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COMPUTATION OF NUMEROUS TOPOLOGICAL INDICES OF LINE GRAPH OF DUTCH WINDMILL GRAPH $(D_N^M)^L$

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ABSTRACT. In this article, we evaluate sum and product versions of the Harmonic index, Symmetric Degree Division index, Inverse Sum index, GA_1, Redefined version of Zagreb index, SK, SK_1, SK_2 index, Balaban index, Revan and Hyper Revan indices of the line graph of Dutch windmill graphs.

1. INTRODUCTION

The Dutch windmill graph is denoted as D_n^m and the graph obtained takes m copies of the C_n with a mutual common vertex. Often the Dutch windmill graph is called as a graph of friendship if $n = 3$ $(i.e.)$ D_3^m. The D_n^m Dutch windmill graph comprises $(n-1)m+1$ nodes and edges of mn. The graphs we considered in this paper are simple finite and connected together. Topological indices are numerical parameters of a graph describing its topology, which is typically invariants of graphs. Several other topological indices have been used in various research, and comprehensive research has been performed on a broad range of graph types on these indices. Inspired by this research, we discuss some topological indices for certainly associated graphs of a particular graph class, namely the Dutch windmill graph.

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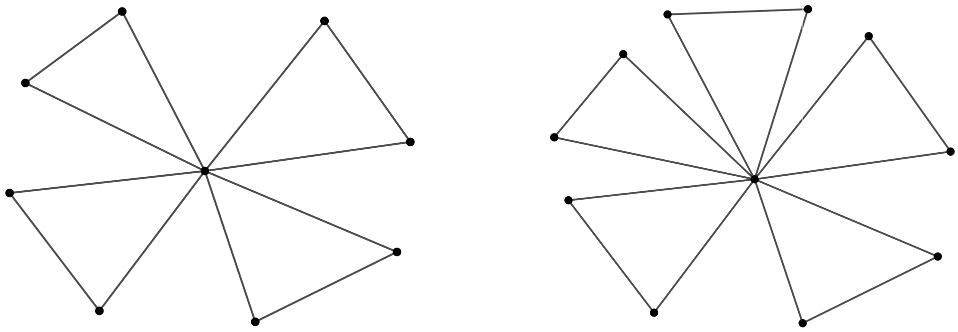
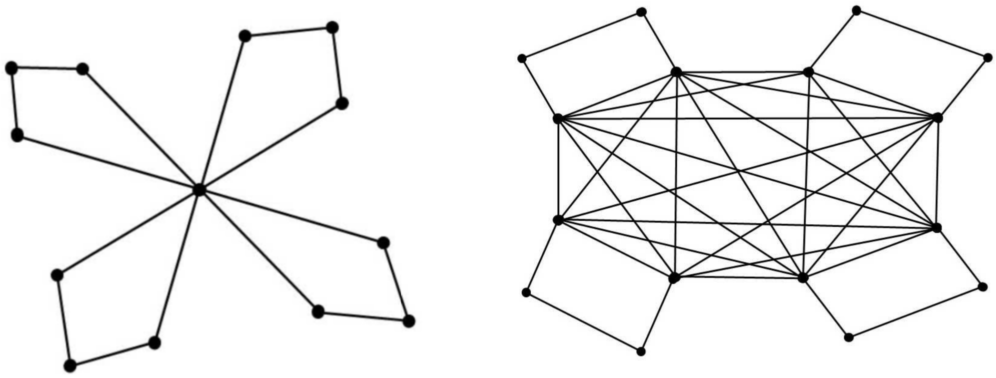


(a) D_3^4 (b) D_3^5

FIGURE 1. Dutch Windmill Graphs



(a) D_4^4 (b) $(D_4^4)^L$

FIGURE 2. Dutch Windmill Graph and its line graph

In this article, we have examined sum and product versions of certain topological indices of the line graphs of Dutch windmill graphs based on the degree of line graph vertices and the edge partitions. Throughout the article, we denote the line graph of D_n^m as $(D_n^m)^L$. We also followed the topological indices notations and the indices formulae from the articles [1, 2, 4–6].

2. Preliminaries

Theorem 2.1. *[7] Let $G(p,q)$ be a graph, then the line graph $L(G)$ of G is a graph with q vertices and the number of edges is*

$$m(L(G)) = \frac{1}{2}\left[\sum_{i=1}^{n} d_i^2\right] - m.$$

Theorem 2.2. *[3] If G is the line graph of a Dutch windmill graph D_n^m, then $V((D_n^m)^L) = mn$ and $E((D_n^m)^L) = 2m^2 + mn - 2m$.*

Note 1. *The $V((D_n^m)^L)$ is mn from that it is clear that $(n-2)m$ vertices are of degree 2, and $2m$ vertices are of degree $2m$.*

Note 2. *The $E((D_n^m)^L)$ is $2m^2 + mn - 2m$ from that the edges are partitioned as given below.*

$(d_u, d_v) : uv \in E(G)$	$(2,2)$	$(2,2m)$	$(2m,2m)$
Number of edges	$m(n-3)$	$2m$	$(2m-1)m$

3. Computed Topological Index Results for the line graph of Dutch Windmill Graphs

Theorem 3.1. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$H((D_n^m)^L) = \frac{m^2n - m^2 + mn + 2m - 1}{2(1+m)}.$$

Proof. From the note (2), we compute the harmonic index of $(D_n^m)^L$ as

$$\begin{aligned} H((D_n^m)^L) &= |E_{(2,2)}|\left[\frac{2}{2+2}\right] + |E_{(2,2m)}|\left[\frac{2}{2+2m}\right] + |E_{(2m,2m)}|\left[\frac{2}{2m+2m}\right] \\ &= m(n-3)\left[\frac{2}{4}\right] + 2m\left[\frac{2}{2(1+m)}\right] + (2m-1)(m)\left[\frac{2}{4m}\right] \\ &= \frac{m^2n - m^2 + mn + 2m - 1}{2(1+m)}. \end{aligned}$$

□

Note 3. *The multiplicative version of $H((D_n^m)^L)$ is*

$$H((D_n^m)^L) = \frac{2m^3n - m^2n + 3m^2 - 6m^3}{2(1+m)}.$$

Theorem 3.2. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$SDD((D_n^m)^L) = 6m^2 - 8m + 2mn + 2.$$

Proof. From the note (2), we compute the symmetric degree division index of $(D_n^m)^L$ as

$$\begin{aligned}
&SDD((D_n^m)^L)\\
&= |E_{(2,2)}|\left[\frac{2^2+2^2}{(2)(2)}\right] + |E_{(2,2m)}|\left[\frac{2^2+(2m)^2}{2(2m)}\right] + |E_{(2m,2m)}|\left[\frac{(2m)^2+(2m)^2}{(2m)(2m)}\right]\\
&= m(n-3)\left[\frac{8}{4}\right] + 2m\left[\frac{4+4m^2}{4m}\right] + (2m-1)(m)\left[\frac{2(2m)^2}{4m^2}\right]\\
&= 2m(n-3) + 2(1+m^2) + 2m(m-1)\\
&= 6m^2 - 8m + 2mn + 2.
\end{aligned}$$

□

Note 4. *The multiplicative version of $SDD((D_n^m)^L)$ is*

$$SDD((D_n^m)^L) = 8m^2\left[2mn - n - 6m - m^2n + 2m^3n + 3m^2 - 6m^3 + 3\right].$$

Theorem 3.3. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$ISI((D_n^m)^L) = \frac{2m^4 + m^3 + m^2n + mn - 3m}{1+m}.$$

Proof. From the note (2), we compute the Inverse sum In-degree index of $(D_n^m)^L$ as

$$\begin{aligned}
&ISI((D_n^m)^L)\\
&= |E_{(2,2)}|\left[\frac{(2)(2)}{(2)+(2)}\right] + |E_{(2,2m)}|\left[\frac{(2)(2m)}{2+(2m)}\right] + |E_{(2m,2m)}|\left[\frac{(2m)*(2m)}{(2m)+(2m)}\right]\\
&= m(n-3)\left[\frac{4}{4}\right] + 2m\left[\frac{4m}{2(1+m)}\right] + (2m-1)(m)\left[\frac{4m^2}{4m}\right]\\
&= 2m(n-3) + \frac{4m^2}{(1+m)} + m^2(m-1)\\
&= \frac{2m^4 + m^3 + m^2n + mn - 3m}{1+m}.
\end{aligned}$$

□

Note 5. *The multiplicative version of $ISI((D_n^m)^L)$ is*

$$ISI((D_n^m)^L) = \frac{4m^5\big[2mn - n - 6m + 3\big]}{1+m}.$$

Theorem 3.4. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$GA_1((D_n^m)^L) = 2m^2 + mn - 4m + (1+m)\sqrt{m}.$$

Proof. From the note (2), we compute the GA_1 index of $(D_n^m)^L$ as

$$\begin{aligned}
GA_1((D_n^m)^L) = {} & |E_{(2,2)}|\left[\frac{2+2}{2\sqrt{(2)*(2)}}\right] + |E_{(2,2m)}|\left[\frac{2+2m}{2\sqrt{(2)(2m)}}\right] \\
& + |E_{(2m,2m)}|\left[\frac{(2m)+(2m)}{2\sqrt{(2m)(2m)}}\right] \\
= {} & m(n-3)\left[\frac{4}{2\sqrt{4}}\right] + 2m\left[\frac{2(1+m)}{2\sqrt{4m}}\right] + (2m-1)(m)\left[\frac{4m}{2\sqrt{4m^2}}\right] \\
= {} & m(n-3) + \sqrt{m}(1+m) + (2m-1)m \\
= {} & 2m^2 + mn - 4m + (1+m)\sqrt{m}.
\end{aligned}$$

□

Note 6. *The multiplicative version of $GA_1((D_n^m)^L)$ is*

$$GA_1((D_n^m)^L) = m\sqrt{m}\big[mn - n - 3m + 2m^2n - 6m^2 + 3\big].$$

Now, we compute the first and third Redefined Zagreb index of $(D_n^m)^L$. Since, the second Redefined Zagreb index is Inverse sum index.

Theorem 3.5. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$ReZ_1((D_n^m)^L) = mn.$$

Proof. From the note (2), we compute the first Redefined Zagreb index of $(D_n^m)^L$. Then

$$\begin{aligned}
&ReZ_1((D_n^m)^L)\\
&= |E_{(2,2)}|\left[\frac{2+2}{2*(2)}\right] + |E_{(2,2m)}|\left[\frac{2+2m}{(2)(2m)}\right] + |E_{(2m,2m)}|\left[\frac{(2m)+(2m)}{(2m)(2m)}\right]\\
&= m(n-3)\left[\frac{4}{4}\right] + 2m\left[\frac{2(1+m)}{2(2m)}\right] + (2m-1)(m)\left[\frac{4m}{4m^2}\right]\\
&= m(n-3) + (1+m) + (2m-1)\\
&= mn.
\end{aligned}$$

□

Note 7. *The multiplicative version of $ReZ_1((D_n^m)^L)$ is*

$$ReZ_1((D_n^m)^L) = 2m^3n + m^2n + 3m - mn - 3m^2 - 6m^3.$$

Theorem 3.6. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$ReZ_3((D_n^m)^L) = 16\big[2m^5 - m^4 + m^3 + m^2 + mn - 3m\big].$$

Proof. From the note (2), we compute the third Redefined Zagreb index of $(D_n^m)^L$. Then

$$\begin{aligned}
ReZ_3((D_n^m)^L) &= |E_{(2,2)}|\big[(2*2)(2+2)\big] + |E_{(2,2m)}|\big[(2*2m)(2+2m)\big]\\
&\quad + |E_{(2m,2m)}|\big[((2m)+(2m))*(2m+2m)\big]\\
&= m(n-3)(16m) + 16m^2(1+m) + (2m-1)(16m^4)\\
&= 16\big[2m^5 - m^4 + m^3 + m^2 + mn - 3m\big].
\end{aligned}$$

□

Note 8. *The multiplicative version of $ReZ_3((D_n^m)^L)$ is*

$$ReZ_3((D_n^m)^L) = 4096m^7\big[2m^2n + mn - 3m - n - 6m^2 + 3\big].$$

Now, we compute the SK, SK_1, SK_2 index of $(D_n^m)^L$.

Theorem 3.7. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$SK((D_n^m)^L) = 2m\big[2m^2 + n - 2\big].$$

Proof. From the note (2), we compute the SK index of $(D_n^m)^L$. Then

$$\begin{aligned} SK((D_n^m)^L) &= |E_{(2,2)}|\left[\frac{2+2}{2)}\right] + |E_{(2,2m)}|\left[\frac{2+2m}{2}\right] + |E_{(2m,2m)}|\left[\frac{(2m)+(2m)}{2}\right] \\ &= m(n-3)\left[\frac{4}{2}\right] + 2m\left[\frac{2(1+m)}{2}\right] + (2m-1)(m)\left[\frac{4m}{2}\right] \\ &= 2m\big[2m^2+n-2\big]. \end{aligned}$$

□

Note 9. *The multiplicative version of $SK((D_n^m)^L)$ is*

$$SK((D_n^m)^L) = 8m^4\big[2m^2n + mn - 6m^2 - n - 3m + 3\big].$$

Theorem 3.8. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$SK_1((D_n^m)^L) = 2m\big[2m^3 + 2m + n - m^2 - 3\big].$$

Proof. From the note (2), we compute the SK_1 index of $(D_n^m)^L$. Then

$$\begin{aligned} SK_1((D_n^m)^L) &= |E_{(2,2)}|\left[\frac{2*2}{2)}\right] + |E_{(2,2m)}|\left[\frac{2*(2m)}{2}\right] + |E_{(2m,2m)}|\left[\frac{(2m)*(2m)}{2}\right] \\ &= m(n-3)\left[\frac{4}{2}\right] + 2m(2m) + (2m-1)(2m^3) \\ &= 2m\big[2m^3 + 2m + n - m^2 - 3\big]. \end{aligned}$$

□

Note 10. *The multiplicative version of $SK_1((D_n^m)^L)$ is*

$$SK_1((D_n^m)^L) = 16\big[2m^7n - m^6n + 3m^6 - 6m^7\big].$$

Theorem 3.9. *If $(D_n^m)^L$ is the line graph of a (D_n^m), then*

$$SK_2((D_n^m)^L) = 8m^4 - 2m^3 + 4m^2 + 4mn - 10m.$$

Proof. From the note (2), we compute the SK_2 index of $(D_n^m)^L$. Then

$$\begin{aligned} SK_2((D_n^m)^L) &= |E_{(2,2)}|\left[\frac{2+2}{2)}\right]^2 + |E_{(2,2m)}|\left[\frac{2+2m}{2}\right]^2 + |E_{(2m,2m)}|\left[\frac{(2m)+(2m)}{2}\right]^2 \\ &= m(n-3)\left[\frac{4}{2}\right]^2 + 2m\left[\frac{2(1+m)}{2}\right]^2 + (2m-1)(m)\left[\frac{4m}{2}\right]^2 \\ &= 8m^4 - 2m^3 + 4m^2 + 4mn - 10m. \end{aligned}$$

□

Note 11. *The multiplicative version of* $SK_2((D_n^m)^L)$ *is*

$$SK_2((D_n^m)^L) = 32m^5\big[3m^2n + 2m^3n - 9m^2 - 6m^3 - n + 3\big].$$

Theorem 3.10. *If* $(D_n^m)^L$ *is the line graph of a* (D_n^m)*, then*

$$J((D_n^m)^L) = \frac{2m + mn - 2m}{2m^2 - 2m + 1}\left[\frac{mn + 2\sqrt{m} - m - 1}{2}\right].$$

Proof. From the note (2), we compute the balaban index of $(D_n^m)^L$. Here the number of vertices in $(D_n^m)^L = nm$ and the number of edges $= 2m^2 + mn - 2m$. Therefore, we have

$$\begin{aligned} J &= \frac{2m^2 + mn - 2m}{2m^2 + mn - 2m - mn + 1}\sum_{uv\in E}\frac{1}{\sqrt{d_ud_v}} \\ &= \frac{2m^2 + mn - 2m}{2m^2 - 2m + 1}\sum_{uv\in E}\frac{1}{\sqrt{d_ud_v}}. \end{aligned}$$

Now, we have

$$\begin{aligned} J(D_n^m) &= \frac{2m^2 + mn - 2m}{2m^2 - 2m + 1}\sum_{uv\in E}\frac{1}{\sqrt{d_ud_v}} \\ &= \frac{2m^2 + mn - 2m}{2m^2 - 2m + 1}\left[|E_{(2,2)}|\left[\frac{1}{\sqrt{2*2}}\right] + |E_{(2,2m)}|\left[\frac{1}{\sqrt{2*(2m)}}\right]\right. \\ &\left. + |E_{(2m,2m)}|\left[\frac{1}{\sqrt{(2m)*(2m)}}\right]\right] \\ &= \frac{2m^2 + mn - 2m}{2m^2 - 2m + 1}\left[\frac{(n-3)*m}{2} + \frac{2m}{\sqrt{4m}} + \frac{(2m-1)*m}{2m}\right] \\ &= \frac{2m + mn - 2m}{2m^2 - 2m + 1}\left[\frac{mn + 2\sqrt{m} - m - 1}{2}\right]. \end{aligned}$$

□

Note 12. *The multiplicative version of* $J((D_n^m)^L)$ *is*

$$J((D_n^m)^L) = \frac{m\sqrt{m}\big[2mn - n - 6m + 3\big]}{4}.$$

Now, we compute the three revan indices and its hyper versions for the line graph of Dutch Windmill graphs. For that we have the $V(D_n^m)^L = mn$ and $E(D_n^m)^L = 2m^2 + mn - 2m$. From the graph structure, $\Delta((D_n^m)^L) = 2m$, where m is the number of copies in D_n^m and $\delta((D_n^m)^L) = 2$. For calculating revan indices we need to find $r(v)$.

For $(D_n^m)^L$, we have the edge partitions based on the edges end point degrees, the partitions are $|E_{(2,2)}|$, $|E_{(2,2m)}|$ and $|E_{(2m,2m)}|$. If $|E_{(2,2)}|$ is considered then $r(v) = 2m$ and suppose $|E_{(2m,2)}|$ is considered then $r(v) = 2$.

Theorem 3.11. *The first Revan index of* $(D_n^m)^L$ *is*

$$R_1((D_n^m)^L) = 4m^2n.$$

Proof. From the note (2), we compute the first Revan index of D_n^m as

$$\begin{aligned}
&R_1((D_n^m)^L)\\
&= |E_{(2,2)}|\big[(2m) + (2m)\big] + |E_{(2,2m)}|\big[(2m) + (2)\big] + |E_{(2m,2m)}|\big[(2) + (2)\big]\\
&= (n-3)(4m^3) + 4m(1+m) + (2m-1)(4m)\\
&= 4m^2n.
\end{aligned}$$

□

Note 13. *The multiplicative version of* $R_1((D_n^m)^L)$ *is*

$$R_1((D_n^m)^L) = (4m)^3\big[m^2n - mn - 3m^2 + 2m^3n - 6m^3 + 3m\big].$$

Theorem 3.12. *The second Revan index of* $(D_n^m)^L$ *is*

$$R_2((D_n^m)^L) = 4m^3n - 12m^3 + 16m^2 - 4m.$$

Proof. From the note (2), we compute the second Revan index of $(D_n^m)^L$ as

$$\begin{aligned}
R_2((D_n^m)^L) &= |E_{(2,2)}|\big[(2m) * (2m)\big] + |E_{(2,2m)}|\big[(2m) * (2)\big] + |E_{(2m,2m)}|\big[(2) * (2)\big]\\
&= (n-3)(4m^3) + 8m^2 + (2m-1)(4m)\\
&= 4m^3n - 12m^3 + 16m^2 - 4m.
\end{aligned}$$

□

Note 14. *The multiplicative version of* $R_2((D_n^m)^L)$ *is*

$$R_2((D_n^m)^L) = 2^7m^6\big[2mn - n - 6m + 3\big].$$

Theorem 3.13. *The first Hyper Revan index of* $(D_n^m)^L$ *is*

$$HR_1((D_n^m)^L) = 16m^3n - 8m + 48m^2 - 40m^3.$$

Proof. From the note (2), we compute the first Revan index of D_n^m as

$$\begin{aligned}
&R_1((D_n^m)^L)\\
&= |E_{(2,2)}|\big[(2m) + (2m)\big]^2 + |E_{(2,2m)}|\big[(2m) + (2)\big]^2 + |E_{(2m,2m)}|\big[(2) + (2)\big]^2\\
&= 8m\big[2m^2(n-3) + (1+m)^2 + 2(2m-1)\big]\\
&= 16m^3n - 8m + 48m^2 - 40m^3.
\end{aligned}$$

□

Note 15. *The multiplicative version of* $HR_1((D_n^m)^L)$ *is*

$$HR_1((D_n^m)^L) = 2^{11}m^5\big[3m^2n - n + 2m^3n - 9m^2 - 6m^3 + 3\big].$$

Theorem 3.14. *The second Hyper Revan index of* $(D_n^m)^L$ *is*

$$HR_2((D_n^m)^L) = 16m^5n - 16m + 32m^2 + 32m^3 - 48m^5.$$

Proof. From the note (2), we compute the second Hyper Revan index of $(D_n^m)^L$ as

$$\begin{aligned}
&HR_2((D_n^m)^L)\\
&= |E_{(2,2)}|\big[(2m) * (2m)\big]^2 + |E_{(2,2m)}|\big[(2m) * (2)\big]^2 + |E_{(2m,2m)}|\big[(2) * (2)\big]^2\\
&= (n-3)m(16m^4) + 2m(16m^2) + (2m-1)(m)(16)\\
&= 16m^5n - 16m + 32m^2 + 32m^3 - 48m^5.
\end{aligned}$$

□

Note 16. *The multiplicative version of* $HR_2((D_n^m)^L)$ *is*

$$HR_2((D_n^m)^L) = 2^{13}m^9\big[2mn - n - 6m + 3\big].$$

4. Conclusion

The computation of various topological indices of multiple graphs provides a broad view for numerous mathematical works, especially in the analysis of molecules in molecular chemistry and they relate along with their properties. We discussed a few topological indices of the line graphs of Dutch windmill graphs in this paper. Throughout this paper we calculated the Harmonic index, Symmetric Degree Division index, Inverse Sum index, GA_1, Redefined version of Zagreb index, SK, $SK1$, $SK2$ index, Balaban index, Revan and hyper Revan indices of the line graph of Dutch windmill graphs.

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