

## SOLVING FUZZY ASSIGNMENT AND FUZZY TRAVELLING SALESMAN PROBLEMS USING R SOFTWARE

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**ABSTRACT.** In this article an interesting procedure is proposed for fuzzy travelling salesman problems and assignment problems using "R" software packages such as "FuzzyNumbers" and "lpSolve" and fuzzy travelling salesman problems using "R" software packages "FuzzyNumbers" and "TSP". To validate the procedure some numerical examples are illustrated and the results are compared with the contemporary procedures.

### 1. INTRODUCTION

To minimize the assigning cost in assignment environment is the objective of the assignment problem. The assumption is each machine can do only one job at a time. Travelling salesman problem is playing vital role in combinatorial optimization problems. Finding minimum weighted Hamiltonian cycle is the fruit of the travelling salesman problem. In real time modeling the decision variables are not certain to measure. When it is vague or ambiguity exists in modeling the parameter, fuzzy notions introduced by L.A. Zadeh [1] can be used. Several authors presented different approaches to solve Fuzzy assignment problem and fuzzy travelling salesman problem [2–10, 13, 14].

In this paper, we applied "FuzzyNumbers" package in R software to map the fuzzy parameters into crisp parameters and used "lpSolve" package to solve it.

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The merit of this method is it uses an open source R software which can be used by all. It gives an advantage of saving the computational time and human resources. Also if the assignment problem has large number of machines and jobs and travelling salesman problem has larger number of cities then it may be difficult to do manually. We can make use of this method for those kind of problems. In this paper, Section 2 deliberates basic definitions connected with the fuzzy theory, mathematical formulations of fuzzy travelling salesman problem and fuzzy assignment problem. In Section 3 fuzzy assignment problems solved using "R" software packages such as "FuzzyNumbers" and "lpSolve" and fuzzy travelling salesman problems using "R" software packages "FuzzyNumbers" and "TSP" are discussed briefly. Standard problems and conclusion are given in the Section 4 and Section 5 respectively.

## 2. PRELIMINARIES

**Definition 2.1.** [11] Let  $X$  is a subset of real numbers is a universe. A fuzzy set  $\tilde{A}$  is a ordered pair of  $(x, \mu_{\tilde{A}}(x))$  where  $\mu_{\tilde{A}}(x)$  is characterized as a mapping from universe to  $[0,1]$ .

**Definition 2.2.** A fuzzy set  $\tilde{A}$  with membership relation  $\mu_{\tilde{A}}(x)$  is piecewise continuous, normal and convex is known as fuzzy number.

**Definition 2.3.** A fuzzy number  $\tilde{A} = (a, b, c, d)$  with membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{d-x}{d-c} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

is denoted as trapezoidal fuzzy number. It becomes triangular fuzzy number if  $b$  and  $c$  are equal.

**Definition 2.4.** [12] Yager centroid index is  $Y(\tilde{A}) = \int_0^1 A_U^\alpha + A_L^\alpha d\alpha$ , where  $A_U^\alpha$  is a Higher  $\alpha$  - cut and  $A_L^\alpha$  is a Lower  $\alpha$  - cut. If  $Y(\tilde{A}) \leq Y(\tilde{B})$ ,  $\tilde{A} \leq \tilde{B}$ .

**2.1. Fuzzy Assignment Problem.** The fuzzy assignment problem can be written in  $n \times n$  cost matrix given below.

$$\begin{array}{c}
 \text{Instrument 1} \quad \text{Instrument 2} \quad \text{Instrument 3} \quad \cdots \quad \text{Instrument } n \\
 \begin{array}{l}
 \text{Job 1} \\
 \text{Job 2} \\
 \text{Job 3} \\
 \cdots \\
 \text{Job } n
 \end{array}
 \left( \begin{array}{ccccc}
 \tilde{C}_{11} & \tilde{C}_{12} & \tilde{C}_{13} & \cdots & \tilde{C}_{1n} \\
 \tilde{C}_{21} & \tilde{C}_{22} & \tilde{C}_{23} & \cdots & \tilde{C}_{2n} \\
 \tilde{C}_{31} & \tilde{C}_{32} & \tilde{C}_{33} & \cdots & \tilde{C}_{3n} \\
 \cdots & \cdots & \cdots & \cdots & \cdots \\
 \tilde{C}_{n1} & \tilde{C}_{n2} & \tilde{C}_{n3} & \cdots & \tilde{C}_{nn}
 \end{array} \right)
 \end{array}$$

Mathematically it can be stated as,

$$\min \tilde{Z} \approx \sum_{i=1}^n \sum_{j=1}^n \tilde{C}_{ij} x_{ij},$$

subject to

$$\sum_{i=1}^n x_{ij} = 1, \sum_{j=1}^n x_{ij} = 1,$$

$$x_{ij} = \begin{cases} 1, & \text{if } i^{\text{th}} \text{ job is assigned to the } j^{\text{th}} \text{ instrument} \\ 0, & \text{otherwise} \end{cases}.$$

**2.2. Fuzzy Travelling Salesman Problem.**

$$\begin{array}{c}
 \text{Place 1} \quad \text{Place 2} \quad \text{Place 3} \quad \cdots \quad \text{Place } n \\
 \begin{array}{l}
 \text{Place 1} \\
 \text{Place 2} \\
 \text{Place 3} \\
 \cdots \\
 \text{Place } n
 \end{array}
 \left( \begin{array}{ccccc}
 \infty & \tilde{C}_{12} & \tilde{C}_{13} & \cdots & \tilde{C}_{1n} \\
 \tilde{C}_{21} & \infty & \tilde{C}_{23} & \cdots & \tilde{C}_{2n} \\
 \tilde{C}_{31} & \tilde{C}_{32} & \infty & \cdots & \tilde{C}_{3n} \\
 \cdots & \cdots & \cdots & \cdots & \cdots \\
 \tilde{C}_{n1} & \tilde{C}_{n2} & \tilde{C}_{n3} & \cdots & \infty
 \end{array} \right)
 \end{array}$$

Mathematically, it can be stated as

$$\min \tilde{Z} \approx \sum_{i=1}^n \sum_{j=1}^n \tilde{C}_{ij} x_{ij},$$

subject to

$$(2.1) \quad \sum_{i=1}^n x_{ij} = 1, \quad \sum_{j=1}^n x_{ij} = 1,$$

$$(2.2) \quad x_{ij} + x_{ji} \leq 1, 1 \leq i \neq j \leq n$$

$$(2.3) \quad x_{ij} + x_{jk} + x_{ki} \leq 2, 1 \leq i \neq j \neq k \leq n$$

$$(2.4) \quad x_{ip_1} + x_{p_1 p_2} + \cdots + x_{p_{n-2} i} \leq n - 2, 1 \leq i \neq p_1 \neq p_2 \neq \cdots \leq n.$$

Constraints (2.1) ensure that each city is visited only once. Constraint (2.2) eliminates all 2-city sub tours. Constraint (2.3) eliminates all 3-city sub tours. Constraints (2.4) eliminate all  $(n - 2)$  city sub tours.

### 3. PROCEDURE TO SOLVE FUZZY ASSIGNMENT PROBLEM AND FUZZY TRAVELLING SALESMAN PROBLEM USING R PACKAGES

This part presents the steps involved in solving for fuzzy travelling salesman problem and fuzzy assignment problem using the R packages "FuzzyNumbers" [15], "lpSolve" [16] and "TSP" [17]. The "FuzzyNumbers" package is used to construct fuzzy number and to give diagrammatic representation by its core, support, and either its left/right side functions or lower/upper  $\alpha$  - cut bounds. This package is applied to get the membership function at various points, expected value, width, and ambiguity of a fuzzy number. Addition, subtraction and multiplication of fuzzy numbers can also be done using this package. The following functions defined in FuzzyNumbers packages:

- The value of a fuzzy number A is defined  $val(A) = \int_0^1 \alpha A_U^\alpha + A_L^\alpha d\alpha$ .
- Expected interval of fuzzy number is  $EI(A) = [\int_0^1 A_U^\alpha d\alpha, \int_0^1 A_L^\alpha d\alpha]$  and is represented by "expectedInterval(A)"
- The expected value of a fuzzy number  $\tilde{A}$  is defined as  $EV(A) = \frac{1}{2}[\int_0^1 A_U^\alpha d\alpha + \int_0^1 A_L^\alpha d\alpha]$  is denoted as "expectedValue(A)" the same as the Yagar's ranking ( $Y(\tilde{A})$ ) of a fuzzy number  $\tilde{A}$ .

So we have applied expected value function to convert fuzzy number  $\tilde{A}$  as a crisp number.

**Step 1:** Enter the cost matrix of the fuzzy assignment problem or fuzzy travelling salesman problem with Triangular or Trapezoidal fuzzy numbers as its parameters.

**Step 2:** Convert each fuzzy number to crisp number by applying "expected-Value" function in the "FuzzyNumbers" package and kept in a separate matrix.

**Step 3:** Use "lpSolve" package to solve the crisp assignment problem or use "TSP" package to solve crisp Travelling salesman problem.

**Step 4:** Deduce the optimal assignments as output.

#### 4. NUMERICAL EXAMPLES

**Example 1.** Consider fuzzy assignment problem

	Instrument 1	Instrument 2	Instrument 3	Instrument 4	Instrument 5
Job 1	(4,6,7,9)	(3,5,7,9)	(5,7,10,12)	(3,4,6,9)	(4,5,7,10)
Job 2	(2,3,5,9)	(5,7,9,13)	(4,6,9,12)	(5,6,7,10)	(2,3,5,7)
Job 3	(7,9,10,12)	(6,7,9,10)	(7,9,10,13)	(6,7,10,13)	(7,10,13,14)
Job 4	(4,5,7,9)	(5,7,12,15)	(7,9,13,15)	(2,9,10,13)	(5,7,10,14)
Job 5	(4,10,13,15)	(3,7,9,13)	(2,3,10,14)	(3,7,10,13)	(4,7,10,14)

Applying "FuzzyNumbers" package we get the crisp assignment problem

	Instrument 1	Instrument 2	Instrument 3	Instrument 4	Instrument 5
Job 1	6.50	6.00	8.50	5.5	6.50
Job 2	4.75	8.50	7.75	7.0	4.25
Job 3	9.50	8.00	9.75	9.0	11.00
Job 4	6.25	9.75	11.00	8.5	9.00
Job 5	10.50	8.00	7.25	9.0	8.75

Using "lpSolve" package to solve the above crisp assignment problem. The output of above crisp problem is provided below.

	Instrument 1	Instrument 2	Instrument 3	Instrument 4	Instrument 5
Job 1	0	0	0	1	0
Job 2	0	0	0	0	1
Job 3	0	1	0	0	0
Job 4	1	0	0	0	0
Job 5	0	0	1	0	0

The optimal assignment is  $A \rightarrow 4, B \rightarrow 5, C \rightarrow 2, D \rightarrow 1, E \rightarrow 3$ . The fuzzy optimum assignment cost is  $(3, 4, 6, 9) + (2, 3, 5, 7) + (6, 7, 9, 10) + (4, 5, 7, 9) + (2, 3, 10, 14) = (17, 22, 37, 49)$  which is same as [14]

**Example 2.** Consider the fuzzy travelling salesman problem [13]

	A	B	C	D
A	$\infty$	(4,6,8,10)	(5,7,9,11)	(6,8,10,12)
B	(4,6,8,10)	$\infty$	(2,4,6,8)	(1,3,5,7)
C	(5,7,9,11)	(2,4,6,8)	$\infty$	(3,5,7,9)
D	(6,8,10,12)	(1,3,5,7)	(3,5,7,9)	$\infty$

The crisp matrix obtained using FuzzyNumbers package is provided below.

	A	B	C	D
A	$\infty$	7	8	9
B	7	$\infty$	5	4
C	8	5	$\infty$	6
D	9	4	6	$\infty$

Applying "TSP" package in R for the above crisp matrix we get the following solution as output.

object of class 'TOUR'

result of method 'nearest\_insertion' for 4 cities

tour length : 25

<labels(a2)

Path is [1] "D" "C" "A" "B"

The output of using nn algorithm in TSP package is provided below.

object of class 'TOUR'

result of method 'nn' for 4 cities tourlength : 25

>labels(a)

[1] "B" "D" "C" "A"

The fuzzy travelling cost is  $(4, 6, 8, 10) + (1, 3, 5, 7) + (3, 5, 7, 9) + (5, 7, 9, 11) = (13, 21, 29, 37)$ .

## 5. CONCLUSION

A novel approach is presented for fuzzy travelling salesman problem and fuzzy assignment problem by combining the packages 'FuzzyNumbers' and 'TSP' and 'FuzzyNumbers' and 'lpSolve' in R software respectively. This algorithm is not complex and easy to apply. The advantages of applying this method is furnished by comparison with the existing methods.

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