

LOGIC METHOD OF CLASSIFICATION OF OBJECTS WITH NON-JOINING CLASSES

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ABSTRACT. The article deals with the logical method of recognition of objects and the construction of corrective functions. To build a logical method of classifying objects, a test algorithm is used and is included in the search for dead-end table tests. And also the method of constructing the optimal continuation of logical corrective functions is applied. An estimate of the complexity of the search algorithm for deadlock tests is calculated.

1. INTRODUCTION

In logical recognition systems, logical methods based on discrete analysis and the propositional calculus based on it are used to construct the recognition algorithms themselves. In general, the logical recognition method involves the presence of logical connections expressed through a system of Boolean equations in which the variables are the logical signs of recognizable objects or phenomena.

Logical signs of recognizable objects can be considered as elementary statements that take two values of truth: truth and falsehood.

Logical, first of all, are signs that do not have a quantitative expression. These signs are judgments of a qualitative nature (the presence or absence of some properties or some elements of recognizable objects or phenomena). In medical

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2020 *Mathematics Subject Classification.* 68T10.

Key words and phrases. Discrete Mathematics, Combinatorics.

diagnostics the logical signs may be the following symptoms: sore throat, cough, runny nose, etc. The type of engine of a recognized aircraft — jet, turboprop or piston — can also be considered as a logical feature. In geology, logical signs may include solubility or insolubility in certain acids or in some mixtures of acids, the presence or absence of odor, color, etc.

Also, the signs that have a quantitative expression may be logical, however not the magnitude of the sign of the recognized object is important, but the only fact of falling or not falling into the specified interval. In practice, logical signs of this kind take place in situations where measurement errors can either be neglected or the intervals of the values of the signs are chosen in such a way that the measurement errors have practically no effect on the reliability of decisions regarding the measurement being falling within a given interval.

2. STATEMENT OF THE PROBLEM

Let M be the set of objects with respect to which classification is performed. The set M can be represented as the sum of the subsets $K_1, K_2, \dots, K_l$, usually called classes. About classes some information $J_0(K_1, K_2, \dots, K_l)$ is given (training information). Objects $S \in M$ are defined by their descriptions $J(S)$. The task is to determine which class the object $S[I]$ belongs to from the description $J(S)$ and initial information J_0 .

We introduce l a dimensional predicate $P(S) = j$, if S it belongs to a class K_j , $j = 1, 2, \dots, l$.

We formulate the classification problem with disjoint classes $K_1, K_2, \dots, K_l$. Let the training information be given in the form of a training table $T_{n,m}$. Table rows are standard object descriptions $S_1, S_2, \dots, S_m$.

The set M are divided into levels. At the zero level is the smallest element $\alpha \in M$. Let the built i level. Level with number $i + 1$ consists of many elements. $B = \{b : \exists c \text{ from } i - \text{th level and } \exists d \in M, \text{ such that } c \leq b \leq d\}$.

The dialing norm $\tilde{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)$ in $\tilde{M}$ is the number $|\tilde{\alpha}| = \sum_{i=1}^n |\alpha_i|$, where $|\alpha_i| = j$, if α_j belongs to the j -th level.

A sequence of sets $\{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_s\}$ is called a chain in $\tilde{M}$ if, for any $j = \overline{1, s-1}$, $\tilde{\alpha}_{j+1}$ immediately follows the set $\tilde{\alpha}_j$ ($|\tilde{\alpha}_{j+1}| = |\tilde{\alpha}_j| + 1, j = \overline{1, s-1}$).

The number of elements of the chain S is called the length of the chain and is denoted by $d(S)$. A single point is a chain of length equal to one. The element of the chain having the smallest norm is called the minimum element of the chain.

The distance $\rho(\tilde{\alpha}, \tilde{\beta})$ between sets $\tilde{\alpha} = (\alpha_1, \dots, \alpha_n)$ and $\tilde{\beta} = (\beta_1, \dots, \beta_n)$ of the structure $\tilde{M}$ is defined as $|p(\tilde{\alpha}, \tilde{\beta})| = \sum_{i=1}^n |\alpha_i| - |\beta_i|$.

A ball of radius r and center $\tilde{\alpha} \in \tilde{M}$ is a set of points $\tilde{\beta}$ of the structure $\tilde{M}$ such as $\rho(\tilde{\alpha}, \tilde{\beta}) \leq r$.

Let a partial order

$$(2.1) \quad 0 < 1, \quad 0 < 2$$

be given on the set $S = (0, 1, 2)$.

In the set about of tuples $S^n = \{\tilde{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n) : \alpha_i \in \{0, 1, 2\}\}$, order (2.1) induces a partial order.

$$\tilde{\beta} = (\beta_1, \dots, \beta_n) \leq \tilde{\gamma} = (\gamma_1, \dots, \gamma_n),$$

if $\beta_i \leq \gamma_i$, by (2.1), where $i = \overline{1, n}$.

Consider the structure S^n . In it there exists a unique minimal element such as $(0, 0, \dots, 0)$ and 2^n incomparable maximal elements of sets, all coordinates of which belong to the set $\{1, 2\}$. We divide S^n into levels.

The level U_j , consists of all sets for which j exactly coordinates takes values from $\{1, 2\}$ and the rest $n - j$ of the coordinates are zero.

Obviously, $|U_j| = c_n^j 2^j$ where $|L|$ is the power of the set L . As usual, by a chain in S^n we mean a set $\{\tilde{\alpha}_{i_1}, \tilde{\alpha}_{i_2}, \dots, \tilde{\alpha}_{i_k}\}$ such as $\tilde{\alpha}_{i_1} \leq \tilde{\alpha}_{i_2} \leq \dots \leq \tilde{\alpha}_{i_k}$ and $\alpha_{ij} \in U_{ij}$, $i = \overline{0, k}$, $(1 \leq k \leq n + 1)$.

The dialing norm $\tilde{\alpha} = (\alpha_1, \alpha_2, \dots, \alpha_n)$ in S^n is a number $|d| = \sum_{j=1}^n |d_j|$ where

$$|d_j| = \begin{cases} 0, & \text{if } \alpha_i = 0, \\ 1, & \text{otherwise.} \end{cases}$$

The number $A_n = \sum_{i=1}^n d_j k^{n-1}$ will be called the number $(\alpha_1, \alpha_2, \dots, \alpha_n) \in \tilde{M}$.

It is said that sets $\tilde{\alpha}$ of the U_k level are ordered in $\tilde{M}$, $(k = \overline{0, n})$ lexicographic (anti-lexicographic) order if they are located at the U_k level in descending (increasing) order of numbers $A_{\tilde{\alpha}}$.

It is believed that the set $\tilde{\alpha}$ immediately follows $\tilde{\beta}$ at the U_k level in, if $A_{\tilde{\beta}} < A_{\tilde{\alpha}}$ and there is no such thing $\tilde{\gamma}$ in U_k as $A_{\tilde{\beta}} < A_{\tilde{\gamma}} < A_{\tilde{\alpha}}$.

Let the sets $\tilde{M}$ at the levels be ordered in lexicographic order.

We say that $\tilde{M}$ it has the property P_l if, from the fact that $(\alpha_1, \alpha_2, \dots, \alpha_m)$ the first m elements of level U_i in $\tilde{M}$, $(\beta_1, \beta_2, \dots, \beta_l)$ are the first l elements, U_{i+1} and $\tilde{\alpha}_m < \tilde{\beta}_i$ it follows that for any $\tilde{\beta}_i$, $(i = \overline{1, l-1})$ there exists $\tilde{\beta}_j$, $j = \overline{1, l}$, such as $\tilde{\alpha}_j < \tilde{\beta}_i$.

Let the sets $\tilde{M}$ at the levels be ordered in anti-lexicographic order.

We say that $\tilde{M}$ it has the property P_{al} if, from the fact that $(\alpha_1, \alpha_2, \dots, \alpha_m)$ the first m elements of level U_i in $\tilde{M}$, $(\beta_1, \beta_2, \dots, \beta_l)$ are the first l elements, U_{i+1} , $i = \overline{0, n-1}$, $\tilde{\alpha}_i \leq \tilde{\beta}_j$ it follows that for any $\tilde{\alpha}_{ii}$, $(i = \overline{1, m-1})$ there exists $\tilde{\beta}_j$, $j = \overline{1, l}$, such as $\tilde{\alpha}_j \leq \tilde{\beta}_i$.

It is easy to see that for the cube E_n^2 the properties P_l , P_{al} . The structure S_n where $S = \{0, 1, 2\}$ and $0 < 1$, $0 < 2$, have the property P_l , but does not have the property P_{al} .

We will consider functions $f(x_1, x_2, \dots, x_n)$ given on sets $\tilde{M}$ and taking values from M .

A function $f(x_1, x_2, \dots, x_n)$ is called monotonic relative order $\leq$ if, for any sets $\tilde{\alpha}$ and $\tilde{\beta}$ in $\tilde{M}$ such as $\tilde{\alpha}_i \leq \tilde{\beta}_j$, the relation $f(\tilde{\alpha}_i) \leq f(\tilde{\beta}_j)$.

Denote by M_n the set of all monotone functions of n variables on the structure $\tilde{M}$.

The collection of all functions that are monotonic with respect to order (1) is called a class σ .

The set $\tilde{\alpha} \in \tilde{M}$ is called the upper zero (lower unit) of the function $f \in M_n$ if, $f(\tilde{\alpha}) = 0$ ($f(\tilde{\alpha}) \neq 0$) and for any set $\tilde{\beta} \in M^n$ from $\tilde{\alpha} \leq \tilde{\beta}$ ($\tilde{\alpha} \geq \tilde{\beta}$), it follows that P_1 .

The upper zero (lower unit) $\tilde{\alpha}$ of a function $f \in M_n$ is called its minimum lower unit M.L.U., if for any upper zero (lower unit) $\tilde{\beta}$ of f function will be $|\tilde{\beta}| < |\tilde{\alpha}|$ ($|\tilde{\beta}| > |\tilde{\alpha}|$).

Let an arbitrary function $f \in M_n$ be defined with the help of some operator A_f , which yields a value $f(\tilde{\alpha})$ for any set $\tilde{\alpha}$.

There are two main tasks associated with monotonic functions and often found in applications of mathematical logic:

The problem of decoding monotone functions. If some (known to us) monotonic function $f \in M_n$ is specified by the operator A_f , then it is required by a minimum number of calls to A_f completely restore the table of values of the monotonic function $f(x_1, x_2, \dots, x_n)$, those determine the value of this function at all points $\tilde{M}$.

Search task m.u.z. monotonic functions. If some function $f \in M_n$ is specified by the operator A_f , then a minimum number of calls will be required to A_f find at least one m.u.z. functions $f(x_1, x_2, \dots, x_n)$. Obviously, a similar task is to search for medical science. Functions $f(x_1, x_2, \dots, x_n)$ with the minimum number of calls to the operator.

Consider the many algorithms $\{F\}$ that allow to solve the decryption problem, those many algorithms that for an arbitrary function $f \in M_n$ using the operator A_f completely restore the table of values $f(x_1, x_2, \dots, x_n)$.

Let be $\varphi(F, f)$ the number of calls to the operator A_f sufficient to restore the table of values of a monotonic function $f(x_1, x_2, \dots, x_n)$ when applying the algorithm F .

Consider the functions $\varphi(F, n) = \max \varphi(F, f)$ and $\varphi(n) = \min \varphi(F, n)$. A function $\varphi(n)$ is the minimum number of calls to the operator A_f sufficient to restore the table of values of a monotonic function using an algorithm that solves the problem.

The function $\varphi(n)$ is called the Shannon function. It was shown in [1] that equality is valid for monotone Boolean functions $f(x_1, x_2, \dots, x_n)$ is fair equality $\varphi(n) = C_n^{[n/2]} + C_n^{[n/2]+1}$.

Let be the set $\{B\}$ of search algorithms for m.u.z. arbitrary function $f \in M_n$ using the operator A_f .

We introduce a function $\mu(B, f)$ - the number of calls to the operator A_f , sufficient to find the m.u.z. functions $f \in M_n$.

The function $\mu(n) = \min_{B \in \{B\}} \max_{f \in \{M_n\}} \mu(B, f)$ is called the Shannon function for the m.u.z.

It is known that many extreme problems come down to the search for m.u.z. discrete monotone functions.

In [2], this problem was solved in the Shannon statement. According to [2], for the search problem the class of all monotone Boolean functions $\mu(n) = C_n^{[n/2]} + 1$. In [2], it was proved that for a class of functions of k -valued logic

that are monotone with respect to the natural order and take two values, the Shannon function $\mu(n) = \omega(E_n^k) + 1$ with $k > 2$, $n \geq 2$ the largest number of pairwise incomparable sets that can be selected in the set of k valued sets E_n^k .

For some subclass M_n^m of monotone Boolean functions

$$\mu(n) = \begin{cases} C_n^{[n/2]} + 1, & \text{if } n - 1 \leq m \leq [n/2], \\ C_n^m + 1, & \text{if } f \leq m \leq [n/2]. \end{cases}$$

Here M_n^m is the set of all monotone Boolean functions $f(x_1, x_2, \dots, x_n)$ such that for any $\tilde{\alpha} \in E_n^2$ from $|\tilde{\alpha}| \geq m + 1$ follows $f(\tilde{\alpha}) = 1$.

For some subclasses of discrete monotone functions, the estimate was proved [4]: $\varphi(n) = O\left(\frac{3^n}{\sqrt{n}}\right)$.

3. METHODS FOR FINDING THE MINIMUM LOWER UNITS OF DISCRETE MONOTONE FUNCTIONS

Let M.L.U. $g \in M_n$ belongs to a set $\bigcup_{i=r_1}^{r_2} U_i$ of structure $\tilde{M}$, where $\tilde{M}$ has properties P_{al} , $0 \leq r_1 \leq r_2 \leq n$.

Consider the description of the algorithm $\tilde{R}_1$ of search of the function such as $g(x_1, x_2, \dots, x_n)$ on the set of sets of levels $U_{r_1}, U_{r_1+1}, \dots, U_{r_2}$ which is a modification of the search algorithms m.u.z. function $g \in M_n$. In this algorithm, the value of the function $g(x_1, x_2, \dots, x_n)$ on the sets $\tilde{M}$ will be considered in the anti-lexicographic order:

- (i) At the first step of the algorithm, we calculate the value of g on the set with the lowest number of the r_2 level. Let's on $i - m$ step compute the values of g on a certain set $\tilde{\alpha}(r_2 - (i + 1))$ of the level $r_1 \leq i \leq r_2$. If $g(\tilde{\alpha}_i) \neq 0$ on the step $(i + 1) - m$ requires the value of g on the set $\tilde{\alpha}_{i+1}$ with the lowest level number $(r - i)$. If $g(\tilde{\alpha}_{i+1}) = 0$, then, we pass also to item 2. If on the step $(r_2 - r_1 + i), g(\tilde{\alpha}_{r_2-r_1+i}) \neq 0$, is received, then the algorithm finishes work.
- (ii) Let $(r + i)$ of $(0 < r < r_2 - r_1)$, steps be taken calculated values $g(\tilde{\alpha}_1) = g(\tilde{\alpha}_2) = \dots g(\tilde{\alpha}_r) = g(\tilde{\alpha}_{r+1}) = 0$, where $\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_{r+1}$ are the sets with the lowest level numbers $U_{r_2}, \dots, U_{r_2-r}$ respectively. Then at $(\tau + 2)$ step $\tilde{\alpha}_{r+1}$, the value g on $\tilde{\alpha}_{r+1}$ set is calculated in lexicographic order at the $(r_2 - \tau)$ level. Let at step $i \geq \tau + 2$ the value at the set $\tilde{\alpha}_j, j$ level is calculated, where $j \geq r_2 - \tau$.

If $g(\tilde{\alpha}_i) = 0$ at $(i + 1)$ step, the value g is calculated on $\tilde{\alpha}_{i+1}$ set immediately following $\tilde{\alpha}_j$ the same j level. If such a set does not exist, then the algorithm terminates. If $g(\tilde{\alpha}_i) \neq 0$ and $\tilde{\alpha}_i \in U_{r_1}$, then the algorithm also finishes work. In the case of $g(\tilde{\alpha}_i) \neq 0$ and $\tilde{\alpha}_i \in U_{r_1}$, $(j < r_1)$ at $(i+1)$ step, the value g is calculated on the set of $\tilde{\alpha}_{i+1}$, $(j - 1)$ level, which immediately follows $\tilde{\alpha}_i$ in lexicographic order with the highest number of $j - 1$ level such as $g(\tilde{\alpha}_i)$ is determined by the monotonicity of $g(\tilde{\alpha}_i) = 0$. In the case when there is no set $\tilde{\alpha}_{i+1}$ satisfying the indicated properties, the algorithm stops.

Let the algorithm take S steps to a stop. Then we get a chain of sets such as $\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_s$ on which the value $g(x_1, x_2, \dots, x_n)$ was sequentially calculated. As a result, M.L.U. $g(\tilde{x})$ is $\tilde{\alpha}_k$ set from this chain with maximum k such as $g(\tilde{\alpha}_k) \neq 0$.

If the structure $\tilde{M}$ has the property P_l and m.u.z. function $g(\tilde{x})$ construct an algorithm R' for searching for m.u.z. function $g(\tilde{x})$. In this case, the lexicographic order of the level sets is $\tilde{M}$.

The algorithm R', R_1' allows you to effectively solve many extreme problems. However, when there is $n \geq 1$ they become inappropriate from the point of view of the given resources. Therefore, the urgent problem is the development of simpler heuristic algorithms that allow you to effectively build the search process of m.u.z., as well as taking into account the results of the previous steps.

Let the structure $\tilde{M}$ of n dimensional sets consist of t levels $U_1, U_2, \dots, U_t$, $|U_1| = \max_{i=1,2,\dots,t} |U_i|$, $L = |U_l|$. Fix numbers $\tau(1 \leq r \leq L), p(l < p < t, 0 < p < l)$.

We consider the sets $M_k \subseteq U_k, |M_k| = r$ such that for any $M_k \subseteq U_k, |M_k| = r$ has a place $\min_{\tilde{\alpha}, \tilde{\beta} \in M_k} p(\tilde{\alpha}, \tilde{\beta}) \geq \min_{\tilde{\alpha}', \tilde{\beta}' \in M_k} p(\tilde{\alpha}', \tilde{\beta}')$, where $k = l - p, l - p + 1, l + p$.

$$Put \ g = \min_{\tilde{\alpha}, \tilde{\beta} \in M_k} p(\tilde{\alpha}, \tilde{\beta}).$$

Let the task Z be to calculate q for sets M of M_k type such as $|M_k| = r$. Denote by $\varphi(k, n, q)$ the number of points at the intersection of the level U_k with a ball of radius q and center $\tilde{\alpha} \in U_k$.

For the number C of intersecting balls of radius q , the estimate $C \leq \frac{|U_k|}{\varphi(k, n, q)}$ is true. Let, $\varphi(k, n, q) = \frac{|U_k|}{r}$ based on this equality, we determine the functional dependence for $q_r = \psi(k, n, r)$ and based on the given one r , we calculate the values $q = \psi(k, n, r)$.

The following statement was obtained in [3]. Let L be an arbitrary subset of vertices E_n^2 , $|L| = mG_n$ group code in E_n^2 with d distance, and $|G_n| = B$. Then there exists the subset $L_d \subseteq L$ all points of which lie at least at d distance from each other and which contains at least $B \frac{m}{2^n}$ points. From the results of [3] it follows that in E_n^2 there is a group code with a distance $2r + 1$ by the number of elements $B(n, r) \geq C \frac{2^n}{n^r}$, $(1 < C < 2)$.

Let $L = U_i \subseteq E_n^2$, $|U_i| = C_n^2$, $i = \overline{0, n}$, $r = 1$, $M \subseteq U_i$ before $\forall \tilde{\alpha}, \tilde{\beta} \in M$, where $i = 0, 2, 4, \dots$

By assertion [2], we can assume that

$$C \frac{C_n^i}{n^i} \leq \frac{B(n, l)}{2^n} C_n^i \leq |M|.$$

In this way,

$$C \frac{C_n^i}{n^l} \leq |M| \leq C' \frac{C_n^i}{n^l}.$$

Consider a parametric search algorithm $A(p, r, h, H)$ M.L.U. discrete monotonic function $f(M_n)$ given on the structure $\tilde{M}$, where p is the level of the research threshold of the lower units, r – the number of chains along which the initial search for the lower units is conducted, h – the research step at the levels and H – the number of steps to achieve the goal.

For all levels U_k of $\tilde{M}$ structure, we find the distance $q_k = [\frac{U_k}{r}]$, $k = 1 - p, l - p + 1, \dots, l + p$.

We construct r chains passing through the points of $\tilde{M}$ set so that for any $\tilde{\alpha}$ and $\tilde{\beta}$ of the various chains $p(\tilde{\alpha}, \tilde{\beta})$, the maximal is $(p(\tilde{\alpha}, \tilde{\beta}) \geq q_k)$.

The construction of chains is carried out by $i = \overline{1, p}$, induction starting from the level U_l and ending U_{l+p} .

Let $M_l = \{\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_r\}$. At the first step, we take $\tilde{\alpha}_1, \tilde{\alpha}_2, \dots, \tilde{\alpha}_r$ for the minimum circuit elements r .

Let the chains of $(i - 1)$ -th length be constructed at the $(i - 1)$ step. For any $\tilde{\alpha}, \tilde{\beta} \in M_{i-1}$, the equality $p(\tilde{\alpha}, \tilde{\beta}) \geq q_{i-1}$ i step. For each set $\tilde{\alpha}_j \in M_{i-1}$, $j = \overline{1, r}$, there are $l = n - i + 1$ comparable sets from i level: $\tilde{\beta}_{i_1}^{\tilde{\alpha}_j}, \tilde{\beta}_{i_2}^{\tilde{\alpha}_j}, \dots, \tilde{\beta}_{i_l}^{\tilde{\alpha}_j}$.

For $\tilde{\alpha}_1$ we choose an arbitrary set $\tilde{\beta}_i^{\tilde{\alpha}_1}$. For $\tilde{\alpha}_2$ we choose $\tilde{\beta}_i^{\tilde{\alpha}_2}$ which does not enter the ball of radius $2q_i$ of $\tilde{\beta}_i^{\tilde{\alpha}_1}$ center. For $\tilde{\alpha}_j$ we choose $\tilde{\beta}_i^{\tilde{\alpha}_j}$, not included in the balls having $2q_i$ with centers $\tilde{\beta}_i^{\tilde{\alpha}_1}, \tilde{\beta}_i^{\tilde{\alpha}_2}, \dots, \tilde{\beta}_i^{\tilde{\alpha}_{j-1}}$. If this does not exist, then select the set $\tilde{\beta}_i^{\tilde{\alpha}_j}$ located at the maximum distance from $\tilde{\beta}_i^{\tilde{\alpha}_1}, \tilde{\beta}_i^{\tilde{\alpha}_2}, \dots, \tilde{\beta}_i^{\tilde{\alpha}_{j-1}}$.

The chains at levels from U_l to U_{l-p} are constructed similarly. Thus, we obtain r chains of $2p$ length. Further, referring to A_f operator, we calculate the function

on the sets of constructed circuits. As a result, for each circuit we find a local minimum.

Among local minima, a unit with a minimum shape is selected. Then we will calculate the function on sets lying one level lower than $\tilde{\alpha}$ and not included in $\{\tilde{\gamma}\}$ set of vertices $\tilde{\gamma}$ comparable with at least one local minimum. Moreover, the value of the function will be calculated not on each set, but through h sets in $(1 \leq h \leq H)$, starting from some fixed set.

If there is $\tilde{\alpha}$ such $f(\tilde{\alpha}) \neq 0$, then we will look for the lower unit among the sets $\tilde{\beta}$ such as $\tilde{\beta} \leq \tilde{\alpha}$. Further research will be carried out in the same way as at the previous level.

When the number of calls to the operator A_f reaches H the search stops. The last found lower unit of function $f(\tilde{x})$ is taken as the minimum.

If the found medical unit does not satisfy the requirements, then by changing the parameters r, H, p, h , we can continue the search until an acceptable unit f is found.

4. IMPLEMENTATION OF ALGORITHMS FOR SOLVING DISCRETE EXTREMAL PROBLEMS

Consider a symmetric Boolean function $f(x_1, \dots, x_n)$ from n variables, the scope of which is sets of levels $U_{p+1}, U_{p+2}, \dots, U_{p+l}$ to $E_n^2, i, p \in \{0, 1, \dots, n\}$. Shortened d.n.f. D_c^f function $f(x_1, \dots, x_n)$ consists of EC corresponding to the maximum intervals in $\bigcup_{j=0}^i U_{p+j}$.

The object of research is d.n.f. D_c , consisting of e.k. abbreviated d.n.f. D_c^f function f , moreover $N_{f'} \subset N_f$.

Are being investigated d.n.f. $D'_{f'}$ with the number of e.k. $N \leq 2^m$ and $m = 5, 6$. In the case when $N = 2^m$, using codes of workplaces we build chains for the parametric algorithm to work according to the selected algorithm parameters.

If $N \neq 2^m$, cyclic codes are used to build circuits.

Table 1 presents the results of a machine experiment of the algorithm $A(p, r, h, H)$ to minimize d.n.f. $D'_{f'}$ function f' :

- (i) in the 1st, column of the table the number of e.k. in the original d.n.f.;
- (ii) in the 2nd, the number of variables used in d.n.f.;
- (ii) in the 3rd, a method for constructing the starting points of chains;

- (iv) in the 4th, the minimum Hamming distance between the chains;
- (v) in the 5th, the initial level, from which the search for the medical unit begins;
- (vi) in the 6th, the number of chains with the Hamming distance indicated in "see table 1".:

TABLE 1. The results of a machine experiment of the algorithm

1	2	3	4	5	6	7	8	9	10	11	12	13
210	15	CYC	10	10	42	169	146	6	105	235	118	29 min 49.90 s
210	15	CYC	6	16	70	169	174	11	105	273	118	31 min 48.79 s
128	8	CYC	2	24	228	71	182	3	56	235	80	9 min 21.78 s
128	8	CYC	2	4	128	120	177	107	56	102	86	54 min 37.19 s
90	6	CYC	10	5	18	74	36	1	19	96	76	3 min 21.78 s
90	6	CYC	4	5	45	80	61	46	19	339	72	6 min 1.3 s
64	6	WP	16	16	301	24	316	83	16	710	72	7 min 49.88 s
64	6	WP	16	16	301	30	315	139	16	689	72	6 min 23.57 s
64	6	WP	4	4	140	52	156	139	17	333	72	3 min 27.68 s
64	6	WP	4	4	140	56	156	139	17	889	72	6 min 23.57 s
64	6	CYC	8	8	16	52	29	11	16	79	72	1 min 37.21 s
64	6	CYC	6	6	21	52	34	12	16	85	72	1 min 35.12 s
64	6	CYC	6	6	21	56	34	15	16	141	72	1 min 54.48 s
64	6	CYC	4	4	32	49	133	1	16	170	72	3 min 39.67 s
64	6	CYC	4	4	32	49	45	1	16	82	72	1 min 40.20 s
64	6	CYC	4	4	32	56	44	26	16	205	72	2 min 18.01 s

- (i) in the 7th, research bandwidth (chain length);
- (ii) in the 8th, the specified number of sets, by which the search lower minimum unit (L.M.U.) for medical units is conducted in the second stage of the algorithm. Starting from the 9th column of the table, the results of a machine experiment are given:
- (iii) in the 9th, column indicates the number of cube sets found on which the value of the function is zero;
- (iv) in the 10th, the number of chains on which the value of the function is equal to one;

- (v) in the 11th, the number of the level at which the medical unit was found L.M.U.;
- (vi) in the 12th, the number of calls to the operator A_f , calculating the value of a function f ;
- (vii) in the 13th, the amount of memory used by the machine;
- (viii) in the 14th, the time of the program.

From table 1 it follows that, based on Reed-Miller codes, more chains are obtained than on the basis of cyclic codes with the same algorithm parameters $A(p, r, h, H)$. However, when researching circuits built using Reed-Miller codes, the program's runtime increases sharply, since a large number of circuits in a cube are investigated E_n^2 . But, on the other hand, the study of circuits according to the Reed-Miller code, despite the increase in the program run time, allows us to determine a larger number of sets in the cube E_n^2 and get the most effective results. In addition, a considerable amount of time is spent on constructing the starting points of the chains using the Reed-Miller codes. Thus, the proposed algorithms R' , R'_1 , allow solving many extreme problems. However, for $n \gg 1$ they become impractical from the point of view of the given resources. Therefore, an urgent problem is the development of simpler heuristic algorithms that make it possible to effectively build the process of searching for m.u.z., as well as taking into account the results of the previous steps. As such an algorithm, the paper proposes a parametric algorithm $A(p, r, h, H)$ for finding the minimum lower units of discrete monotone functions. Moreover, with the growth of the parameters, the complexity of the algorithm grows and the exact solution is approached. On the basis of changing the parameters of the algorithm, its complexity is regulated.

CONCLUSION

The solution of many discrete extremal problems reduces to solving the problems of deciphering or searching for the minimum lower units of discrete monotone functions, for which an effective parametric method was developed in this paper and machine results were obtained.

In this paper, algorithms are proposed for solving certain classes of discrete extremal problems to find the exact optimum. When obtaining the algorithms, the procedures for decoding and finding the maximum upper zero of discrete

monotone functions (m.u.z. D.M.F.) were used. Methods for solving problems using the procedures for decoding and searching for m.u.z. discrete monotone functions. Methods for solving problems of decoding and searching for m.u.z. D.M.F. multivalued functions, an approximate parametric algorithm for solving these problems is constructed. A class of problems is investigated that can be reduced to decoding a monotone function given on a finite structure, or searching for a m.u.z. D.M.F.

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