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CUBIC DIFFERENCE LABELING FOR GRACEFUL TREE CONSTRUCTED FROM CATERPILLAR

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ABSTRACT. Let V and E represent the collection of vertices and edges denoted by $|E| = e$ and $|V| = n$ respectively in a connected, undirected, finite graph, $G = (V, E)$. G attains cubic difference labeling such that the edge set of G has labeled a weight defined by the absolute cubic difference of its end-vertices resulting in distinct weights with existence of an injection $f : V(G) \to \{0, 1, ..., p-1\}$. A graph which premits cubic difference labeling is said to be a cubic difference graph. In this paper, cubic difference labeling for graceful tree constructed from caterpillars are discussed.

1. INTRODUCTION

A cubic difference labeling of a graph G with size n is a function f, such that an injection exists from $V(G)$ to the set $\{0, 1, 2, \ldots, n\}$ when each edge uv of G has labeled the weight $|\,[f(u)]^3 - [f(v)]^3\,|$, resulting in distinct weights [7–9]. J.Shima introduced the notion of square difference labeling [5, 6, 11]. Already, the existence of cubic difference labeling for paths, cycles, stars, fan graph, wheel graphs, crown graphs, helm graphs, dragon graphs, coconut trees and shell graphs were discussed by J. Shiama [11]. Graph labeling are widely used in Mobile telecommunication, military offices, Biometric and communication network [7, 8].

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Definition 1.1. *[1,2] **Caterpillar Tree:** A tree in which all the vertices are within distance of a central path is said to be a caterpillar tree. A tree is a caterpillar if all nodes of degree≥ 3 are surrounded by atmost two nodes of degree two or greter.*

Definition 1.2. *[3, 4, 10] A graph which satisfies the cubic difference labeling is called the cubic difference graph.*

2. Main Result

Lemma 2.1. *The vertex labels of vertices of T^* defined by the labeling function f^* are distinct.*

Theorem 2.1. *Every Graceful Tree T^* admits cubic difference labeling.*

Proof. Let T_1 be a caterpillar with m_1 edges with graceful labeling function f_1. Consider (V_{11}, V_{12}) as the bipartition of vertex set $V(T_1)$ such that the assignment of vertex labels in V_{11} defined by f_1 is less than (or) equal to α_1, where α_1 represent the width of α-labeling of T_1. Let T_2 be a caterpillar which has m_2 edges with graceful labeling function f_2. Consider (V_{21}, V_{22}) as the bipartition of vertex set $V(T_2)$ such that the labels of vertices in V_{21} defined by f_2 is less than (or) equal to α_2, where α_2 represents the width of α-labeling of T_2. Let g_1 be the cubic difference labeling function for caterpillar T_1 and g_2 be the cubic difference labeling function for caterpillar T_2.

Let us define the vertex cubic difference labeling of T_1 as $g_1(x_i) = i,\ for\ \ 0 \leq i \leq n$ and define vertex cubic difference labeling of T_2 as $g_2(w_i) = i,\ for\ \ 0 \leq i \leq n$ and define vertex cubic difference labeling of T^* as $g^*(u_i) = i,\ for\ \ 0 \leq i \leq n.$

Let as define edge cubic difference labeling of T_1 as $g_1(uv) = |[g_1(u)]^3 - [g_1(v)]^3|$ for any edge $uv \in E(T_1)$ and define edge cubic difference labeling of T_2 as $g_2(sj) = |[g_2(j)]^3 - [g_2(s)]^3|$ for any edge $sj \in E(T_2)$ and define edge cubic difference labeling of T^* as $g^*(tk) = |[g^*(t)]^3 - [g^*(k)]^3|$ for any edge $tk \in E(T^*)$.

A new tree $T^* = (T_1 \cup T_2) + e$ is constructed from T_1 and T_2 by adding a new edge between the vertex label 0 of T_1 and the vertex with label $(m_2 + 1)$ of T_2. We observe that T^* contains $(|V_1| + |V_2|)$ vertices and $(m_1 + m_2 + 1)$ edges. It is clear from Lemma 2.1 that, vertex labels of T^* are distinct. Also, from the definition of cubic difference labeling, edge labels of T_1 and T_2 are distinct.

Thus edge labels of T^* formed from T_1 and T_2 are also distinct. Hence the proof. □

3. Illustrative Example

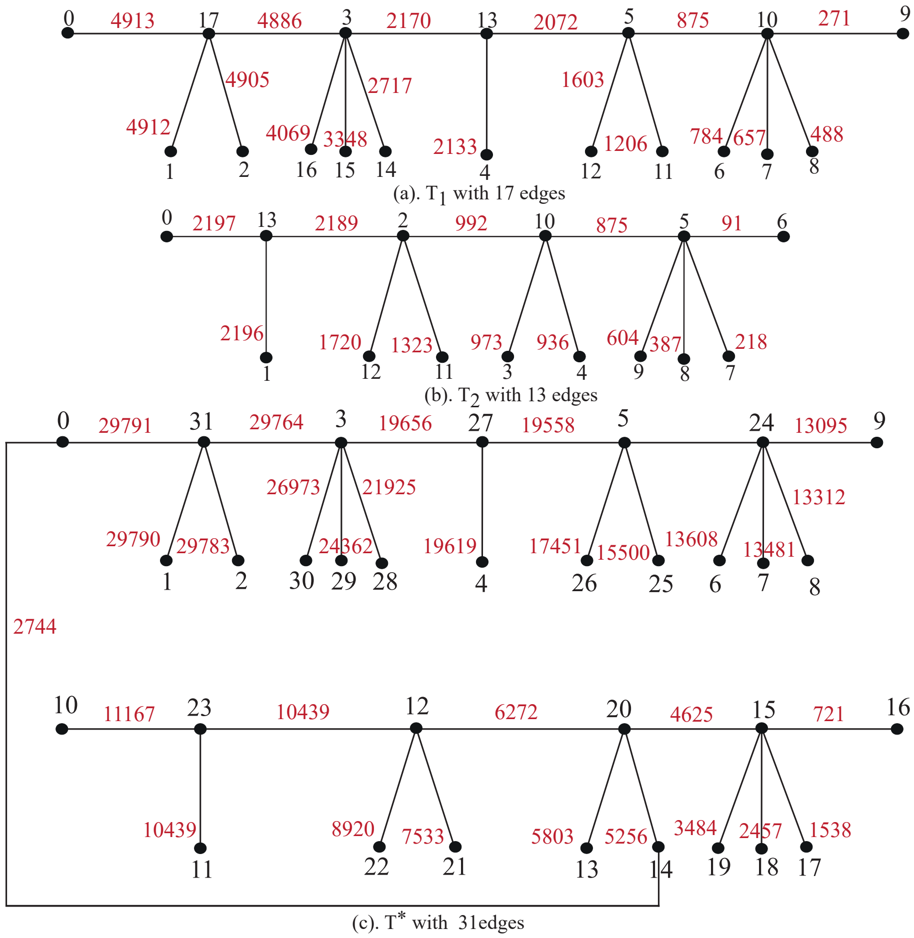

FIGURE 1. Graceful Tree T^* with 31 edges

4. Conclusion

In this paper, we have constructed graceful tree from a pair of caterpillars provided that edge set may not be equal. The graceful tree thus constructed can be modified by replacing new edge joined already between the caterpillars T_1 and T_2 with another edge with the condition that induced new edge label is also $m_2 + 1$. Thus, with the same caterpillar as input, we can construct different trees. We can, thus construct different graceful trees in which all the trees have $|V_1 + V_2|$ vertices and $|m_1| + |m_2| + 1$ edges.

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