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QUASI-VALUATION MAPS BASED ON SUBALGEBRA AND IDEAL CONCEPT IN BM-ALGEBRA

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ABSTRACT. The notion of SQV-map and IQV-map in a BM-algebra are introduced and the relation between them are studied. Using these notions, the binary operation in BM-algebra is found to be uniformly continuous by constructing (pseudo) metric spaces. A quotient BM-algebra is constructed using the congruence relation induced by the IQV-map on it. The relation between the induced congruence relation and an ideal obtained from an IQV-map are studied. The condition under which two BM-algebras become homeomorphic is also derived by defining suitable IQV-maps on them.

1. INTRODUCTION

Logic plays an important role in foundations of mathematics. Study on the structure of logical algebras in different aspects (see [2]), may help the researchers who use algebra and logic to construct their models. In this article, we consider a structure called BM-algebra which is a class of abstract algebra, introduced by Kim and Kim [3]. Megalai and Tamilarasi [4] renamed this algebraic structure as TM-algebra. Several authors have considered different characterizations of BM-algebra ([9], [10], [7]) and made fuzzifications to BM/TM-algebra ([1], [5], [6], [8]) over the range of [0,1]. As a generalization of it, algebraic systems with maps over the range $\mathbb{R}$ namely quasi-valuation can be considered.

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In this paper, we put forth the concept of quasi-valuation maps based on an ideal (briefly, IQV-maps) and a subalgebra (briefly, SQV-maps) in BM-algebras, and then we investigate the relations between them. We obtain conditions for a real-valued function on a BM-algebra to be an IQV-map. Using the notion of an IQV-map, we construct a (pseudo) metric on a BM-algebra, and prove that the binary operation in it is uniformly continuous. We introduce a congruence relation from an IQV-map and construct a quotient structure which is again a BM-algebra. Further we obtain an isometry between two BM-algebras by defining suitable IQV-maps and a homomorphism.

2. QV-MAPS ON BM-ALGEBRAS

Definition 2.1. *A BM-algebra is a triple* $(\mathcal{A}, *, \theta)$ *where* $\mathcal{A}(\neq \phi)$ *is a set with a fixed element* θ *and a binary operation* $*$*, satisfying the following two conditions: (1)* $a * \theta = a$ *and (2)* $(a * b) * (a * c) = c * b$ *for all* $a, b, c \in \mathcal{A}$.

We introduce the concept of a quasi-valuation map based on a subalgebra (briefly, SQV-map) in a BM-algebra.

Definition 2.2. *By a SQV-map of a BM-algebra* $\mathcal{A}$*, we mean a real valued map* $\rho : \mathcal{A} \to \mathbb{R}$ *such that the following condition is satisfied:*

$$\rho(a * b) \geq \rho(a) + \rho(b) \text{ for all } a, b \in \mathcal{A}. \tag{2.1}$$

Example 1. *Consider the BM-algebra* $(\mathcal{A}, *, \theta)$ *where* $\mathcal{A} = \{\theta, p, q\}$ *with the binary operation* $*$ *given by* $\theta * \theta = p * p = q * q = \theta, \theta * p = p * q = q * \theta = q$ *and* $\theta * q = p * \theta = q * p = p$*. Let* ρ *be a real-valued map on* $\mathcal{A}$ *defined by* $\rho(\theta) = -2, \rho(p) = -3$ *and* $\rho(q) = -4$*. Then* ρ *is a SQV-map of* $\mathcal{A}$.

Proposition 2.1. *For any SQV-map* ρ *of a BM-algebra* $\mathcal{A}$*, we have for all* $a \in \mathcal{A}$

(1) $\rho(a) \leq (\frac{1}{2})\rho(\theta)$;
(2) $\rho(a) \leq 0$.

Proof.

(1) For any $a \in \mathcal{A}$, we have $\rho(\theta) = \rho(a * a) \geq \rho(a) + \rho(a)$. Then $2\rho(a) \leq \rho(\theta)$ which implies $\rho(a) \leq (\frac{1}{2})\rho(\theta)$.
(2) For any $a, b \in \mathcal{A}$, we have $\rho(b) = \rho(a * (a * b)) \geq \rho(a) + \rho(a) + \rho(b) = 2\rho(a) + \rho(b)$ and so $2\rho(a) \leq 0$, which implies $\rho(a) \leq 0$. □

Definition 2.3. *By a quasi-valuation map based on an ideal (briefly, IQV-map) of a BM-algebra $\mathcal{A}$, we mean a real valued map $\rho : \mathcal{A} \to \mathbb{R}$ such that the following two conditions are satisfied: $\rho(\theta) = 0$ and*

$$\rho(a) \geq \rho(a * b) + \rho(b) \text{ for all } a, b \in \mathcal{A}. \tag{2.2}$$

This ρ is called an I-valuation map (IV-map) of $\mathcal{A}$ if $\rho(a) = 0 \implies a = \theta, \forall a \in \mathcal{A}$.

Example 2. *Let $(\mathcal{A}, *, \theta)$ be a BM-algebra where $\mathcal{A} = \{\theta, p, q\}$ and binary operation $*$ defined as in Example 1. Consider a real valued map ρ on $\mathcal{A}$ defined by $\rho(\theta) = 0, \rho(p) = -1$ and $\rho(q) = -2$. Then ρ is an IQV-map of $\mathcal{A}$.*

Theorem 2.1. *If $\mathcal{A}$ is a BM-algebra with an IQV-map ρ satisfying the property $\rho(\theta * a) \geq \rho(a)$ for all $a \in \mathcal{A}$, then ρ is a SQV-map of $\mathcal{A}$.*

Proof. Let $a, b \in \mathcal{A}$, then

$$\begin{aligned} \rho(a * b) &\geq \rho((a * b) * a) + \rho(a) \\ &= \rho((a * a) * b) + \rho(a) \\ &= \rho(\theta * b) + \rho(a) \\ &\geq \rho(a) + \rho(b). \end{aligned}$$

Thus ρ is a SQV-map of $\mathcal{A}$. □

Proposition 2.2. *We have the following properties for any IQV-map ρ.*

(1) $\rho(a * b) + \rho(b * a) \leq 0$ *for all* $a, b \in \mathcal{A}$.
(2) $\rho(a * b) \geq \rho(a * c) + \rho(c * b)$ *for all* $a, b, c \in \mathcal{A}$.

Proof.

(1) We have $\rho(a * b) \leq \rho(a) - \rho(b)$ and $\rho(b * a) \leq \rho(b) - \rho(a)$. Thus we get $\rho(a * b) + \rho(b * a) \leq 0$.
(2) Using (2.1), we have $\rho(a*b) \geq \rho((a*b)*(a*c)) + \rho(a*c) = \rho(c*b) + \rho(a*c)$ for all $a, b, c \in \mathcal{A}$. □

For any real-valued function ρ on $\mathcal{A}$, consider the following two sets: $I_\rho := \{a \in \mathcal{A} | \rho(a) = 0\}$ and $J_\rho := \{a \in \mathcal{A} | \rho(a) \geq 0\}$.

Theorem 2.2. *If ρ is an IQV- map of a BM-algebra $\mathcal{A}$, then the set J_ρ is an ideal of $\mathcal{A}$ called the ideal induced by ρ.*

Proof. Clearly, $\theta \in J_\rho$. Let $a, b \in \mathcal{A}$ be such that $a * b \in J_\rho$ and $b \in J_\rho$. Then, $\rho(a*b) \geq 0$ and $\rho(b) \geq 0$. Therefore $\rho(a) \geq \rho(a*b) + \rho(b) \geq 0$ and so $a \in J_\rho$. Hence J_ρ is an ideal of $\mathcal{A}$. □

Theorem 2.3. *If ρ is an IQV- map of a BM-algebra $\mathcal{A}$ with the condition $J_\rho = \{\theta\}$, then the set I_ρ is an ideal of $\mathcal{A}$.*

Proof. Clearly, $\theta \in I_\rho$. Let $a, b \in \mathcal{A}$ be such that $a * b \in I_\rho$ and $y \in I_\rho$. Then $\rho(a * b) = 0$ and $\rho(b) = 0$. Using equation (2.2) we get $\rho(a) \geq \rho(a * b) + \rho(b) = 0$. But we have assumed that $J_\rho = \{\theta\}$. Thus $\rho(a) = 0$. Therefore $a \in I_\rho$ so that I_ρ is an ideal of $\mathcal{A}$. □

Define a mapping $d_\rho : \mathcal{A} \times \mathcal{A} \to \mathbb{R}$, with $(a, b) \mapsto -\rho(a * b) - \rho(b * a)$, for a real-valued function ρ on a BM-algebra $\mathcal{A}$.

Lemma 2.1. *If $\mathcal{A}$ is a BM-algebra with an IQV-map ρ, then d_ρ is a pseudo-metric induced by ρ on $\mathcal{A}$, and so $(\mathcal{A}, d_\rho)$ is a pseudo-metric space.*

Proof. Let $\mathcal{A}$ is a BM-algebra with an IQV-map ρ. It follows from Proposition 2.2 (1) that $d_\rho(a, b) \geq 0$ for all $a, b \in \mathcal{A}$. It is obvious that $d_\rho(a, a) = 0$ and $d_\rho(a, b) = d_\rho(b, a)$ for all $a, b \in \mathcal{A}$. Let $a, b, c \in \mathcal{A}$. By using Proposition 2.2 (2), we get

$$\begin{aligned} d_\rho(a, b) + d_\rho(b, c) &= (-\rho(a * b) - \rho(b * a)) + (-\rho(b * c) - \rho(c * b)) \\ &\geq -\rho(a * c) - \rho(c * a) \text{ by Proposition 2.2 (2)} \\ &= d_\rho(a, c). \end{aligned}$$

Therefore $(\mathcal{A}, d_\rho)$ is a pseudo-metric space. □

Theorem 2.4. *Let $\mathcal{A}$ is a BM-algebra with an IQV-map ρ. If ρ satisfies the condition $J_\rho = \{\theta\}$, then $(\mathcal{A}, d_\rho)$ is a metric space.*

Proof. We have $(\mathcal{A}, d_\rho)$ is a pseudo-metric space by Lemma 2.1, since ρ is an IQV-map of a BM-algebra $\mathcal{A}$. Since $J_\rho = \{\theta\}$ we get $\rho(a) \leq 0\ \forall a \in \mathcal{A}$. Let $a, b \in \mathcal{A}$ be such that $d_\rho(a, b) = 0$. Then $0 = d_\rho(a, b) = -\rho(a*b) - \rho(b*a)$. ie, $\rho(a*b) + \rho(b*a) = 0$ and so $\rho(a * b) = 0$ and $\rho(b * a) = 0$ since $\rho(a) \leq 0$. It follows that $a * b = \theta$ and $b * a = \theta$ which implies $a = b$. Therefore $(\mathcal{A}, d_\rho)$ is a metric space. □

Proposition 2.3. *Let ρ be an IQV- map of a BM-algebra $\mathcal{A}$. Then every pseudo-metric d_ρ induced by ρ satisfies the following inequalities for all $a, b, x, y \in \mathcal{A}$.*

(1) $d_\rho(a, b) = d_\rho(a * x, b * x) = d_\rho(x * a, x * b)$

(2) $d_\rho(a*b, x*y) \leq d_\rho(a*b, x*b) + d_\rho(x*b, x*y)$

Proof.

(1) Consider $a, b, x \in \mathcal{A}$. Since $(a*x)*(b*x) = a*b$ and $(b*x)*(a*x) = b*a$, we get $\rho(a*b) = \rho((a*x)*(b*x))$ and $\rho(b*a) = \rho((b*x)*(a*x))$.

$$\begin{aligned} d_\rho(a,b) &= -\rho(a*b) - \rho(b*a) \\ &= -\rho((a*x)*(b*x)) - \rho((b*x)*(b*x)) \\ &= d_\rho(a*x, b*x). \end{aligned}$$

Similarly, $d_\rho(a,b) = d_\rho(x*a, x*b)$.

(2) Using Proposition 2.2 (2), we get $\rho((a*b)*(x*y)) \geq \rho((a*b)*(x*b)) + \rho((x*b)*(x*y))$ and $\rho((x*y)*(a*b)) \geq \rho((x*y)*(x*b)) + \rho((x*b)*(a*b))$ for all $a, b, x, y \in \mathcal{A}$. Therefore for all $a, b, x, y \in \mathcal{A}$,

$$\begin{aligned} d_\rho(a*b, x*y) &= -\rho((a*b)*(x*y)) - \rho((x*y)*(a*b)) \\ &\leq -(\rho((a*b)*(x*b)) + \rho((x*b)*(x*y))) \\ &\quad - (\rho((x*y)*(x*b)) + \rho((x*b)*(a*b))) \\ &= d_\rho(a*b, x*b) + d_\rho(x*b, x*y). \end{aligned}$$

□

Let $(\mathcal{A}_1, *_1, \theta_1)$ and $(\mathcal{A}_2, *_2, \theta_2)$ be BM-algebras. Define a binary operation $\odot$ on $\mathcal{A}_1 \times \mathcal{A}_2$ by $(a,b) \odot (x,y) = (a *_1 x, b *_2 y)$ for all $(a,b), (x,y) \in \mathcal{A}_1 \times \mathcal{A}_2$. Then $(\mathcal{A}_1 \times \mathcal{A}_2, \odot, (\theta_1, \theta_2))$ is a BM-algebra.

Lemma 2.2. *If d_ρ is a pseudo-metric on a BM-algebra $\mathcal{A}$ with the real valued function ρ defined on $\mathcal{A}$, then $(\mathcal{A} \times \mathcal{A}, d_\rho^*)$ is a pseudo-metric space, where $d_\rho^*((a,b),(x,y))$ $= \max\{d_\rho(a,x), d_\rho(b,y)\}$ for all $(a,b),(x,y) \in \mathcal{A} \times \mathcal{A}$.*

Proof. Let d_ρ is a pseudo-metric on $\mathcal{A}$. Obviously for all $(a,b),(x,y) \in \mathcal{A} \times \mathcal{A}$, we get $d_\rho^*((a,b),(x,y)) \geq 0$. We have, for any $(a,b),(x,y) \in \mathcal{A} \times \mathcal{A}$, $d_\rho^*((a,b),(a,b)) = \max\{d_\rho(a,a), d_\rho(b,b)\} = 0$ and

$$\begin{aligned} d_\rho^*((a,b),(x,y)) &= \max\{d_\rho(a,x), d_\rho(b,y)\} \\ &= \max\{d_\rho(x,a), d_\rho(y,b)\} \\ &= d_\rho^*((x,y),(a,b)). \end{aligned}$$

Now suppose $(a,b),(x,y),(u,v) \in \mathcal{A} \times \mathcal{A}$. Then $d_\rho^*((a,b),(u,v)) + d_\rho^*((u,v),(x,y))$

$$= \max\{d_\rho(a,u), d_\rho(b,v)\} + \max\{d_\rho(u,x), d_\rho(v,y)\}$$
$$\geq \max\{d_\rho(a,u) + d_\rho(u,x), d_\rho(b,v) + d_\rho(v,y)\}$$
$$\geq \max\{d_\rho(a,x), d_\rho(b,y)\} = d^*_\rho((a,b),(x,y)).$$

Therefore $(\mathcal{A}\times\mathcal{A}, d^*_\rho)$ is a pseudo-metric space. □

Theorem 2.5. *Let $\rho : \mathcal{A} \to \mathbb{R}$ be an IQV- map of a BM-algebra $\mathcal{A}$ with the condition $J_\rho = \{\theta\}$. Then $(\mathcal{A}\times\mathcal{A}, d^*_\rho)$ is a metric space.*

Proof. By Lemma 2.1, we know that d_ρ is a pseudo-metric on a BM-algebra $\mathcal{A}$ where $\rho : \mathcal{A} \to \mathbb{R}$ is an IQV- map of $\mathcal{A}$ with $J_\rho = \{\theta\}$. Also from Lemma 2.2 we get $(\mathcal{A}\times\mathcal{A}, d^*_\rho)$ is a pseudo-metric space. Let $(a,b),(x,y) \in \mathcal{A}\times\mathcal{A}$ be such that $d^*_\rho((a,b),(x,y)) = 0$. Therefore $0 = d^*_\rho((a,b),(x,y)) = \max\{d_\rho(a,x), d_\rho(b,y)\}$, and so $d_\rho(a,x) = 0 = d_\rho(b,y)$ since $d_\rho(a,b) \geq 0$ for all $(a,b) \in \mathcal{A}\times\mathcal{A}$. Hence $0 = d_\rho(a,x) = -\rho(a*x) - \rho(x*a)$ and $0 = d_\rho(b,y) = -\rho(b*y) - \rho(y*b)$. It follows from the assumption that $\rho(a*x) = 0 = \rho(x*a)$ and $\rho(b*y) = 0 = \rho(y*b)$. So since ρ is an IQV- map, we get $a*x = \theta = x*a$ and $b*y = \theta = y*b$. Using property given in Definition 2.1, we have $x = a$ and $y = b$, and so $(a,b) = (x,y)$. Hence $(\mathcal{A}\times\mathcal{A}, d^*_\rho)$ is a metric space. □

Theorem 2.6. *If $\rho : \mathcal{A} \to \mathbb{R}$ is an IQV- map of a BM-algebra $\mathcal{A}$ with the condition $J_\rho = \{\theta\}$, then the binary operation $*$ in the BM-algebra $\mathcal{A}$ is uniformly continuous.*

Proof. For any $\epsilon > 0$, if $d^*_\rho((a,b),(x,y)) < \epsilon/2$, then $d_\rho(a,x) < \epsilon/2$ and $d_\rho(b,y) < \epsilon/2$. Using Proposition 2.3, we get

$$\begin{aligned} d_\rho(a*b, x*y) &\leq d_\rho(a*b, x*b) + d_\rho(x*b, x*y) \\ &= d_\rho(a,x) + d_\rho(b,y) \\ &< \epsilon/2 + \epsilon/2 = \epsilon. \end{aligned}$$

Hence the binary operation $* : \mathcal{A}\times\mathcal{A} \to \mathcal{A}$ is uniformly continuous. □

Theorem 2.7. *Let ρ be an IQV- map of a BM-algebra $\mathcal{A}$. Define a relation τ_ρ on $\mathcal{A}$ by $(a,b) \in \tau_\rho \iff \rho(a*b) + \rho(b*a) = 0$ for all $a,b \in \mathcal{A}$. Then τ_ρ is a congruence relation on $\mathcal{A}$. We say τ_ρ is the congruence relation induced by ρ.*

Proof. Since τ_ρ induced by a pseudo-metric, it is an equivalence relation on $\mathcal{A}$. Assume $a,b,x,y \in \mathcal{A}$ be such that $(a,b) \in \tau_\rho$ and $(x,y) \in \tau_\rho$. Then, $\rho(a*b)+\rho(b*a) =$

0 and $\rho(x * y) + \rho(y * x) = 0$. It follows that

$$-\rho((a * x) * (b * y)) - \rho((b * y) * (a * x)) = d_\rho(a * x, b * y) \leq d_\rho(a, b) = -\rho(a * b) - \rho(b * a) = 0.$$

Therefore, $\rho((a * x) * (b * y)) + \rho((b * x) * (a * x)) = 0$, and so $(a * x, b * y) \in \tau_\rho$. Hence τ_ρ is a congruence relation on $\mathcal{A}$. □

Definition 2.4. *Let ρ be an IQV- map of a BM-algebra $\mathcal{A}$ and τ_ρ be a congruence relation on $\mathcal{A}$ induced by ρ. For any $a \in \mathcal{A}$, the set $a_\rho := \{b \in \mathcal{A} | (a, b) \in \tau_\rho\}$ is called an equivalence class of a. Let the set of all equivalence classes be denoted by $\mathcal{A}_\rho$; that is, $\mathcal{A}_\rho := \{a_\rho | a \in \mathcal{A}\}$. Now define a binary operation "$*_\rho$" on $\mathcal{A}_\rho$ as: $a_\rho *_\rho b_\rho = (a * b)_\rho \; \forall \; a_\rho, b_\rho \in \mathcal{A}_\rho$. The resulting algebra $(\mathcal{A}_\rho, *_\rho, \theta_\rho)$ is called the quotient algebra of $\mathcal{A}$ induced by ρ.*

Theorem 2.8. *Let ρ be an IQV- map of a BM-algebra $\mathcal{A}$. Then the quotient algebra $(\mathcal{A}_\rho, *_\rho, \theta_\rho)$ induced by ρ is a BM-algebra.*

Proof. Since ρ is an IQV- map of $\mathcal{A}$, the operation $*_\rho$ is well-defined. For any $a_\rho, b_\rho, c_\rho \in \mathcal{A}_\rho$, we have $a_\rho *_\rho a_\rho = (a * a)_\rho = \theta_\rho$ and $(a_\rho *_\rho b_\rho) *_\rho (a_\rho *_\rho c_\rho) = ((a * b) * (a * c))_\rho = (c * b)_\rho = c_\rho *_\rho b_\rho$. Hence, $(\mathcal{A}_\rho, *_\rho, \theta_\rho)$ is a BM-algebra. □

Remark 2.1. *$d_\rho^\# : \mathcal{A}_\rho \times \mathcal{A}_\rho \to \mathbb{R}$ defined by $d_\rho^\#(a_\rho, b_\rho) = d_\rho(a, b)$ is a well defined metric on $\mathcal{A}_\rho$. Then the natural projection map $\pi : \mathcal{A} \to \mathcal{A}_\rho$ defined by $\pi(a) = a_\rho$ is an isometry (ie, preserves distances). Moreover, the quotient topology on $\mathcal{A}_\rho$ coincide with the metric topology induced by $d_\rho^\#$.*

Proposition 2.4. *Suppose $(\mathcal{A}, *, \theta)$ and $(\mathcal{B}, *_1, \theta_1)$ be two BM-algebras. Let $\Phi : \mathcal{A} \to \mathcal{B}$ be a BM-homomorphism, and $\sigma : \mathcal{B} \to \mathbb{R}$ be an IQV- map on $\mathcal{B}$. Then $\rho = \sigma \circ \Phi$ is an IQV- map on $\mathcal{A}$.*

Proof. Let $a, b \in \mathcal{A}$. Then $\rho(a) = \sigma \circ \Phi(a) = \sigma(\Phi(a)) \geq \sigma(\Phi(a) *_1 \Phi(b)) + \sigma(\Phi(b)) = \sigma(\Phi(a * b)) + \sigma(\Phi(b)) = \rho(a * b) + \rho(b)$ and $\rho(\theta) = \sigma \circ \Phi(\theta) = \sigma(\Phi(\theta)) = \sigma(\theta_1) = 0$. Hence ρ is an IQV- map on $\mathcal{A}$. □

Proposition 2.5. *Suppose $(\mathcal{A}, *, \theta)$ and $(\mathcal{B}, *_1, \theta_1)$ be two BM-algebras with a BM-homomorphism Φ and $\sigma : \mathcal{B} \to \mathbb{R}$ be an IQV- map on $\mathcal{B}$. Let $\rho = \sigma \circ \Phi$. Then Φ preserves distances under the pseudo metric induced by ρ and σ.*

Proof. We know that ρ and σ are IQV- maps on $\mathcal{A}$ and $\mathcal{B}$ respectively. Therefore pseudo-metrics induced by ρ and σ are $d_\rho(a,b) = -\rho(a*b) - \rho(b*a)\ \forall a,b \in \mathcal{A}$ and $d_\sigma(u,v) = -\sigma(u *_1 v) - \sigma(v *_1 u)\ \forall u,v \in \mathcal{B}$ respectively. Then

$$\begin{aligned} d_\rho(a,b) &= -\rho(a*b) - \rho(b*a) \\ &= -\sigma(\Phi(a*b)) - \sigma(\Phi(b*a)) \\ &= -\sigma(\Phi(a) *_1 \Phi(b)) - \sigma(\Phi(b) *_1 \Phi(a)) \\ &= d_\sigma(\Phi(a), \Phi(b))\ \forall a,b \in \mathcal{A}. \end{aligned}$$

□

Corollary 2.1. *If Φ is a bijection, then it is an isometry and hence is a homeomorphism.*

Proof. Suppose $a \in \mathcal{A}$. Let $\epsilon \in \mathbb{R}^+$ and $\delta = \epsilon$. Then $d_\rho(a,b) < \delta$ for some $b \in \mathcal{A}$ $\implies d_\sigma(\Phi(a), \Phi(b)) < \delta = \epsilon$. Thus, Φ is continuous on $\mathcal{A}$. Similarly since Φ^{-1} is also an isometry, it is also continuous on $\mathcal{B}$. Hence Φ is a homeomorphism. □

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