,τ) derivation.

Let N be a Γ-nearring and s:N$\rightarrow$N be an additive map.Then s is called a derivation if $s(y\alpha z) = s(y)\alpha z + y\alpha s(z), y,z \in N, \alpha \in \Gamma$ and s is called a Jordan derivation, if $s(y\alpha y) = s(y)\alpha y + y\alpha s(y), y \in N, \alpha \in \Gamma$.

Let N be a Γ-nearring and g: N$\rightarrow$N be an additive map. Then g is called a generalized (σ,τ) derivation if there exists a derivation s: N$\rightarrow$N such that $g(y\alpha y) = g(y)\alpha\sigma(z) + \tau(y)\alpha d(z)$, for all $y,z \in N, \alpha \in \Gamma$ Finally, g is called a Jordan generalized derivation, if there exists a derivation s: N$\rightarrow$N such that $g(y^2) = g(y)\alpha\sigma(y) + \tau(y)\alpha s(y)$, for all $y,z \in N, \alpha \in \Gamma$.

Lemma 1.1. *Let N be a Γ-nearring and if p, q, r $\in$ M and $\alpha,\beta \in \Gamma$. Then*

(i) $g(p\alpha q+q\alpha p) = g(p)\alpha\sigma(p)+g(p)\alpha\sigma(q)+g(q)\alpha\sigma(p)+g(q)\alpha\sigma(q)+\tau(p)\alpha\sigma(p)+\tau(p)\alpha\sigma(q)\tau(q)\alpha\sigma(p) + \tau(q)\alpha\sigma(p)$.
(ii) $g(p\alpha q\beta p+q\beta p\alpha p) = g(p)\alpha\sigma(q)\beta p+\tau(p)\beta s(q)\alpha p+p\alpha q\beta s(p)+g(q)\beta p\alpha\sigma(p)+\tau(q)\beta p\alpha s(q) + q\alpha q\beta s(p)$.
(iii) *In particular,* $g(p\alpha q\alpha p) = p\alpha g(q)\alpha\sigma(p) + p\alpha\tau(q)\alpha s(p) + s(p)\alpha q\alpha p$.
(iv) $g(p\alpha q\alpha r + r\alpha q\alpha p) = g(p)\alpha q\alpha r + g(r)\alpha q\alpha\sigma(p) + p\alpha s(q)\tau(r) + g(r)\alpha q\alpha p + g(p)\alpha q\alpha\sigma(r) + r\alpha s(q)\alpha\tau(p)$.

Proof.

(i) $g(p\alpha q+q\alpha p) = g((p+q)\alpha(p+q)) = g(p+q)\alpha\sigma(p+q)+\tau(p+q)\alpha s(p+q) = g(p)\alpha\sigma(p)+g(p)\alpha\sigma(q)+g(q)\alpha\sigma(p)+g(q)\alpha\sigma(q)+\tau(p)\alpha\sigma(p)+\tau(p)\alpha\sigma(q)+\tau(q)\alpha\sigma(p) + \tau(q)\alpha\sigma(q)$.
(ii) Replacing q with $p\beta q + q\beta p$ in (i)
$g(p\alpha(p\beta q+q\beta p)+(p\beta q+q\beta p)\alpha p)) = g(p\alpha p\beta q+p\alpha q\beta p+p\beta q\alpha p+q\beta p\alpha p) = g(p)\alpha\sigma(q)\beta p + \tau(p)\beta s(q)\alpha p + p\alpha q\beta s(p) + g(q)\beta p\alpha\sigma(p) + \tau(q)\beta p\alpha s(q) + q\alpha q\beta s(p)g(p)\alpha q\alpha p)$.

(iii) $g(p\alpha q\alpha p) = g(p\alpha(q\alpha p)) = p\alpha g(q)\alpha\sigma(p) + p\alpha\tau(q)\alpha s(p) + s(p)alphaq\alpha p.$

(iv) $g(p\alpha(q\alpha p)) = p\alpha g(q)$. If we change p with $p\alpha r$ in (iii), we get $g((p\alpha r)\alpha q\alpha(p\alpha r)) = g(p\alpha r)\alpha\sigma(p)\alpha(p\alpha r) + \tau(p\alpha r)\alpha s(q)\alpha(p\alpha r) + p\alpha q\alpha s(p\alpha r) = (g(p)\alpha\sigma(r)\sigma(p\sigma r) + \tau(p)\alpha s(r))\alpha\sigma(q)\alpha(p\alpha r) + \tau(p\alpha r)\alpha s(q)\alpha(p\alpha r) + p\alpha q\alpha(s(p)\alpha r + p\alpha s(r) = (g(p)\alpha\sigma(r)\alpha\sigma(q)\alpha(p\alpha r) + \tau(p)\alpha d(r)\alpha\sigma(q)\alpha(p\alpha r) + \tau(p\alpha r)\alpha s(q)\alpha(p\alpha r) + p\alpha q\alpha s(p)\alpha r + p\alpha q p\alpha s(r).$

□

Lemma 1.2. *If* $\delta_\alpha(p,q) = g(p\alpha q) - g(p)\alpha\sigma(q) - \tau(p)\alpha s(q)$ *for* $p, q \in N$ *and* $\alpha \in \Gamma$. *Then*

(i) $\delta_\alpha(p,q) + \delta_\alpha(q,p) = 0$

(ii) $\delta_alpha(p,q+r) = \delta_alpha(p,q) + \delta_\alpha(p,r)$

(iii) $\delta_alpha(p+q,r) = \delta_\alpha(p,r) + \delta_\alpha(q,r)$

(iv) $\delta - alpha + \beta(p,q) = \delta_\alpha(p,q) + \delta\beta(p,q), forallp, q, \in Mand\alpha, \beta \in \Gamma.$

Proof.

(i) $\delta_\alpha(p,q) = g(p\alpha q) - g(p)\alpha\sigma(q) - \tau(p)\alpha s(q)$ $\delta_\alpha(q,p) = g(q\alpha p) - g(q)\alpha\sigma(p) - \tau(q\alpha s(p)$. By Lemma 1.1 (i) gives $\delta_\alpha(p,q) + \delta_\alpha(q,p) = 0$.

(ii) $\delta_\alpha(p,q+r) = g(p\alpha(q+r)) - g(p)\alpha\sigma(q+r) - \tau(p\alpha s(q+r) = g(p\alpha q) + g(p\alpha r) - g(p)\alpha(\sigma(q)+\sigma(r)) - \tau(p)\alpha(s(q)+s(r)) = g(p\,alphaq) + g(p\alpha r) - g(p)\alpha\sigma(q) - g(p\alpha\sigma(r) - \tau(p)\alpha s(q) - \tau(p)\alpha s(r)$
$= g(p\alpha q) - g(p)\alpha\sigma(q) - \tau(p)\alpha s(q) + g(p\alpha r) - g(p)\alpha\sigma(r) - \tau(p)\alpha s(r)$
$= \delta_\alpha(p,q) + \delta_\alpha(p,q).$

(iii) $\delta_\alpha(p+q,r) = g((p+q)\alpha r)) - g(p+q)\alpha\sigma(r) - \tau(p+q)\alpha s(r)$
$= g(p\alpha r) + g(q\alpha r) - (g(p)+g(q))\alpha\sigma(r) - (\tau(p)+\tau(q))\alpha s(r) = g(p\alpha r) + g(q\alpha r) - g(p)\alpha\sigma(r) - g(q)\alpha\sigma(r) - \tau(p)\alpha s(r) - \tau(q)\alpha s(r) = g(p\alpha r) - g(p)\alpha\sigma(r) - \tau(p\alpha s(r) + g(q\alpha(r) - g(q)\alpha\sigma(r) - \tau(q)\alpha s(r) = \delta_\alpha(p,r) + \delta_\alpha(q,r).$

(iv) $\delta_{\alpha+\beta}(p,q) = \delta_\alpha(p,q) + \delta_\beta(p,q) = g(p\alpha q) - g(p)\alpha\sigma(q) - \tau(p)\alpha s(q) + g(p\beta q) - g(p)\beta\sigma(q) - \tau(p)\beta s(q) = \delta_alpha(p,q) + \delta\beta(p,q).$

□

Lemma 1.3. *Let N be a two torsion free* Γ*-nearring and if* $p, q, r \in N$ *and* $\alpha, \beta \in \Gamma$*, then*

(i) $\delta_alpha(p,q)\alpha[p,q]\alpha = 0(when[p,q]\alpha = p\alpha q - q\alpha p)$

(ii) $\delta_\alpha(p,q)\beta x\beta[p,q]\alpha = 0foranyy \in N.$

Proof.

(i) change r by $p\alpha q$ in Lemma 1.1 (iv), we obtain
$g(p\alpha q\alpha p\alpha q + p\alpha q\alpha q\alpha p) = g(p\alpha q + p\alpha q) = g(p\alpha q) + g(p\alpha q) = g(p)\alpha\sigma(q) + \tau(p)\alpha s(q) + g(p)\alpha\sigma(q) + \tau(p)\alpha s(q)$.

(ii) one can set$p = p\alpha q\beta y\beta q\alpha p + q\alpha p\beta y\beta p\alpha q$. Then, since
$g(p) = g(p\alpha(q\beta y\beta q)\alpha p + q\alpha(p\beta y\beta p)\alpha q$ and $g(p) = g(p\alpha q)\beta y\beta(q\alpha p) + (q\alpha p)\beta y\beta(p\alpha q)$ by Lemma 1.1 (iii) and 1.1 (iv).

□

2. Main result

Theorem 2.1. *Let N be a two torsion free Gamma nearring. If N has two elements p and q so that for any $\alpha \in \Gamma$ one can have $r\gamma y\gamma[p,q]_\alpha = 0$ or $[p,q]_\alpha\gamma y\gamma$ r=0 implies r=0 $\forall y \in N\gamma\ in\Gamma$ then every Jordan generalized(σ,τ) derivation on N is a generalized (σ,τ) derivation.*

Proof. Let m,n be fixed elements of N satisfying the property $r\gamma y\gamma[m,n]_\alpha = 0$ or $[m,n]_\alpha\gamma y\gamma r = 0$ implies r=0. Then by Lemma 1.3 (ii), one can obtain

$$\delta_\alpha(m,n) = 0 \quad \forall\alpha \in \Gamma. \tag{2.1}$$

The main objective is to show $\delta_{alpha}(p,q) = 0, p, q \in \Gamma$. In Lemma 1.3. (ii), one can change p by $p+m$, we have:

$$\delta_\alpha(p+m,q)\beta y\beta[p+m,q]_\alpha = 0$$
$$\delta_\alpha(p+m,q)\beta y\beta((p+m)\alpha q - q\alpha(p+m) = 0\,.$$

By Lemma 1.2 (iii), one can have

$$\delta_\alpha(p,q)\beta y\beta(p,q)\alpha + \delta_\alpha(p,q)\beta y\beta(m,q)\alpha + \delta_\alpha(m,q)\beta y\beta(p,q)\alpha + \delta_alpha(m,q)\beta y\beta(m,q)\alpha = 0.$$

By Lemma 1.3. (ii), as $\delta_\alpha(p,q)\beta y\beta[p,q]\alpha = 0$ and $\delta_\alpha(m,q)\beta y\beta[m,q]\alpha = 0$, so that one can see

$$\delta_\alpha(p,q)\beta y\beta[m,q]\alpha + \delta_\alpha(m,q)\beta y\beta[p,q]\alpha = 0 \quad y,p,q \in N \quad \text{and} \quad \alpha,\beta \in \Gamma. \tag{2.2}$$

Now change q by $q+n$ in (2.2) and by Lemma 1.2 (ii), one can see

$$\begin{aligned}
&\delta_\alpha(p,q+n)\beta y\beta[m,q+n]\alpha+\delta\alpha(m,q+n)\beta y\beta[p,q+n]\alpha=0\\
&\delta_\alpha(p,q)\beta y\beta[m,q]\alpha+\delta_\alpha(p,q)\beta y\beta[m,n]\alpha+\delta(p,n)\beta y\beta[m,q]\alpha+\\
&+\delta_\alpha(p,n)\beta y\beta[m,n]\alpha+\delta_\alpha(m,q)\beta y\beta[p,q]\alpha+\delta_\alpha(m,q)\beta y\beta[p,n]\alpha+\\
&+\delta_\alpha(m,n)\beta y\beta[p,q]\alpha+\delta_\alpha(m,n)\beta x\beta[p,n]\alpha=0\,,
\end{aligned}$$

or by first and second equations, one can see

$$\begin{aligned}
&\delta_\alpha(p,q)\beta y\beta[m,n]\alpha+\delta_\alpha(p,n)\beta y\beta[m,q]\alpha+\\
&\delta_\alpha(m,n)\beta y\beta[m,n]\alpha+\delta_\alpha(m,q)\beta y\beta[p,n]\alpha=0.
\end{aligned}\tag{2.3}$$

Changing p by m in (2.3) and by (2.1) together with the result that N is a two torsion free, one can see

$$\delta_\alpha(m,q)\beta y\beta[m,n]\alpha=0,\ \ q,y\in N\ \ \text{and}\ \ \alpha,\beta\in\Gamma.$$

Hence by hypothesis, one can have

$$\delta_\alpha(m,q)=0,\quad q\in N,\alpha\in\Gamma.\tag{2.4}$$

Further change q by n in (2.2), by (2.1) and the hypothesis

$$\delta_\alpha(p,n)=0,\ \ p\in N,\alpha\in\Gamma.\tag{2.5}$$

That is $g(p\alpha n)-g(p)\alpha\sigma(n)-\tau(p)\alpha s(n)$.

Substituting equation (2.4) and (2.5) in (2.3), one can have

$$\delta_\alpha(p,q)\beta y\beta[m,n]\alpha=0.$$

Now by hypothesis, one can see $\delta_\alpha(p,q)=0,p,q\in N,\alpha\in\Gamma$. That is g is a generalized(σ,τ) derivation on N. □

3. CONCLUSIONS AND FUTURE RESEARCH

The above talk focuses on (σ,τ)-Jordan Generalized Derivations in Gamma Nearrings and three lemmas which are prerequisites to establish a main result on (σ,τ)-Jordan Generalized Derivations in Gamma Nearrings. In this discourse we crave to present some fundamental characteristic properties of Gamma nearrings and an evolution in the conjecture of Gamma nearrings. The innovatory proofs of three lemmas are proposed here so that it opens a way for young researchers.In this discourse we crave to present some fundamental characteristic properties of Gamma nearrings and an evolution in the conjecture of Gamma nearrings. The innovatory proofs of three lemmas are proposed here so that it opens a way for young researchers.

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