Advances in Mathematics: Scientific Journal **9** (2020), no.10, 8367–8374
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.10.66

VARIETY OF RATIONAL RESOLVING SETS OF CORONA PRODUCT OF GRAPHS

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ABSTRACT. A subset S of $V(G)$ is called a resolving set if for each pair of vertices u, v in $V - S$, there exists a vertex $w \in S$ such that $d(u, w) \neq d(v, w)$ and is called a rational resolving set of G, if for each pair u, $v \in V - S$, there is a vertex $w \in S$ such that $d(u/w) \neq d(v/w)$, where $d(x/w)$ denotes the mean of the distances from the vertex w to all those $y \in N[x]$. A rational resolving set is called minimal rational resolving set if no proper subset of it is a rational resolving set. In this paper we study the varieties of minimal rational resolving sets of graph G and compute their minimum and maximum cardinality for certain classes of corona product of graphs.

1. INTRODUCTION

Many networks are represented by a graph, in which vertex play an important role and it depends on its neighbors. In the development of smart cities one of the most important task is to identify every building uniquely and to be given a unique ID, which helps to avail numerous public digital services quickly. For example a fire engine or a police vehicle or an ambulance can reach the destination building quickly with the help of a unique ID. This concept helps to find and optimize the number of source places required for the unique representation of

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2010 *Mathematics Subject Classification.* 05C12, 05C38.
Key words and phrases. corona product, resoloving set, rational resoloving set, rational metric dimension.

the destination with the help of graph theory, since many networks are represented by graphs. To obtain the position of a vertex in the network, we need to select the landmarks in such a way that the distance of the vertex from the landmark and the distance of its neighborhood vertices from the landmark are considered. This problem is equivalent to finding a minimal rational resolving set of the constructed graph G.

A subset S of $V(G)$ is called a resolving set if for each pair of vertices u, v in $V - S$ there exists a vertex w in S such that $d(u,w) \neq d(v,w)$. The minimum cardinality of such a set S is called metric dimension of a graph G, denoted by $\beta(G)$. Metric dimension was defined by F. Harary et al. [2] and Peter J. Slater [6]. B. Sooryanarayana [7] and various authors in articles [8,9] have obtained many results on metric dimension. Rational metric dimension of graphs were originally proposed by A. Raghavendra et al. [10]. M. M. Padma and M. Jayalakshmi [4,5] introduced and developed the concept of r_r, r_r^*, R_r, R_r^* sets of graphs. We use the standard terminology, the terms not defined here may be found in [1,3].

2. r_r, r_r^*, R_r, R_r^* Sets of a Graph

Let $G(V,E)$ be a simple, connected, non trivial, finite, undirected graph. Let $N(u) = \{w : uw \in E(G)\}$ be the open neighbourhood and $N[u] = N(u) \cup \{u\}$ be the closed neighbourhood of a vertex u. For the vertex u of G and an ordered subset $S = \{s_1, s_2, ..., s_k\}$ of V, associate a vector denoted and defined by $\Gamma(u/S) = (d(u/s_1), d(u/s_2), ..., d(u/s_k))$, where $d(u/v) = \frac{\sum_{u_i \in N[u]} d(u_i,v)}{deg(u)+1}$. Then S is said to be a *rational resolving set* if $\Gamma(x/S) \neq \Gamma(y/S)\ \forall\ x,y \in V - S$ and the minimum cardinality of such a set S is called the *rational metric dimension* or the *lower r_r number*, denoted by $rmd(G)$ or $l_{r_r}(G)$. The associated set is called the *rational metric basis* or an rmb set. The maximum cardinality of a minimal r_r set of graph G is said to be an *upper r_r number* of G, denoted by $u_{r_r}(G)$.

A subset S of $V(G)$ is said to be an *r_r^* set* if S is an r_r set and $\bar{S} = V - S$ is also an r_r set. The minimum and maximum cardinality of a minimal *r_r^* set* of graph G are called respectively, the *lower r_r^* number* and the *upper r_r^* number* of G and are denoted by $l_{r_r^*}(G)$ and $u_{r_r^*}(G)$.

A subset S of $V(G)$ is said to be an *R_r set* if S an r_r set and $\bar{S} = V - S$ is not an r_r set. The minimum and maximum cardinality of minimal R_r sets of G are called respectively, the *lower* R_r *number* and the *upper* R_r *number* of G, denoted by $l_{R_r}(G)$ and $u_{R_r}(G)$.

A subset S of $V(G)$ is said to be an R_r^* *set* if both S and $\bar{S} = V - S$ are not r_r sets. The minimum and maximum cardinality of minimal R_r^* *sets* of G are called respectively, the *lower* R_r^* *number* and the *upper* R_r^* *number* of G, denoted by $l_{R_r^*}(G)$ and $u_{R_r^*}(G)$.

Remark 2.1. *For any graph $G(V, E)$, we use the convention that if r_r^*, R_r, R_r^* sets does not exist, then their cardinality is zero.*

3. r_r, r_r^*, R_r, R_r^* Sets of Corona product of Graphs

The corona of two graphs G_1 and G_2 having the order n_1, n_2 and size m_1, m_2 respectively is the graph obtained by taking a copy of G_1 and n_1 copies of G_2, and joining i^{th} vertex of G_1 with an edge to every vertex in the i^{th} copy of G_2. It is denoted by $G_1 \odot G_2$.

Let P_n be a path with $V(P_n) = \{v_i/1 \leq i \leq n\}$. The comb graph is defined as $G = P_n \odot K_1$ with $V(G) = \{v_i \cup u_i/1 \leq i \leq n\}$ and $E(G) = \{v_iu_i/1 \leq i \leq n\} \cup \{v_iv_{i+1}/1 \leq i \leq n-1\}$.

Lemma 3.1. *For the comb graph $G = P_n \odot K_1$ with $n \geq 3$,*

(i) *The singleton sets $\{u_1\}, \{v_1\}, \{v_2\}$ and by symmetry $\{u_n\}, \{v_n\}, \{v_{n-1}\}$ are minimal r_r sets.*

(ii) *The singleton set $\{v_i\}$ with $3 \leq i \leq n-2$ and $\{u_i\}$ with $2 \leq i \leq n-1$ are non r_r sets.*

(iii) *Any two element subset of $H = \{v_i \cup u_j$ / with $3 \leq i \leq n-2$, $2 \leq j \leq n-1$, $i \neq j\}$ of a comb graph G, is a minimal r_r set with maximum cardinality.*

Proof. The proof follows, since for any integer i with $1 \leq i \leq n$, we have

$$d(v_i/v_j) = \begin{cases} \frac{2}{3} & \text{if } \ i = j = 1. \\ \frac{3}{4} & \text{if } \ i = j \ \text{ and } 2 \leq i \leq n-1. \\ |i-j| & \text{if } \ i = 1, n \ \text{ and } j \neq i. \\ |i-j| + \frac{1}{4} & \text{if } \ 2 \leq i, j \leq n-1 \ \text{ and } i \neq j. \end{cases}$$

$$d(u_i/u_j) = \begin{cases} \frac{1}{2} & \text{if } \ i = j. \\ \frac{5}{2} & \text{if } \ |i-j| = 1. \\ |i-j| + \frac{3}{2} & \text{if } \ |i-j| \geq 2. \end{cases}$$

$$d(u_i/v_j) = \begin{cases} |i-j| + \frac{1}{2} & \text{if } 1 \le i, j \le n. \end{cases}$$

$$d(v_i/u_j) = \begin{cases} |i-j| + 1 & \text{if } i = 1, n. \\ |i-j| + \frac{5}{4} & \text{otherwise.} \end{cases}$$

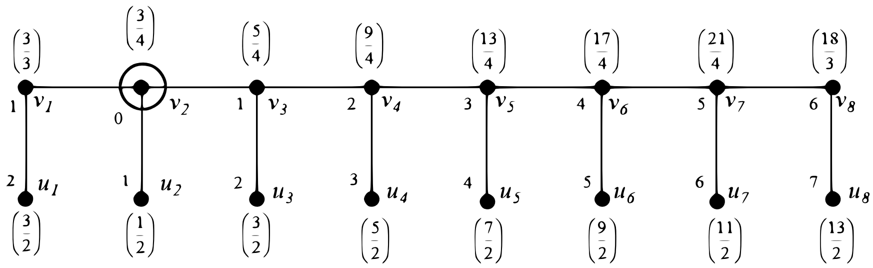

FIGURE 1. A minimal r_r set $\{v_2\}$ of $P_8 \odot K_1$.

□

Theorem 3.1. *For a comb graph G with $n \ge 3$,*

(i) $l_{r_r}(G) = 1, u_{r_r}(G) = 2$.
(ii) $l_{r_r^*}(G) = 1, u_{r_r^*}(G) = 2$.
(iii) $l_{R_r}(G) = u_{R_r}(G) = 2n - 2$.
(iv) $l_{R_r^*}(G) = u_{R_r^*}(G) = 0$.

Proof. Let $G = P_n \odot K_1$.

(i) From Lemma 3.1, The singleton set $\{u_1\}$ of $V(G)$ is a minimal r_r set with minimum cardinality and any two element subset of $H = \{v_i \cup u_j / 3 \le i \le n-2,\ 2 \le j \le n-1,\ i \ne j\}$ of $V(G)$, is a minimal r_r set with maximum cardinality. , $l_{r_r}(G) = 1$ and $u_{r_r}(G) = 2$.

(ii) $S = \{u_1\}$ is an r_r set and $\bar{S} = \{V - S\}$ is also an r_r set as it contain the end vertex u_n. Hence, S is an r_r^* set with minimum cardinality. Therefore, $l_{r_r^*}(G) = 1$.

From Lemma 3.1, for any two element subset of $S = \{v_i \cup u_j / 3 \le i \le n-2,\ 2 \le j \le n-1,\ i \ne j\}$ of $V(G)$, $\bar{S}$ contain an end vertex and Hence, is an r_r set. Therefore, S is an r_r^* set with maximum cardinality and $u_{r_r^*}(G) = 2$.

(iii) From Lemma 3.1, every two element subset $\{v_i \cup u_i / 3 \le i \le n-2\}$ is a non r_r set and every k-element subset of $V(G)$ for $k \ge 3$ is an r_r set.

Hence, $S \subseteq V(G)$ is a minimal R_r set, only if $\bar{S} = \{v_i \cup u_i \,/\, 3 \leq i \leq n-2\}$. Therefore, $l_{R_r}(G) = u_{R_r}(G) = 2n-2$.

(iv) For any k-element subset S of $V(G)$ with $1 \leq k < n-1$, either S or $V - S$ contain atleast one end vertex which imply either S or $V - S$ is always an r_r set. Therefore, there exists no R_r^* set for G and Hence, $l_{R_r^*}(G) = u_{R_r^*}(G) = 0$.

Hence the proof. □

Let C_n be the cycle with vertex set $V(C_n) = \{v_i/1 \leq i \leq n\}$. The sun(sunlet) graph is defined as $C_n \odot K_1$. Let $G = C_n \odot K_1$, $n \geq 3$ with $V(C_n \odot K_1) = \{v_i \cup u_i/1 \leq i \leq n\}$, $E(C_n \odot K_1) = E(C_n) \cup \{v_i u_i/1 \leq i \leq n\}$.

Lemma 3.2. *For the sunlet graph G, $rmd(G) = 2$.*

Proof. From the vertex v_i or u_i with $1 \leq i \leq n$, the rational metric distance of u_j (or v_j) which are at equidistant is the same. Hence, minimum of two vertices are required to rational resolve and except diagonally opposite or $\{u_i, v_i\}$, rational resolves G. Therefore, $rmd(G) = 2$, which leads to the following theorem. □

Theorem 3.2. *For the sunlet graph G,*

(i) $l_{r_r}(G) = u_{r_r}(G) = 2$.

(ii) $l_{r_r^*}(G) = u_{r_r^*}(G) = 2$.

(iii) $l_{R_r}(G) = u_{R_r}(G) = \begin{cases} 2n-2 & \textit{if} \quad n \quad \textit{is odd or } n = 4. \\ 2n-4 & \textit{if} \quad n \quad \textit{is even} \end{cases}$

(iv) $l_{R_r^*}(G) = u_{R_r^*}(G) = \begin{cases} 4 & \textit{if} \quad n = 4. \\ 0 & \textit{if} \quad n \neq 4. \end{cases}$

Lemma 3.3. *For $G = K_n \odot K_1$, $n \geq 4$, $rmd(G) = n-1$.*

Proof. Let $V = \{v_i \cup u_i/1 \leq i \leq n\}$ be the vertex set and $E(G) = E(K_n) \cup \{v_i u_i/1 \leq i \leq n\}$ be the edge set of G. With respect to u_1, $d(v_i/u_1)$ remains same and $d(u_i/u_1)$ remains same for every i with $2 \leq i \leq n$. Also with respect to v_1, $d(v_i/v_1)$ remains same and $d(u_i/v_1)$ remains same for every i with $2 \leq i \leq n$. Hence, minimum $n-1$ vertices are required to rational resolve G. Therefore, $rmd(G) = n-1$, which leads to the following theorem. □

Theorem 3.3. *For $G = K_n \odot K_1$,*

(i) $l_{r_r}(G) = u_{r_r}(G) = n-1$.

(ii) $l_{r_r^*}(G) = u_{r_r^*}(G) = n - 1$.
(iii) $l_{R_r}(G) = u_{R_r}(G) = n + 2$.
(iv) $l_{R_r^*}(G) = u_{R_r^*}(G) = 0$.

Lemma 3.4. *For any graph G of order n, the corona product $G \odot \bar{K_m}$ with $n \geq 2$, $m > 1$, $rmd(G \odot \bar{K_m}) = n(m-1)$.*

Proof. Let G be a graph of order n with the vertex set $V(G) = \{v_1, v_2, v_3, ..., v_n\}$. By the definition of corona product $G \odot \bar{K_m}$, a vertex v_i of G is attached with m pendent edges and let $u_{i1}, u_{i2}, u_{i3}, ..., u_{im}$ be the associated pendent vertices for every i with $1 \leq i \leq n$. Then the subgraph induced by the vertex set $\{v_i, u_{i1}, u_{i2}, ..., u_{im}\}$ is a star graph with v_i as the central vertex and hence, $G \odot \bar{K_m}$ has n copies of induced star graph $K_{1,m}$. For a star graph $K_{1,m}$, $m - 1$ pendent vertices are required to rational resolve and any $(m-1)$ element subset S of $V(K_{1,m})$ containing only pendent vertices is a minimal r_r set. Hence, $m - 1$ vertices from each copy of $K_{1,m}$ are required and also sufficient to rational resolve $G \odot \bar{K_m}$. Therefore, $rmd(G \odot \bar{K_m}) = n(m-1)$. □

Theorem 3.4. *For $G \odot \bar{K_m}$ with $n \geq 3$, $m > 1$,*

(i) $l_{r_r}(G \odot \bar{K_m}) = u_{r_r}(G \odot \bar{K_m}) = n(m-1)$.
(ii) $l_{r_r^*}(G \odot \bar{K_m}) = u_{r_r^*}(G \odot \bar{K_m}) = 0$.
(iii) $l_{R_r}(G \odot \bar{K_m}) = u_{R_r}(G \odot \bar{K_m}) = n(m-1)$.
(iv) $l_{R_r^*}(G \odot \bar{K_m}) = u_{R_r^*}(G \odot \bar{K_m}) = 2$.

Proof. Consider the corona product $G \odot \bar{K_m}$.

(i) From Lemma 3.4, set S containing $n(m-1)$ elements of $V(G \odot \bar{K_m})$ is a minimal r_r set with minimum cardinality, which imply $l_{r_r}(G \odot \bar{K_m}) = m - 1$. Also for induced star subgraph $K_{1,m}$ of $G \odot \bar{K_m}$, there exists no minimal r_r set with cardinality greater than $m - 1$ and hence, there exists no minimal r_r set with cardinality greater than $n(m-1)$ for $G \odot \bar{K_m}$. Therefore, $u_{r_r}(G \odot \bar{K_m}) = n(m-1)$.
(ii) From each induced star subgraph $K_{1,m}$ of $G \odot \bar{K_m}$, any r_r set of $K_{1,m}$ must contain minimum $m - 1$ elements. Equivalently, any r_r set S of $V(G \odot \bar{K_m})$, must contain $n(m-1)$ elements. Hence, for any r_r set S of $V(G \odot \bar{K_m})$, $\bar{S}$ can not contain $n(m-1)$ elements. Therefore, there exists no r_r^* set for $G \odot \bar{K_m}$ and hence, $l_{r_r^*}(G \odot \bar{K_m}) = u_{r_r^*}(C_n \odot \bar{K_m}) = 0$.

(iii) Any r_r set of $V(G \odot \bar{K_m})$ must contain minimum $n(m-1)$ elements and hence, for any r_r set S, $\bar{S}$ is not an r_r set. Also any r_r set of $V(G \odot \bar{K_m})$ with cardinality greater than $n(m-1)$ is not minimal. Therefore, any r_r set of $V(G \odot \bar{K_m})$ with $n(m-1)$ elements is a minimal R_r set with minimum and maximum cardinality which imply $l_{R_r}(G \odot \bar{K_m}) = u_{R_r}(G \odot \bar{K_m}) = n(m-1)$.

(iv) Any r_r set S of $V(G \odot \bar{K_m})$, has to contain minimum $(m-1)$ elements from each induced star subgraph. Hence, any 2-element subset S of $V(G \odot \bar{K_m})$ containing two pendent vertices from any one of the induced star subgraph is not an r_r set of $V(G \odot \bar{K_m})$. Also for such S, $\bar{S}$ is not an r_r set of $V(G \odot \bar{K_m})$ as one of its induced star subgraph contain $(m-2)$ elements and hence, S is a minimal R_r^* set with minimum and maximum cardinality. Therefore, $l_{R_r^*}(G \odot \bar{K_m}) = u_{R_r^*}(G \odot \bar{K_m}) = 2$.

Hence the result. □

Corollary 3.1. *For $C_n \odot \bar{K_m}$ with $n \geq 3$, $m > 1$,*

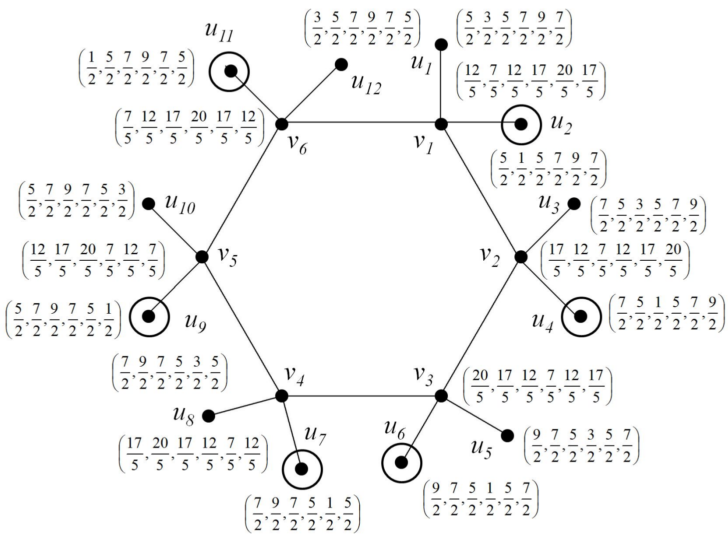

FIGURE 2. A minimal r_r set $\{u_2, u_4, u_6, u_7, u_9, u_{11}\}$ of $C_6 \odot \bar{K_2}$.

(i) $l_{r_r}(C_n \odot \bar{K_m}) = u_{r_r}(C_n \odot \bar{K_m}) = n(m-1)$.
(ii) $l_{r_r^*}(C_n \odot \bar{K_m}) = u_{r_r^*}(C_n \odot \bar{K_m}) = 0$.
(iii) $l_{R_r}(C_n \odot \bar{K_m}) = u_{R_r}(C_n \odot \bar{K_m}) = n(m-1)$.
(iv) $l_{R_r^*}(C_n \odot \bar{K_m}) = u_{R_r^*}(C_n \odot \bar{K_m}) = 2$.

ACKNOWLEDGEMENT

Authors are very much thankful to Dr. B. Sooryanarayana, Professor, Department of Mathematics, the Principal and the Management of Dr. Ambedkar Institute of Technology, Bangalore for their constant support and encouragement during the preparation of this paper.

REFERENCES

[1] F. HARARY: *Graph theory*, Narosa Publishing House, New Delhi, 1969.

[2] F. HARARY, R. A. MELTAR: *On the metric dimension of a graph*, Ars Combin., **2** (1976), 191-195.

[3] N. HARTSFIELD, G. RINGEL: *Pearls in Graph Theory*, Academic Press, USA, 1994.

[4] M. M. PADMA, M. JAYALAKSHMI: *Boundry values of r_r, r_r^*, R_r, R_r^* sets of certain classes of graphs*, Journal of Applied Mathematics and Bioinformatics, **6**(2) (2016), 143-154.

[5] M. M. PADMA, M. JAYALAKSHMI: *k- local resolving and rational resolving sets of graphs*, International Journal of Engineering Sciences and Management, **2** (2020), 15 -20.

[6] P. J. SLATER: *Dominating and references in a graph*, J. Math. Phys. Sci, **22** (1988), 445-455.

[7] B. SOORYANARAYANA: *On the metric dimension of graph*, Indian J. Pure Appl. Math., **29**(4) (1998), 413-415.

[8] F. HARARY, R. A. MELTER: *On the metric dimension of a graph*, Ars Combin., **2** (1976), 191-195.

[9] C. HERNANDO, M. MORA, I. M. PELAYO, C. SEARA, J. CACERES, M. L. PUERTAS: *On the metric dimension of some families of graphs*, Electron. Notes. Discrete Math., **22** (2005), 129-133.

[10] A. RAGHAVENDRA, B. SOORYANARAYANA, C. HEGDE: *Rational metric dimension of graphs,* Commun. Optim. theory, **8** (2014), 1-16.

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