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TRANSLATION OF Q-FUZZY SUBRING AND ANTI Q-FUZZY SUBRING

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ABSTRACT. In this paper, the concept of Q-fuzzy translation of Q-fuzzy subring (ideal), anti Q-fuzzy subring are presented and further, some important notions and basic algebraic properties are discussed. Also we studied the homomorphic behavior of Q-fuzzy translation of Q-fuzzy subring and Anti Q-fuzzy subring.

1. INTRODUCTION

The pioneering work of Zadeh on the fuzzy subset of a set in [8] and Rosenfeld on fuzzy subgroups of a group in [4] led to the fuzzification of algebraic structures. For example in [2, 3], Liu introduced the notion of fuzzy ideal of a ring. Biswas in [1] gave the idea of Anti fuzzy subgroups. The notion of translates of fuzzy set has been defined by the author in [5]. The same author in [6] introduced the translates of anti fuzzy subrings and ideals. The notion of translation of anti S-fuzzy subhemiring was introduced by the authors in [7]. In this paper, we study the translation of Q-fuzzy subring and anti Q-fuzzy subring.

2. PRELIMINARIES

Definition 2.1. *Let X be a non-empty set and Q be a non-empty set. A Q-fuzzy subset A of X is a function $A : X \times Q \to [0, 1]$.*

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Definition 2.2. *Let R be a ring. A Q-fuzzy subset A of R is said to be a Q-fuzzy subring (QFSR) of R if it satisfies the following conditions.*

(i) $A(x-y,q) \geq \min\{A(x,q), A(y,q)\}$
(ii) $A(xy,q) \geq \min\{A(x,q), A(y,q)\}$, *for all* $x, y \in R$ *and* $q \in Q$.

Definition 2.3. *Let R be a ring. A Q-fuzzy subset A in R is called anti Q-fuzzy subring (AQFSR) of R if*

(i) $A(x-y,q) \leq \max\{A(x,q), A(y,q)\}$
(ii) $A(xy,q) \leq \max\{A(x,q), A(y,q)\}$, *for all* $x, y \in R$.

Definition 2.4. *A Q-fuzzy subset A of R is called*

(a) *Q-fuzzy left ideal (QFLI) of R if*
  (i) $A(x-y,q) \geq \min\{A(x,q), A(y,q)\}$
  (ii) $A(xy,q) \geq A(y,q)$, *for all* $x, y \in R$ *and* $q \in Q$.
(b) *Q-fuzzy right ideal (QFRI) of R if*
  (i) $A(x-y,q) \geq \min\{A(x,q), A(y,q)\}$
  (ii) $A(xy,q) \geq A(x,q)$
(c) *Q-fuzzy ideal (QFI) of R if*
  (i) $A(x-y,q) \geq \min\{A(x,q), A(y,q)\}$
  (ii) $A(xy,q) \geq \max\{A(x,q), A(y,q)\}$, *for all* $x, y \in R$ *and* $q \in Q$.

Definition 2.5. *A Q-fuzzy subset A of a ring R is called*

(a) *Anti fuzzy left ideal (AFLI) of R if*
  (i) $A(x-y,q) \leq \max\{A(x,q), A(y,q)\}$
  (ii) $A(xy,q) \leq A(y,q)$, *for all* $x, y \in R$ *and* $q \in Q$.
(b) *Anti fuzzy right ideal (AFRI) of R if*
  (i) $A(x-y,q) \leq \max\{A(x,q), A(y,q)\}$
  (ii) $A(xy,q) \leq A(x,q)$, *for all* $x, y \in R$ *and* $q \in Q$.
(c) *Anti fuzzy ideal (AFI) of R if*
  (i) $A(x-y,q) \leq \max\{A(x,q), A(y,q)\}$
  (ii) $A(xy,q) \leq \min\{A(x,q), A(y,q)\}$, *for all* $x, y \in R$ *and* $q \in Q$.

Definition 2.6. *Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two subring. Then the function $f : R \to R'$ is called a ring homomorphism if it satisfies the following conditions.*

(i) $f(x+y) = f(x) + f(y)$
(ii) $f(xy) = f(x) \cdot f(y)$, *for all* $x, y \in R$.

Definition 2.7. *Let* $(R, +, \cdot)$ *and* $(R', +, \cdot)$ *be any two subrings. Then the function* $f : R \to R'$ *is called a ring anti-homomorphism if it satisfies the following conditions.*

(i) $f(x + y) = f(y) + f(x)$

(ii) $f(xy) = f(y) \cdot f(x)$,*for all* $x, y \in R$.

Definition 2.8. *Let* A *be a* Q*-fuzzy subset of* R *and* $\alpha \in [0, 1 - \sup\{A(x, q) : x \in R, 0 < A(x, q) < 1\}]$. *Then*
$T = T^A_\alpha$ *is called a* Q*-fuzzy translation of* A *if* $T(x, q) = A(x, q) + \alpha$, *for all* $x \in R$.

3. Translation of Q-fuzzy Subring and Anti Q-fuzzy Subring

Theorem 3.1. *If* A *is a QFSR of the ring* R *then* $T = T^A_\alpha$ *is also QFSR of* R.

Proof. Let $x, y \in R$ and $q \in Q$ be any elements, we have

$$\begin{aligned} T(x - y, q) &= A(x - y, q) + \alpha \geq \min\{A(x, q), A(y, q)\} + \alpha \\ &= \min\{A(x, q) + \alpha, A(y, q) + \alpha\} = \min\{T(x, q), T(y, q)\} \\ T(xy, q) &= A(xy, q) + \alpha \geq \min\{A(x, q), A(y, q)\} + \alpha \\ &= \min\{A(x, q) + \alpha, A(y, q) + \alpha\} = \min\{T(x, q), T(y, q). \end{aligned}$$

Therefore T is a QFSR of R. □

Theorem 3.2. *If* A *is a QFI of the ring* R, *then* $T = T^A_\alpha$ *is also QFI of* R.

Proof. Let $x, y \in R$ and $q \in Q$ be any elements, then we have

(3.1)

$$\begin{aligned} T(x - y, q) &= A(x - y, q) + \alpha \geq \min\{A(x, q), A(y, q)\} + \alpha \\ &= \min\{A(x, q) + \alpha, A(y, q) + \alpha\} = \min\{T(x, q), T(y, q)\} \\ T(xy, q) &= A(xy, q) + \alpha \geq A(y, q) + \alpha = T(y, q) \qquad \text{(as } A \text{ is } QFLI \text{ of } R) \end{aligned}$$

Also

(3.2) $$T(xy, q) = A(xy, q) + \alpha \geq A(x, q) + \alpha = T(x, q) \qquad \text{(as } A \text{ is } QFRI \text{ of } R)$$

From (3.1) and (3.2) we get

$$T(xy, q) \geq \max\{T(x, q), T(y, q)\}$$

Thus T is a QFI of R. □

Theorem 3.3. *If A is AQFSR of R, then $T = T_\alpha^A$ is also AQFSR of R.*

Proof. It is trivial. □

Theorem 3.4. *If A is AQFI of R, then $T = T_\alpha^A$ is also AQFI of R.*

Proof. It is trivial. □

Theorem 3.5. *If T_1 and T_2 are two Q-fuzzy translation of Q-fuzzy subring A of a ring R then their intersection $T_1 \cap T_2$ is also a Q-fuzzy translation of R.*

Proof. Let T_1 and T_2 be two Q-fuzzy translation of Q-fuzzy subring A of R. Then

$$\begin{aligned}(T_1 \cap T_2)(x,q) &= T_1(x,q) \cap T_2(x_2,q) = (A(x,q) + \alpha_1) \cap (A(x,q) + \alpha_2) \\ &= \min\{A(x,q) + \alpha_1, A(x,q) + \alpha_2\} = A(x,q) + \min(\alpha_1, \alpha_2).\end{aligned}$$

Therefore $T_1 \cap T_2$ is Q-fuzzy translation of Q-fuzzy subring A of R. □

Theorem 3.6. *The intersection of a family of Q-fuzzy translation of Q-fuzzy subring A of a ring R is a Q-fuzzy translation of R.*

Proof. It is trivial. □

Theorem 3.7. *If T_1 and T_2 are two Q-fuzzy translation of QFSR A of R then their union $T_1 \cup T_2$ is also a Q-fuzzy translation of R.*

Proof. Let T_1 and T_2 be two Q-fuzzy translation of Q-fuzzy subring A of R. Then

$$\begin{aligned}(T_1 \cup T_2)(x,q) &= T_1(x,q) \cup T_2(x_2,q) = (A(x,q) + \alpha_1) \cup (A(x,q) + \alpha_2) \\ &= \max\{A(x,q) + \alpha_1, A(x,q) + \alpha_2\} = A(x,q) + \max(\alpha_1, \alpha_2).\end{aligned}$$

Therefore $T_1 \cup T_2$ is a Q-fuzzy translation of Q-fuzzy subring A of R. □

Theorem 3.8. *The union of a family of Q-fuzzy translation of Q-fuzzy subring A of a ring R is a Q-fuzzy translation of R.*

Proof. It is trivial. □

Theorem 3.9. *Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two rings and Q be a non-empty set. If $f : R \to R'$ is a homomorphism, then the Q-fuzzy translation is a homomorphism, then the Q-fuzzy translation of a Q-fuzzy subring A of R under the homomorphic image is a Q-fuzzy subring of $f(R) = R'$.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two rings and Q be a non-empty set and $f:R\to R'$ be a homomorphism.Let $T=T_\alpha^A$ be the Q-fuzzy translation of a Q-fuzzy subring A of R and $V=f(T_\alpha^A)$ be the homomorphic image of T under f. We have to verify that V is a Q-fuzzy subring of R'. Let $f(x,q),f(y,q)\in R'$. Then

$$\begin{aligned}V(f(x)-f(y),q)&=V(f(x-y),q)\geq T_\alpha^A(x-y,q)=A(x-y,q)+\alpha\\&\geq (A(x,q)\wedge A(y,q))+\alpha=(A(x,q)+\alpha)\wedge(A(y,q)+\alpha)\\&=T_\alpha^A(x,q)\wedge T_\alpha^A(y,q).\end{aligned}$$

Therefore $V(f(x)-f(y),q)\geq V(f(x),q)\wedge V(f(y),q)$,

$$V(f(x)f(y),q)=V(f(xy),q)\geq T_\alpha^A(xy,q)=T_\alpha^A(x,q)\wedge T_\alpha^A(y,q),$$

and further, $V(f(x)f(y),q)\geq V(f(x),q)\wedge V(f(y),q)$. Thus V is Q-fuzzy subring of R'. □

Theorem 3.10. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two rings. If $f:R\to R'$ is a homomorphism then the Q-fuzzy translation of a Q-fuzzy subring V of R' under the homomorphic pre-image is a Q-fuzzy subring of R.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two rings and $f:R\to R'$ be a homomorphism. Let T_α^V be the translation of Q-fuzzy subring V of R' and A be the homomorphic pre-image of T_α^V under f. We have to prove that A is a Q-fuzzy subring of R. Let x and y be in R and q be in Q. Then

$$\begin{aligned}A(x-y,q)=&T_\alpha^V(f(x-y),q)=T_\alpha^V(f(x)-f(y),q)\\=&V[f(x)-f(y),q]+\alpha\geq[V(f(x),q)\wedge V(f(y),q)]+\alpha\\\geq&[V(f(x),q)+\alpha]\wedge[V(f(y),q)+\alpha]\\=&T_\alpha^V(f(x),q)\wedge T_\alpha^V(f(y),q)=A(x,q)\wedge A(y,q).\end{aligned}$$

Similarly, $A(xy,q)\geq A(x,q)\wedge A(y,q)$. Therefore, A is a Q-fuzzy subring of R. □

Theorem 3.11. *Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two rings. If $f:R\to R'$ is an anti-homomorphism, then the translation of a Q-fuzzy subring A of R under the anti-homomorphic image is a Q-fuzzy subring of $f(R)=R'$.*

Proof. Let $(R,+,\cdot)$ and $(R',+,\cdot)$ be any two rings and $f:R\to R'$ be an anti-homomorphism. That is $f(x+y)=f(y)+f(x)$ and $f(xy)=f(y)\cdot f(x)$, for all $x,y\in R$. Let T_α^A be the translation of a Q-fuzzy subring A of R and V be the anti-homomorphic image of T_α^A under f. We have to prove that V is a Q-fuzzy subring of $f(R) = R'$. Now, for $f(x)$ and $f(y)$ in R' and q in Q we have

$$\begin{aligned} V[((f(x) - f(y)), q) &= V[f(y-x), q] \geq T_\alpha^A(y-x, q) = A(y-x, q) + \alpha \\ &\geq (A(y,q) \wedge A(x,q)) + \alpha = (A(x,q) + \alpha) \wedge (A(y,q) + \alpha) \\ &= T_\alpha^A(x,q) \wedge T_\alpha^A(y,q) = V[f(x), q] \wedge V[f(y), q] \end{aligned}$$

for all $f(x), f(y) \in R'$ and $q \in Q$. Further,

$$\begin{aligned} V[((f(x) \cdot f(y)), q) &= V[f(yx), q] \geq T_\alpha^A(yx, q) = A(yx, q) + \alpha \\ &\geq (A(y,q) \wedge A(x,q)) + \alpha = (A(x,q) + \alpha) \wedge (A(y,q) + \alpha) \\ &= T_\alpha^A(x,q) \wedge T_\alpha^A(y,q) = V[f(x), q] \wedge V[f(y), q] \end{aligned}$$

for all $f(x), f(y) \in R'$ and $q \in Q$. Therefore, V is a Q-fuzzy subring of the ring R'. □

Theorem 3.12. *Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two rings. If $f : R \to R'$ is an anti-homomorphism, then the translation of a Q-fuzzy subring V of $f(R) = R'$ under the anti-homomorphic pre-image is a Q-fuzzy subring of R.*

Proof. Let $(R, +, \cdot)$ and $(R', +, \cdot)$ be any two rings and $f : R \to R'$ be an anti-homomorphism. That is $f(x+y) = f(y) + f(x)$ and $f(xy) = f(y) \cdot f(x)$, for all $x, y \in R$. Let T_α^V be the translation of a Q-fuzzy subring V of $f(R) = R'$ and A be the anti-homomorphic pre-image of T_α^V under f. We have to prove that A is a Q-fuzzy subring of R. Let x and y be in R and q in Q then

$$\begin{aligned} A(x-y, q) &= T_\alpha^V(f(x-y), q) = T_\alpha^V(f(y) - f(x), q) = V[f(y) - f(x), q] + \alpha \\ &\geq [V[f(x), q] \wedge V[f(y), q]] + \alpha = (V[f(x), q] + \alpha) \wedge (V[f(y), q] + \alpha) \\ &= T_\alpha^V(f(x), q) \wedge T_\alpha^V(f(y), q) = A(x,q) \wedge A(y,q) \end{aligned}$$

for all x and y in R and q in Q. Also,

$$\begin{aligned} A(xy, q) &= T_\alpha^V(f(xy), q) = T_\alpha^V(f(y)f(x), q) = V[f(y) \cdot f(x), q] + \alpha \\ &\geq [V[f(x), q] \wedge V[f(y), q]] + \alpha = (V[f(y), q] + \alpha) \wedge (V[f(x), q] + \alpha) \\ &= T_\alpha^V(f(x), q) \wedge T_\alpha^V(f(y), q) = A(x,q) \wedge A(y,q) \end{aligned}$$

for all x and y in R and q in Q. Therefore, A is a Q-fuzzy subring of the ring R. □

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