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# SOME MAP GLUING THEOREMS AND CHARACTERIZATIONS OF SOFT UNIFORM CONTINUITY

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ABSTRACT. In this paper, we extend the theory of soft uniform continuity and soft dense sets. We give characterizations of soft uniform continuous maps in terms of sequences of soft points and in terms of its restriction to soft sequentially compact soft subsets of domain. We also obtain some map gluing theorems of soft uniformly continuous maps in terms of soft dense subsets.

## 1. INTRODUCTION

The soft set theory, introduced by Molodtsov in [8], is a new mathematical tool, which aims to describe phenomena and concepts of vagueness, uncertainty and ambiguous meaning. Soft set theory provides a general framework with the involvement of parameters. Applications of set theory in other disciplines and real life problems are progressing rapidly showing the rich potential of the concept of soft sets. Since its advent a lot of work has been done in this field by various authors in [1, 5, 12, 13], etc. The concept of soft points that we use in this paper is defined in [10]. The notion of soft uniformly continuous maps and soft compact sequential metric space were introduced in [2].

In [11], we introduced the concept of soft dense sets and give characterizations of soft continuity between soft metric spaces in terms soft dense subsets of domain space. Soft continuity of a soft map in terms of the soft continuity

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of its restrictions to members of a soft cover of the domain having soft dense intersection were also obtained [Theorem 2.2 below]. These results lead us to find the applications of soft dense sets to soft uniform continuity also. In this paper, we give characterizations of soft uniformly continuous maps in terms of the sequences of soft points and soft dense subsets. We discuss some map gluing theorems which describe the soft uniformly continuous maps in terms soft dense subsets. We also obtain analogs of some of the results in [11] for soft uniform continuity.

In section 2, we recall the basic concept related to soft metric spaces. In section 3, we give characterization of soft uniformly continuous maps in terms of sequences of soft points [Theorem 3.1 below] and in terms of its restriction to soft sequentially compact soft subsets of domain [Theorem 3.2 below]. Next, we obtain a map gluing theorem on soft uniformly continuous maps in terms of soft dense subsets [Theorem 3.3 below]. We give another characterizations of soft uniform continuity of a soft map [Theorem 3.4 and 3.5 Theorem below].

### 2. PRELIMINARIES

Throughout this paper, X refers to an initial universal set and A is the set of all parameters for X. Also,  $\tilde{X}$  and  $(\tilde{X}, \tilde{d}, A)$  will denote the absolute soft set and soft metric space with soft metric  $\tilde{d}$  respectively. Detailed study of soft real numbers, soft continuity and soft uniform continuity with related concepts is given in [2–4, 6, 9]. We have also discussed some basic definitions and related concepts of soft set theory in [11]. Therefore, we recall some other useful definitions related to soft uniform continuity and soft dense subsets which we shall use in our results below.

Let  $(\widetilde{X}, \widetilde{d}, A)$  and  $(\widetilde{Y}, \widetilde{\rho}, B)$  be two soft metric spaces. The mapping  $(\varphi, e)$ :  $(\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  is a soft mapping defined in [7], where  $\varphi : X \to Y$  and  $e : A \to B$  are two mappings

**Definition 2.1.** [2] A soft mapping  $(\varphi, e)$  is soft uniformly continuous if for given any  $\tilde{\epsilon} > \bar{0}$ , there exists  $\tilde{\delta} > \bar{0}$  (depending only on  $\tilde{\epsilon}$ ) such that for any soft points  $P_a^x, P_b^y \in \tilde{X}, \tilde{d}(P_a^x, P_b^y) \leq \tilde{\delta}$  implies  $\tilde{\rho}((\varphi, e)(P_a^x), (\varphi, e)(P_b^y)) \geq \tilde{\epsilon}$ .

**Definition 2.2.** [2] A soft metric space  $(\widetilde{X}, \widetilde{d}, A)$  is called soft sequential compact metric space if every soft sequence has a soft subsequence that converges in  $\widetilde{X}$ .

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**Definition 2.3.** [11] A soft set (Z, A) of  $\widetilde{X}$  is called soft dense in  $\widetilde{X}$  if  $\overline{(Z, A)} = \widetilde{X}$ . In other words, for every  $P_a^x \in \widetilde{X}$  and  $\widetilde{\epsilon} > \overline{0}$  there exists soft point  $P_b^y \in (Z, A)$  such that  $\widetilde{d}(P_b^y, P_a^x) \in \widetilde{\epsilon}$ .

Throughout this paper, (Z, A) will denote an arbitrarily fixed soft dense subset of  $\tilde{X}$ . The following results will be utilized in this paper.

**Theorem 2.1.** [11] A soft set (Z, A) of  $\widetilde{X}$  is said to be soft dense in  $\widetilde{X}$  if and only if for every  $P_a^x \in \widetilde{X}$  there exists a sequence  $\{P_{a_n}^{d_n}\}_n$  of soft points in (Z, A) converging to  $P_a^x$ .

**Theorem 2.2.** [11] Let  $(\tilde{X}, \tilde{d}, A)$  and  $(\tilde{Y}, \tilde{\rho}, B)$  be two soft metric spaces. Let  $(\varphi, e) : (\tilde{X}, \tilde{d}, A) \to (\tilde{Y}, \tilde{\rho}, B)$  be a soft mapping and  $\tilde{X} = (M, A) \cup (N, A)$  where  $(M, A) \cap (N, A)$  is soft dense in  $\tilde{X}$ . Then  $(\varphi, e)$  is soft continuous if  $(\varphi, e)|_{(M,A)}$  and  $(\varphi, e)|_{(N,A)}$  are both soft continuous.

**Theorem 2.3.** [11] Let  $(\varphi, e) : (\widetilde{Z}, \widetilde{d}_z, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  be a soft continuous mapping where (Z,A) is soft dense in  $\widetilde{X}$ . Then there exist a largest soft subset (W, A) of  $\widetilde{X}, (Z, A) \subseteq (W, A)$  such that  $(\varphi, e)$  can be extended to a soft map  $(f, e) : (\widetilde{W}, \widetilde{d}_w, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  which is soft continuous.

## 3. Results

We begin with the following characterization of soft uniformly continuous maps in terms of the sequences of soft points:

**Theorem 3.1.** Let  $(\widetilde{X}, \widetilde{d}, A)$  and  $(\widetilde{Y}, \widetilde{\rho}, B)$  be two soft metric spaces. A soft map  $(\varphi, e)$  is soft uniformly continuous if and only if for any sequences  $\{P_{a_n}^{x_n}\}_n$  and  $\{P_{b_n}^{y_n}\}_n$  of soft points in  $\widetilde{X}$ ,  $\widetilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \to \overline{0}$  implies  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \to \overline{0}$ .

Proof. Suppose  $\widetilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \to \overline{0}$  implies  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \to \overline{0}$  for all sequences  $\{P_{a_n}^{x_n}\}_n$  and  $\{P_{b_n}^{y_n}\}_n$  of soft points in  $\widetilde{X}$ . Let  $(\varphi, e)$  is not soft uniformly continuous, then there exists an  $\widetilde{\epsilon} > \overline{0}$  such that for every positive integer n, there exist soft points  $P_{a_n}^{x_n}$  and  $P_{b_n}^{y_n}$  in  $\widetilde{X}$  satisfying  $\widetilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \in \overline{\frac{1}{n}}$  and  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \geq \widetilde{\epsilon}$ . Therefore, there exist sequences  $\{P_{a_n}^{x_n}\}_n$  and  $\{P_{b_n}^{y_n}\}_n$  of soft ponts in  $\widetilde{X}$  such that  $\widetilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \to \overline{0}$  but  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n}))$ 

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does not converge to  $\overline{0}$ , which contradicts our assumption. Hence  $(\varphi, e)$  is soft uniformly continuous.

Conversely, Let  $(\varphi, e)$  is soft uniformly continuous,  $\tilde{\epsilon} \geq \bar{0}$  and  $\{P_{a_n}^{x_n}\}_n$  and  $\{P_{b_n}^{y_n}\}_n$  be sequences of soft points such that  $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \to \bar{0}$ . Since  $(\varphi, e)$  is soft uniformly continuous, there exist  $\tilde{\delta} = \tilde{\delta}(\tilde{\epsilon}) \geq \bar{0}$  such that for any pair of points  $P_a^x$ ,  $P_b^y$  in  $\tilde{X}$ ,  $\tilde{d}(P_a^x, P_b^y) \geq \tilde{\delta}$  implies  $\tilde{\rho}((\varphi, e)(P_a^x), (\varphi, e)(P_b^y)) \geq \tilde{\epsilon}$ . Also, we have  $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \to \bar{0}$  implies that there exists a positive integer  $n_0$  such that  $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \to \tilde{\delta}$  for all  $n \geq n_0$  and so,  $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \geq \tilde{\epsilon}$  for all  $n \geq n_0$ . Therefore,  $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \to \bar{0}$ .

Following theorem is a map gluing theorem of soft uniform continuity of a soft map in terms of its restriction to soft sequentially compact soft subsets of  $\tilde{X}$ :

**Theorem 3.2.** Let  $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  be a soft mapping and  $\widetilde{X} = (F, A)$  $\widetilde{\cup} (G, A)$  where (F, A) is arbitrary and (G, A) is soft sequentially compact. Then  $(\varphi, e)$  is soft uniformly continuous if  $(\varphi, e)|_{(F,A)}$  is soft uniformly continuous and  $(\varphi, e)$  is soft continuous at each point of (G, A).

*Proof.* Let (*φ*, *e*) is not soft uniformly continuous. Since (*φ*, *e*)|<sub>(*F*,*A*)</sub> and (*φ*, *e*)|<sub>(*G*,*A*)</sub> are both soft uniformly continuous, then by above theorem there exists an *ϵ* ≥ 0 such that for any positive integer n, there exist soft points  $P_{a_n}^{x_n}$  and  $P_{b_n}^{y_n}$  in (*F*, *A*) and (*G*, *A*) respectively satisfying  $\tilde{d}(P_{a_n}^{x_n}, P_{b_n}^{y_n}) \in \frac{1}{n}$  and  $\tilde{\rho}((\varphi, e)(P_{a_n}^{x_n}), (\varphi, e)(P_{b_n}^{y_n})) \ge \tilde{\epsilon}$ . Then as (*G*, *A*) is soft sequential compact,  $\{P_{b_n}^{y_n}\}_n$  has a subsequence  $\{P_{b_n_k}^{y_{n_k}}\}$  converging to a soft point  $P_b^y$  in (*G*, *A*), i.e.  $\tilde{d}(P_{b_{n_k}}^{y_{n_k}}, P_b^y) \to \bar{0}$ . It follows that corresponding subsequence  $\{P_{a_{n_k}}^{x_{n_k}}\}$  of  $\{P_{a_n}^{x_n}\}_n$  also converges to  $P_b^y$  since  $\tilde{d}(P_{a_{n_k}}^{x_{n_k}}, P_{b_{n_k}}^{y_{n_k}}) \in \frac{1}{n}$  and  $\tilde{d}(P_{b_{n_k}}^{y_{n_k}}, P_b^y) \to \bar{0}$ . Now, by soft continuity of (*φ*, *e*) at  $P_b^y \in (G, A)$ , we have (*φ*, *e*)( $P_{a_{n_k}}^{x_n}$ ) → (*φ*, *e*)( $P_{b_n}^{y_n}$ ), (*φ*, *e*)( $P_{b_n}^{y_n}$ ))  $\cong \tilde{\epsilon}$  for every n. Hence (*φ*, *e*) is soft uniformly continuous.

Following theorem is another map gluing theorem on soft uniformly continuous maps which characterizes soft uniform continuity in terms of soft dense subsets of  $\tilde{X}$ .

**Theorem 3.3.** Let  $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  be a soft map. Then  $(\varphi, e)$  is soft uniformly continuous if and only if  $(\varphi, e)|_{(Z,A)}$  is soft uniformly continuous and

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 $(\varphi, e)$  is soft continuous at each point of  $(Z, A)^c$  for some soft dense subset (Z, A) of  $\widetilde{X}$ .

 $\begin{array}{l} \textit{Proof. Let } \widetilde{\epsilon} > \overline{0}. \textit{ Since } (\varphi, e)|_{(Z,A)} \textit{ is soft uniformly continuous, there exists } \widetilde{\delta} > \overline{0} \\ \textit{ such that for any pair of soft points } P_e^t \textit{ and } P_f^s \textit{ in } (Z, A), ~ \widetilde{\rho}((\varphi, e)(P_e^t), (\varphi, e)(P_f^s)) \\ \widetilde{\epsilon} & \widetilde{\epsilon} \\ \widetilde{\epsilon} \\ \overrightarrow{\epsilon} \\ \textit{ whenever } \widetilde{d}(P_e^t, P_f^s) ~ \widetilde{\epsilon} \\ \widetilde{\delta}. \textit{ Let } P_a^x, ~ P_b^y ~ \widetilde{\epsilon} \\ \widetilde{\lambda} \\ \textit{ such that } \widetilde{d}(P_a^x, P_b^y) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta}. \textit{ Since } \\ (Z, A) \textit{ is dense in } \widetilde{X}, \textit{ there exist } P_e^t \textit{ and } P_f^s \textit{ in } (Z, A) \textit{ such that } \widetilde{d}(P_a^x, P_b^t) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \emph{ and } \widetilde{d}(P_b^y, P_f^s) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta}. \textit{ Now as } (\varphi, e) \textit{ is soft continuous at each point of } (Z, A)^c, \\ \widetilde{\rho}((\varphi, e)(P_a^x), (\varphi, e)(P_e^t)) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta} \\ \emph{ implies } \widetilde{\rho}((\varphi, e)(P_e^t), (\varphi, e)(P_f^s)) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta} \\ \emph{ implies } \widetilde{\rho}((\varphi, e)(P_e^t), (\varphi, e)(P_f^s)) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta} \\ \emph{ we have, } \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta} \\ \emph{ (} P_a^x, P_b^y) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \overrightarrow{\delta} \\ \emph{ we have, } \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) ~ \widetilde{\epsilon} \\ \widetilde{\delta} \\ \emph{ (} P_e^x), (\varphi, e)(P_f^s)) + \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_f^s)) ~ \widetilde{\epsilon} \\ \overrightarrow{\delta} \\ \emph{ implies } \widetilde{\epsilon} \\ \emph{ (} \varphi, e)(P_e^t), (\varphi, e)_n(P_f^s)) + \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^x)) ~ \widetilde{\epsilon} \\ \overrightarrow{\delta} \\ \emph{ (} \varphi, e)(P_e^x), (\varphi, e)_n(P_f^s)) + \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_a^s)) ~ \widetilde{\epsilon} \\ \overrightarrow{\delta} \\ \emph{ (} \varphi, e)(P_e^t), (\varphi, e)_n(P_f^s)) + \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_f^s)) ~ \widetilde{\epsilon} \\ \overrightarrow{\delta} \\ \emph{ (} \varphi, e)(P_e^t), (\varphi, e)_n(P_f^s)) + \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_f^s)) ~ \widetilde{\epsilon} \\ \overrightarrow{\delta} \\ \emph{ (} \varphi, e)(P_e^t), (\varphi, e)_n(P_f^s)) + \widetilde{\rho}((\varphi, e)(P_b^y), (\varphi, e)(P_f^s)) ~ \widetilde{\epsilon} \\ \overrightarrow{\delta} \\ \emph{ (} \varphi, e)(P_e^t), (\varphi, e)_n(P_f^s)) = \widetilde{\rho}(\varphi, e)(P_f^s) ~ \widetilde{\epsilon} \\ \emph{ (} \varphi, e)(P_e^t), (\varphi, e)(P_f^s)) = \widetilde{\epsilon} \\ \emph{ (} \varphi, e)(P_f^s) = \widetilde{\epsilon} \\ \emph{ (} \varphi, e)(P$ 

By using Theorem 2.2, we get the following corollary of above theorem

**Corollary 3.1.**  $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  be a soft map,  $\widetilde{X} = (F, A) \widetilde{\cup} (G, A)$ where  $(F, A) \widetilde{\cap} (G, A)$  is soft dense in  $\widetilde{X}$ . Then  $(\varphi, e)$  is soft uniformly continuous if  $(\varphi, e)|_{(F,A)\widetilde{\cap}(G,A)}$  is soft uniformly continuous and  $(\varphi, e)|_{(F,A)}$  and  $(\varphi, e)|_{(G,A)}$  are both soft continuous.

*Proof.* By Theorem 2.2, if  $(\varphi, e)|_{(F,A)}$  and  $(\varphi, e)|_{(G,A)}$  are both soft continuous then  $(\varphi, e)$  is soft continuous. Then by above theorem,  $(\varphi, e)$  is soft uniformly continuous

The following theorem is another characterization of soft uniform continuity of a soft map

**Theorem 3.4.** A soft map  $(\varphi, e)$  is soft uniformly continuous if and only if for any sequence of soft points  $\{P_{a_n}^{d_n}\}_n$  and  $\{P_{b_n}^{x_n}\}_n$  in (Z, A) and  $\widetilde{X}$  respectively,  $\widetilde{d}(P_{a_n}^{d_n}, P_{b_n}^{x_n}) \to \overline{0}$  implies  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{d_n}), (\varphi, e)(P_{b_n}^{x_n})) \to \overline{0}$ .

Proof. Let  $\widetilde{d}(P_{a_n}^{d_n}, P_{b_n}^{x_n}) \to \overline{0}$  implies  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{d_n}), (\varphi, e)(P_{b_n}^{x_n})) \to \overline{0}$  for all sequence of soft points  $\{P_{a_n}^{d_n}\}_n$  and  $\{P_{b_n}^{x_n}\}_n$  in (Z, A) and  $\widetilde{X}$  respectively. Now assume  $\{P_{e_n}^{x_n}\}_n$  and  $\{P_{f_n}^{y_n}\}_n$  be sequence of soft points in  $\widetilde{X}$  such that  $\widetilde{d}(P_{e_n}^{x_n}, P_{f_n}^{y_n}) \to \overline{0}$ . Now, as (Z, A) is soft dense in  $\widetilde{X}$ , for each n, there exist soft points  $P_{a_n}^{d_n}$  and  $P_{b_n}^{t_n}$  in (Z, A) such that  $\widetilde{d}(P_{e_n}^{x_n}, P_{a_n}^{d_n}) \to \overline{0}$  and  $\widetilde{d}(P_{b_n}^{t_n}, P_{f_n}^{y_n}) \to \overline{0}$ . Therefore, we get a sequence of soft points  $\{P_{a_n}^{d_n}\}_n$  and  $\{P_{b_n}^{t_n}\}_n$  in (Z, A) such that  $\widetilde{d}(P_{a_n}^{d_n}, P_{b_n}^{d_n})$ .

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 $\begin{array}{l} \rightarrow \ \bar{0}. \ \text{Then by assumption } \widetilde{\rho}((\varphi,e)(P_{a_n}^{d_n}),(\varphi,e)(P_{b_n}^{t_n})) \rightarrow \ \bar{0}. \ \text{It follows that if} \\ \widetilde{d}(P_{e_n}^{x_n},P_{f_n}^{y_n}) \rightarrow \ \bar{0} \ \text{then } \widetilde{\rho}((\varphi,e)(P_{e_n}^{x_n}),(\varphi,e)(P_{f_n}^{y_n})) \ \widetilde{\leq} \ \widetilde{\rho}((\varphi,e)(P_{e_n}^{x_n}),(\varphi,e)(P_{a_n}^{d_n})) + \\ \widetilde{\rho}((\varphi,e)(P_{a_n}^{d_n}),(\varphi,e)_n(P_{b_n}^{t_n})) + \ \widetilde{\rho}((\varphi,e)(P_{b_n}^{t_n}),(\varphi,e)(P_{f_n}^{y_n})) \ \widetilde{\leq} \ \widetilde{\underline{\epsilon}} \ + \ \widetilde{\underline{\epsilon}} \ = \ \widetilde{\epsilon} \qquad \Box \end{array}$ 

Combining Theorem 3.3 and Theorem 3.4 above, we get the following characterizations of soft uniform continuity of soft maps which is analog of Theorem 3.5 of [11] for soft uniformly continuous maps

**Theorem 3.5.** For any soft map,  $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ , the following are equivalent

- (1)  $(\varphi, e)$  is soft uniformly continuous.
- (2) for any sequence of soft points  $\{P_{a_n}^{d_n}\}_n$  and  $\{P_{b_n}^{x_n}\}_n$  in (Z, A) and  $\widetilde{X}$  respectively,  $\widetilde{d}(P_{a_n}^{d_n}, P_{b_n}^{x_n}) \to \overline{0}$  implies  $\widetilde{\rho}((\varphi, e)(P_{a_n}^{d_n}), (\varphi, e)(P_{b_n}^{x_n})) \to \overline{0}$ .
- (3)  $(\varphi, e)|_{(Z,A)} : (\tilde{Z}, \tilde{d}_Z, A) \to (\tilde{Y}, \tilde{\rho}, B)$  is soft uniformly continuous and  $(\varphi, e)$  is soft continuous at each point of  $(Z, A)^c$ .

From above theorem, we get the following corollary

**Corollary 3.2.** Let  $(\varphi, e) : (\widetilde{Z}, \widetilde{d}_z, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  be a soft continuous mapping where (Z,A) is soft dense in  $\widetilde{X}$ . Then there exist a largest soft subset (W, A) of  $\widetilde{X}$ ,  $(Z, A) \subseteq (W, A)$  such that  $(\varphi, e)$  can be extended to a soft map  $(f, e) : (\widetilde{W}, \widetilde{d}_w, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  which is soft uniformly continuous.

Proof. Let  $(Z', A) = (Z, A) \widetilde{\cup} P_a^x$  and  $(W, A) = \underset{P_a^x \in S}{\widetilde{\cup}} P_a^x$  where S be set of all soft points  $P_a^x$  in  $\widetilde{X}$  for which there exists a soft continuous extension of  $(\varphi, e)$ ,  $(f_x, e) : (\widetilde{Z'}, \widetilde{d}_{z'}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ . Then by Theorem 2.3, (W, A) is the largest soft subset of  $\widetilde{X}$ ,  $(Z, A) \widetilde{\subseteq} (W, A)$  such that  $(\varphi, e)$  can be extended to a soft map  $(f, e) : (\widetilde{W}, \widetilde{d}_w, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$  defined by  $(f, e)(P_a^x) = (f_x, e)(P_a^x)$  and (f, e) is soft continuous. Now by Theorem 3.5, we get (f, e) is soft uniformly continuous.

In the following corollary of above Theorem 3.5, we get analog of Corollary 3.2 in [11] for soft uniform continuity

**Corollary 3.3.** Let  $\{(F, A)_{\alpha} \mid \alpha \in \Lambda\}$  be soft cover of  $\widetilde{X}$ , that is  $\widetilde{X} = \bigcup_{\alpha \in \Lambda} (F, A)_{\alpha}$ such that  $\widetilde{\cap}_{\alpha} (F, A)_{\alpha}$  is soft dense in  $\widetilde{X}$  then a map  $(\varphi, e) : (\widetilde{X}, \widetilde{d}, A) \to (\widetilde{Y}, \widetilde{\rho}, B)$ is soft uniformly continuous if  $(\varphi, e)|_{(F,A)_{\alpha}}$  is soft uniformly continuous for some  $\alpha$ and soft continuous for every  $\alpha$ .

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*Proof.* Since  $(\varphi, e)|_{(F,A)_{\alpha}}$  is soft uniformly continuous for some  $\alpha$  and since by Corollary 3.2 of [11], the soft map  $(\varphi, e)$  is soft continuous. Then by above Theorem 3.5,  $(\varphi, e)$  is soft uniformly continuous.

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