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AN EQUITABLE TOTAL COLORING OF SOME CLASSES OF PRODUCT OF GRAPHS

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ABSTRACT. An equitable total coloring of a graph G is an assignment of colors to all the elements (vertices, edges) of the graph G such that adjacent or incident elements receive the different color and for any two color classes different by at most one. In this paper, we prove some theorems on equitable total coloring for strong products of path and cycle.

1. INTRODUCTION

All graphs considered here are finite, simple and undirected. Let $G = (V(G), E(G))$ be a graph with the sets of vertices $V(G)$ and edges $E(G)$ respectively. An equitable total coloring of G is a mapping $f : V(G) \cup E(G) \to \{1, 2, \dots, k\}$, where k is a proper coloring satisfying the following conditions.

(1) $f(u) \neq f(v)$ for two adjacent vertices $u, v \in V(G)$,
(2) $f(e) \neq f(e^{'})$ for two adjacent edges $e, e^{'} \in E(G)$,
(3) $f(v) \neq f(e)$ for any vertex $v \in V(G)$ and for any edge $e \in E(G)$ incident to v,
(4) $||T_i| - |T_j|| \leq 1; i, j = 1, 2, \dots, k$. $T_i = V_i \cup E_i; 1 \leq i \leq k$.

A total coloring of graph G is a coloring of the vertices and the edges of G such that any two adjacent or incident elements (vertices, edges) have different color. The total chromatic number of a graph G denoted by $\chi^{''}(G)$, is the minimum

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number of colors that required in a total coloring. Total Coloring Conjecture formulated by Behzad [1] and Vizing [7], says that $\Delta(G)+1 \leq \chi''(G) \leq \Delta(G)+2$ for a simple graph G. A total coloring of graph G is said to be equitable if the number of elements (vertices, edges) are colored with each color different by at most one. The minimum number of colors that required for an equitable total coloring of graph G is called the equitable total chromatic number of G and is denoted by $\chi_e''(G)$. In 1973, Meyer [6] introduced the concepts of an equitable coloring and the equitable chromatic number. In 1994, Fu [4] investigated the concept of an equitable total coloring and the equitable total chromatic number. He raised the conjecture that for any simple graph G, then $\chi_e''(G) \leq \Delta(G) + 2$. Manikandan et al. [5] has proved the equitable coloring of some convex polytope graphs.

Theorem 1.1. *[2] For cycle graph C_n with maximum degree $\Delta(G)$, then*

$$\chi_e''(C_n) = \begin{cases} \Delta(G)+1, & \text{if } n \equiv 0 \text{ mod 3} \\ \Delta(G)+2, & \text{otherwise.} \end{cases}$$

In this paper, we study the conjecture on equitable total coloring of strong product of path and cycle.

2. Results and discussion

Definition 2.1. *[3] Consider G and H be two graphs. The strong product $G \boxtimes H$, defined by $V(G \boxtimes H) = \{(g,h) | g \in V(G), h \in V(H)\}$ and $E(G \boxtimes H) = E(G \boxdot H) \cup E(G \times H)$.*

Theorem 2.1. *The equitable total coloring of $P_n \boxtimes P_m$ and its chromatic number is $\Delta(P_n \boxtimes P_m)+1$ for all $n, m \geq 3$ and $n, m \in \mathbb{Z}^+$.*

Proof. Here $\Delta(P_n \boxtimes P_m)+1$ is the equitable total chromatic number of $P_n \boxtimes P_m$. In this graph of strong product having $n(R_1, R_2, \ldots, R_n)$ rows and $m(C_1, C_2, \ldots, C_m)$ columns. We divide $\Delta(P_n \boxtimes P_m)+1$ into three equal parts, say X_1, X_2 and X_3. There are three cases to be considered.

Case (i): If $n \equiv 0(mod\ 3)$ and $m \geq 3$.

Color all the vertices in R_1 using X_1 color and Color all the elements in R_2 with X_2 and R_3 with X_3 color. Then assign the colors to all the elements in the remaining rows $R_4, R_5, \ldots, R_{n-1}$ of $P_n \boxtimes P_m$ using X_1, X_2 and X_3 colors cyclically using a repeated pattern with the minor adjustment to the repetition to attains the equitable total coloring conditions. Color all the vertices in R_n using X_3 color. Then assign the colors to all the edges between R_1 and R_2 using X_3 color and the edges between R_2 and R_3 with X_1 colors and the edges between R_3 and R_4 with X_2 colors. Now, we color the remaining edges between $(R_4, R_5), (R_5, R_6), \dots, (R_{n-1}, R_n)$ using the colors with the color X_3, X_1 and X_2 cyclically using a repeated pattern with the minor adjustment to the repetition.

Finally, we color the remaining edges in R_1 and R_n using X_2 color which satisfy the conditions of equitable total coloring.

Case (ii): If $n \equiv 1(mod\ 3)$ and $m \geq 3$.

By case (i), assign colors to all the vertices of R_1, all the elements in $R_2, R_3, \dots, R_{n-1}$ and edges between $(R_1, R_2), (R_2, R_3), \dots, (R_{n-1}, R_n)$ of $P_n \boxtimes P_m$. Now, color all the vertices in R_n using X_1 color. Finally, we color the remaining edges in R_1 and R_n using $\Delta(P_n \boxtimes P_m) + 1$ color according to satisfy the conditions of equitable total coloring.

Case (iii): If $n \equiv 2(mod\ 3)$ and $m \geq 3$.

According to case (i), assign colors to all the vertices of R_1, all the elements $R_2, R_3, \dots, R_{n-2}$ and edges between $(R_1, R_2), (R_2, R_3), \dots, (R_{n-2}, R_{n-1})$ of $(P_n \boxtimes P_m)$. Now, color all the vertices in R_{n-1} using X_1 color and R_n using X_2 color. Then color all the edges in R_{n-1} using X_3 color and at each maximum degree vertex in R_{n-1} use the remaining colors of X_1 and X_3 to the edges which are incident from R_n. Finally, we color the remaining edges using $\Delta(P_n \boxtimes P_m) + 1$ color according to satisfy the conditions of equitable total coloring.

Hence, The equitable total chromatic number is $\Delta(P_n \boxtimes P_m) + 1$. □

Theorem 2.2. *The equitable total chromatic number of $P_n \boxtimes C_m$ for all $n, m \geq 3$ and $n, m \in \mathbb{Z}^+$*

$$\chi''_e(P_n \boxtimes C_m) = \begin{cases} \Delta(P_n \boxtimes C_m) + 1, & \text{if } m \equiv 0 (mod\ 3) \\ \Delta(P_n \boxtimes C_m) + 2, & \text{otherwise.} \end{cases}$$

Proof. There are two cases to be considered.

Case (1): If $m \equiv 0(mod\ 3)$

Here $\Delta(P_n \boxtimes C_m) + 1$ is the equitable total chromatic number of $P_n \boxtimes C_m$. In this graph of strong product having n $(R_1, R_2, R_3, \dots, R_n)$ rows and m $(C_1, C_2, \dots, C_m)$ columns. We divide $\Delta(P_n \boxtimes C_m) + 1$ into three equal parts, say X_1, X_2 and X_3.

Subcase (1.1): If $n \equiv 0(mod\ 3)$
Color all the vertices in R_1 using X_1 color and Color all the elements in R_2 with X_2 and R_3 with X_3 color. Then assign the colors to all the elements in the remaining rows $R_4, R_5, \dots, R_{n-1}$ of $(P_n \boxtimes C_m)$ using X_1, X_2 and X_3 colors cyclically and Color all the vertices in R_n using X_3 color. Then assign the colors to all the edges between R_1 and R_2 using X_3 color and the edges between R_2 and R_3 with X_1 color and the edges between R_3 and R_4 with X_2 color. Then we color the remaining edges between $(R_4, R_5), (R_5, R_6), (R_6, R_7), \dots, (R_{n-1}, R_n)$ using the colors of X_3, X_1 and X_2 cyclically.

Finally, we color the remaining edges in R_1 and R_n using X_2 color which satisfy the conditions of equitable total coloring.

Subcase (1.2): If $n \equiv 1(mod\ 3)$
By subcase (1.1), assign color to all the vertices in R_1, all the elements in $R_2, R_3, \dots, R_{n-1}$ and edges between $(R_1, R_2), (R_2, R_3), \dots, (R_{n-1}, R_n)$ of $(P_n \boxtimes C_m)$. Now, color all the vertices in R_n using X_1 color. Finally, we color the remaining edges in R_1 and R_n using $\Delta(P_n \boxtimes C_m) + 1$ colors according to satisfy the conditions of equitable total coloring.

Subcase (1.3): If $n \equiv 2(mod\ 3)$
According to subcase (1.1), assign color to all the vertices in R_1, all the elements in $R_2, R_3, \dots, R_{n-2}$ and edges between $(R_1, R_2), (R_2, R_3), \dots, (R_{n-2}, R_{n-1})$ of $(P_n \boxtimes C_m)$. Now, color all the vertices in R_{n-1} using X_1 color and R_n using X_2 color. Then color all the edges in R_{n-1} using X_3 color and at each maximum degree vertex in R_{n-1} use the remaining colors of X_1 and X_3 to the edges which are incident from R_n.
Finally, we color the remaining edges using $\Delta(P_n \boxtimes C_m) + 1$ colors according to satisfy the conditions of equitable total coloring.

Hence, the equitable total chromatic number of $(P_n \boxtimes C_m)$ is $\Delta(P_n \boxtimes C_m) + 1$.

Case (2): If $m \not\equiv 0(mod\ 3)$
Here, $\Delta(P_n \boxtimes C_m) + 2$ is the equitable total chromatic number of $(P_n \boxtimes C_m)$. In this graph of strong product having n $(R_1, R_2, \dots, R_n)$ rows and m $(C_1, C_2, \dots, C_m)$ columns. We divide $\Delta(P_n \boxtimes C_m) + 2$ into four parts, say X_1, X_2, X_3 and X_4. X_1 with three colors, X_2 with three colors, X_3 with two colors and X_4 with the remaining colors of $\Delta(P_n \boxtimes C_m) + 2$.

Subcase (2.1): If n **is odd and** m **is even:**
Color all the vertices of $R_2, R_4, R_6, \dots, R_{n-1}$ and all the edges of $R_3, R_5, R_7,$

$\ldots, R_{n-2}$ using X_3 color. Then color all the edges of $R_2, R_4, R_6, \ldots, R_{n-1}$ and all the vertices of $R_3, R_5, R_7, \ldots, R_{n-2}$ using X_4 color. Then assign the colors to all the edges between R_1 and R_2 using X_1 color and the edges between R_2 and R_3 with X_2 color. Then color the remaining edges between (R_3, R_4), $(R_4, R_5), \ldots, (R_{n-1}, R_n)$ using the colors of X_1 and X_2 cyclically. Finally, we color the remaining elements in R_1 and R_n using $\Delta(P_n \boxtimes C_m) + 2$ colors according to satisfying the equitable total coloring conditions.

Subcase (2.2): If both n and m are odd:

Assign X_3 color to all the vertices of $R_2, R_4, R_6, \ldots, R_{n-1}$ from C_1 to C_{m-1} and all the edges of $R_3, R_5, R_7, \ldots, R_{n-2}$ from C_1 to C_m. Then color all the edges of $R_2, R_4, R_6, \ldots, R_{n-1}$ from C_1 to C_m and all the vertices of $R_3, R_5, R_7, \ldots, R_{n-2}$ from C_1 to C_{m-1} using X_4 color. Color the edges of C_m using one of the color of X_3 and one of the color of X_4 and assign the remaining color of X_3 and X_4 in the edges of C_1. Then color the edges between $(R_1, R_2), (R_2, R_3), (R_3, R_4), (R_4, R_5)$, $\ldots, (R_{n-1}, R_n)$ using the colors of X_1 and X_2 cyclically. Now, we color the remaining elements in $R_2, R_3, R_4, \ldots, R_{n-1}$ using the missing colors of both X_1 and X_2. Finally, we color the remaining elements in R_1 and R_n using $\Delta(P_n \boxtimes C_m) + 2$ colors according to satisfying the equitable total coloring conditions.

Subcase (2.3): If both n and m are even:

Color all the vertices of $C_1, C_3, C_5, \ldots, C_{m-1}$ from R_1 to R_{n-2} and all the edges of $C_2, C_4, C_6, \ldots, C_m$ from R_1 to R_{n-1} using X_3 color. Then color all the edges of $C_1, C_3, C_5, \ldots, C_{m-1}$ from R_1 to R_{n-1} and all the vertices of $C_2, C_4, C_6, \ldots, C_m$ from R_1 to R_{n-2} using X_4 color. Color the edges of R_{n-1} using one of the color of X_3 and one of the color of X_4 and assign the remaining color of X_3 and X_4 in the edges of R_n. Then color the edges between $(C_1, C_2), (C_2, C_3), (C_3, C_4)$, $\ldots, (C_m, C_1)$ except the edges of R_1 using the colors of X_1 and X_2 cyclically using a repeated pattern with the minor adjustment to the repetition to attains the equitable total coloring conditions. Now, we color the vertices of R_{n-1} and the remaining edges between (R_{n-1}, R_n) using the missing colors of both X_1 and X_2. Finally, we color the remaining elements in R_1, R_n using $\Delta(P_n \boxtimes C_m) + 2$ colors according to satisfying the equitable total coloring conditions.

Subcase (2.4): If n is even and m is odd:

Assign X_3 color to all the vertices of $R_2, R_4, R_6, \ldots, R_{n-2}$ from C_1 to C_{m-1} and all the edges of $R_3, R_5, R_7, \ldots, R_{n-1}$ from C_1 to C_m. Then color all the edges of $R_2, R_4, R_6, \ldots, R_{n-2}$ from C_1 to C_m and all the vertices of $R_3, R_5, R_7, \ldots, R_{n-1}$

from C_1 to C_{m-1} using X_4 color. Color the edges of C_m using one of the color of X_3 and one of the color of X_4 and assign the remaining color of X_3 and X_4 in the edges of C_1.

Then assign the colors to the remaining edge of $(P_n \boxtimes C_m)$ in the following way: at each maximum degree vertex in R_p from C_2 to $C_{\frac{(m+1)}{2}}$, using X_1 color to the edges which are incident from R_{p-1} and use X_2 color to the edges which are incident from R_{p+1}. Then at each maximum degree vertex in R_p from $C_{\frac{(m+3)}{2}}$ to C_m using X_2 color to the edges which are incident from R_{p-1} and use X_1 color to the edges which are incident from R_{p+1}. Both the color will be given cyclically in order from $p = 2, 4, \ldots, n-2$. At each vertex in R_n from C_2 to $C_{\frac{(m+1)}{2}}$, use X_1 color to the edges which are incident from R_{n-1} and from $C_{\frac{(m+3)}{2}}$ to C_m use X_2 color to the edges which are incident from R_{n-1}. Then color the vertices of C_m in $R_2, R_4, R_6, \ldots, R_{n-2}$ using the missing colors of X_1 and in $R_3, R_5, R_7, \ldots, R_{n-1}$ using the missing colors of X_2. Now, we color the remaining edges between (C_1, C_2) and (C_m, C_1) using the missing colors of both X_1 and X_2 colors.

Finally, we color the remaining elements in R_1, R_n using $\Delta(P_n \boxtimes C_m)+2$ colors according to satisfying the equitable total coloring conditions.
Therefore, the equitable total chromatic number of $(P_n \boxtimes C_m)$ is $\Delta(P_n \boxtimes C_m)+2$. Hence, the equitable total chromatic number of $P_n \boxtimes C_m$ for all $n, m \geq 3$ and $n, m \in \mathbb{Z}^+$ $\chi_e''(P_n \boxtimes C_m) = \begin{cases} \Delta(P_n \boxtimes C_m)+1, & \text{if } m \equiv 0(mod\ 3) \\ \Delta(P_n \boxtimes C_m)+2, & \text{otherwise.} \end{cases}$ □

Theorem 2.3. *The equitable total coloring of $C_n \boxtimes C_m$ for all $n, m \geq 3$ and $n, m \in \mathbb{Z}^+$*

$$\chi_e''(C_n \boxtimes C_m) = \begin{cases} \Delta(C_n \boxtimes C_m)+1, & \textit{if } n, m \equiv 0(mod\ 3) \\ \Delta(C_n \boxtimes C_m)+2, & \textit{otherwise if both } n, m \textit{ are not prime.} \end{cases}$$

Proof. There are two cases as follows:
Case (1): If $n, m \equiv 0(mod\ 3)$
Here, $\Delta(C_n \boxtimes C_m)+1$ is the equitable total chromatic number. In this graph of strong product having $n(R_1, R_2, \ldots, R_n)$ rows and $m(C_1, C_2, \ldots, C_m)$ columns. We divide $\Delta(C_n \boxtimes C_m)+1$ into three equal parts, say X_1, X_2, X_3.

Color all the elements in R_1 using X_1 color and R_2 with X_2 color and R_3 with X_3 color. Then assign the colors to all the element in the remaining rows $R_4, R_5, \ldots, R_n$ of $(C_n \boxtimes C_m)$ using X_1, X_2 and X_3 colors cyclically. Then assign the colors to all the edges between R_1 and R_2 using X_3 color and the edges between

R_2 and R_3 with X_1 color and the edges between R_3 and R_4 with X_2 color. Then, we color the remaining edges between $(R_4, R_5), (R_5, R_6), \ldots, (R_n, R_1)$, using the colors of X_3, X_1 and X_2 cyclically which satisfy the conditions of equitable total coloring.

Therefore, The equitable total chromatic number of $(C_n \boxtimes C_m)$ is $\Delta(C_n \boxtimes C_m) + 1$.

Case (2): If both n, m are not prime:

Here, $\Delta(C_n \boxtimes C_m) + 2$ is the equitable total chromatic number. In this graph of strong product having $n(R_1, R_2, \ldots R_n)$ rows and $m(C_1, C_2, \ldots, C_m)$ columns.

Subcase (2.1): If both n and m are even:

We divide $\Delta(C_n \boxtimes C_m) + 2$ into four parts, say X_1, X_2, X_3 and X_4. X_1 with three colors, X_2 with three colors, X_3 with two colors and X_4 with the remaining colors of $\Delta(C_n \boxtimes C_m) + 2$.

Color all the vertices of $R_1, R_3, R_5, \ldots, R_{n-1}$ and all the edges of $R_2, R_4, R_6, \ldots, R_n$ using X_3 color. Then color all the edges of $R_1, R_3, R_5, \ldots, R_{n-1}$ and all the vertices of $R_2, R_4, R_6, \ldots, R_n$ using X_4 color. Then assign the colors to all the edges between R_1 and R_2 using X_1 color and the edges between R_2 and R_3 with X_2 color. Then color the remaining edges between $(R_3, R_4), (R_4, R_5), \ldots, (R_n, R_1)$ using the colors of X_1 and X_2 cyclically which satisfy the conditions of equitable total coloring.

Subcase (2.2): If n is even and m is odd:

We divide $\Delta(C_n \boxtimes C_m) + 2$ into four parts, say X_1, X_2, X_3 and X_4. X_1 with three colors, X_2 with three colors, X_3 with two colors and X_4 with the remaining colors of $\Delta(C_n \boxtimes C_m) + 2$.

Assign X_3 color to all the vertices of $R_1, R_3, R_5, \ldots, R_{n-1}$ from C_1 to C_{m-1} and all the edges of $R_2, R_4, R_6, \ldots, R_n$ from C_1 to C_m. Then color all the edges of $R_1, R_3, R_5, \ldots, R_{n-1}$ from C_1 to C_m and all the vertices of $R_2, R_4, R_6, \ldots, R_n$ from C_1 to C_{m-1} using X_4 color. Color the edges of C_m using one of the color of X_3 and one of the color of X_4 and assign the remaining color of X_3 and X_4 in the edges of C_1. Then color the edges between $(R_1, R_2), (R_2, R_3), (R_3, R_4), \ldots, (R_n, R_1)$ using the colors of X_1 and X_2 cyclically. Now, we color the remaining vertices in C_m and edges between (C_m, C_1) using the missing colors of both X_1 and X_2 which satisfy the conditions of equitable total coloring.

Subcase (2.3): If n is odd and m is even:

We divide $\Delta(C_n \boxtimes C_m) + 2$ into four parts, say X_1, X_2, X_3 and X_4. X_1 with three colors, X_2 with three colors, X_3 with two colors and X_4 with the remaining colors of $\Delta(C_n \boxtimes C_m) + 2$.

Color all the vertices of $C_1, C_3, C_5, \ldots, C_{m-1}$ from R_1 to R_{n-1} and all the edges of $C_2, C_4, C_6, \ldots, C_m$ from R_1 to R_n using X_3 color. Then color all the edges of $C_1, C_3, C_5, \ldots, C_{m-1}$ from R_1 to R_n and all the vertices of $C_2, C_4, C_6, \ldots, C_m$ from R_1 to R_{n-1} using X_4 color. Color the edges of R_n using one of the color of X_3 and one of the color of X_4 and assign the remaining color of X_3 and X_4 in the edges of R_1. Then color the edges between $(C_1, C_2), (C_2, C_3), (C_3, C_4), \ldots, (C_m, C_1)$ using the colors of X_1 and X_2 cyclically. Now, we color the vertices of R_n and the remaining edges between (R_n, R_1) using the missing colors of both X_1 and X_2 which satisfy the conditions of equitable total coloring.

Subcase (2.4): If $n \equiv 0(mod\ 3)$ and m is odd:

We divide $\Delta(C_n \boxtimes C_m) + 2$ into four parts, say X_1, X_2, X_3 and X_4. X_1 with three colors, X_2 with three colors, X_3 with three colors and X_4 with the remaining colors of $\Delta(C_n \boxtimes C_m) + 2$.

Color all the elements in R_1 using X_1 and X_4 color and R_2 with X_2 and X_4 color and R_3 with X_3 and X_4 color. Then assign the colors to all the element in the remaining rows $R_4, R_5, \ldots, R_n$ of $(C_n \boxtimes C_m)$ using $(X_1, X_4), (X_2, X_4)$ and (X_3, X_4) colors cyclically. Then assign the colors to all the edges between R_1 and R_2 using X_3 and X_4 colors. To color the edges between R_2 and R_3 use X_1 color. To color the edges between R_3 and R_4 assign X_2 and the X_4 colors. The edges between R_4 and R_5 use X_3 and the edges between R_5 and R_6 assign X_1 and X_4 colors. To color the edges between R_6 and R_7 assign X_2 color. Use the above process cyclically for all the edges between $(R_7, R_8), (R_8, R_9), \ldots, (R_n, R_1)$ which satisfy the conditions of equitable total coloring.

Subcase (2.5): If $m \equiv 0(mod\ 3)$ and n is odd:

We divide $\Delta(C_n \boxtimes C_m) + 2$ into four parts, say X_1, X_2, X_3 and X_4. X_1 with three colors, X_2 with three colors, X_3 with three colors and X_4 with the remaining colors of $\Delta(C_n \boxtimes C_m) + 2$.

Color all the elements in C_1 using X_1 and X_4 color and C_2 with X_2 and X_4 color and C_3 with X_3 and X_4 color.Then assign the colors to all the element in the remaining columns $C_4, C_5, \ldots, C_n$ of $(C_n \boxtimes C_m)$ using $(X_1, X_4), (X_2, X_4)$ and (X_3, X_4) colors cyclically. Then assign the colors to all the edges between C_1 and C_2 using X_3 and X_4 colors. To color the edges between C_2 and C_3 use X_1 color. To color the edges between C_3 and C_4 assign X_2 and X_4 colors. The edges

between C_4 and C_5 use X_3 and the edges between C_5 and C_6 assign X_1 and X_4 colors. To color the edges between C_6 and C_7 assign X_2 color. Use the above process cyclically for all the edges between $(C_7, C_8), (C_8, C_9), \ldots, (C_n, C_1)$ which satisfy the conditions of equitable total coloring.
Therefore, the equitable total chromatic number of $(C_n \boxtimes C_m)$ is $\Delta(C_n \boxtimes C_m) + 2$. □

Open problem: If m and n are prime, then the equitable total chromatic number $(C_n \boxtimes C_m)$?

REFERENCES

[1] M. BEHZAD: *Graphs and their chromatic number*, Thesis, Michigan state University, 1965.

[2] M. A. GANG, M. A. MING: *The equitable total chromatic number of some join graphs*, open journal of Applied Sciences, 2012.

[3] R. HAMMACK, W. IMRICH, S. KLAVZAR: *Handbook of Product Graphs*, CRC Press, Taylor and Francis Group, Boca Raton, 2011.

[4] H.-L. FU: *Some results on equalized total coloring*, Congr. Numer., **102** (1994), 111–119.

[5] K. MANIKANDAN, T. HARIKRISHNAN: *Equitable Coloring of Some Convex Polytopegraphs*, Int. J. Appl. Comput. Math, **4** (2018), Article number 119.

[6] W. MEYER: *Equitable coloring*, Amer. Math. Monthly, **80** (1973), 920–922.

[7] V. G. VIZING: *Some unsolved problems in graph theory*, Uspekhi mat. Navk., **23** (1968), 117–134.

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