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PROPERTIES OF SOLUTIONS OF EQUATION OF REGULATORY MECHANISMS OF CARDIAC ACTIVITY

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ABSTRACT. The purpose of this article is qualitative analysis of the mathematical model of the regulatory mechanisms of cardiac activity. Mathematical model is in form of functional-differential equation with delay argument. In this paper analyzed the properties of solutions of equation of mathematical model such as existence, continuity, non-negativity, uniqueness, and limitation of solutions. As a result, the ability of this equation to correctly describe the properties of the biological system under consideration is mathematically substantiated.

1. INTRODUCTION

In recent years, mathematical and computer modeling of biological systems based on the laws of regulatorika has been developing. In this task, it is important to take into account the specifics of the processes occurring in the considering biological system, the spatio-temporal relationship between the elements of the system [1-6]. Since the functioning of biological systems consists of nonlinear complex processes, mathematical models of these system cannot be studied with the use of existing traditional classical methods. Therefore, a qualitative analysis of the equations is necessary [7-9]. With a qualitative analysis of the

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equation, it can be determined that the characteristic solutions can correctly reflect the activity and modes of the considering living system. Before obtaining analytical solutions (if any) or numerical solutions of the equation, for a qualitative analysis of the mathematical model of biological objects based on the theory and methods of qualitative analysis, the features of the characteristic solutions of the model are determined.

2. Equation Of Mathematical Model

Let's consider a qualitative analysis of equation (2.1) of regulatory mechanisms of cardiac activity.

$$\begin{aligned}&\frac{1}{h}\frac{dZ(\theta)}{d\theta}\\ &= \frac{AZ^6(\theta-1)(1+BZ^6(\theta-1))^2}{((1+BZ^6(\theta-1))^2+CZ^6(\theta-1))((1+BZ^6(\theta-1))^2+DZ^6(\theta-1))}\\ &- b_1Z(\theta),\end{aligned} \tag{2.1}$$

$$\theta > 1,$$

$$Z(\theta) = \varphi(\theta), \quad \theta \in [0,1],$$

where $Z(\theta)$-cardiac myocardial activity; A, B, C, D, b_1 - non negative parameters.

All parameters of equation (2.1) are non-negative. Only in this case, the solutions of the equation representing the activity of the excitation of conduction system of the heart can make biological sense. We begin the qualitative analysis of equation (2.1) by examining the existence, continuity, non-negativity, uniqueness, and limitation of solutions [8-11]. Otherwise, if the solutions do not meet these conditions, does not make sense from a biological point of view.

3. Existence And Continuity Of The Solutions

Using the sequential integration method given in the [1,12,13], we prove the existence and continuity of the solutions of equation (2.1). In order to obtain solutions of functional-differential equations with delayed arguments by using the method of sequential integration, the initial conditions are required to be given in the form of a continuous function in the intersection [10,11,13].

Therefore, for the initial condition $Z(\theta)$ of equation (2.1), let it be in the form of a continuous function defined on a segment, and not at a certain point

$$Z(\theta) = \varphi(\theta), \quad \theta \in [0, 1].$$

Using the initial continuous function $\varphi(\theta)$ given in the interval $[0, 1]$, we can find solutions $Z(\theta)$ in the case $\theta > 1$. In this case, the solutions of the previous segment are taken as the initial value for finding solutions in each segment. In this case, to find a solution to the function $Z(\theta)$ on the interval $(1, 2]$, expression (2.1) can be written as follows:

$$\begin{aligned} &\frac{1}{h}\frac{dZ(\theta)}{d\theta} \\ &= \frac{A\varphi^6(\theta-1)(1+B\varphi^6(\theta-1))^2}{((1+B\varphi^6(\theta-1))^2+C\varphi^6(\theta-1))((1+B\varphi^6(\theta-1))^2+D\varphi^6(\theta-1))} \\ &- b_1 Z(\theta). \end{aligned} \tag{3.1}$$

To find solutions (3.1), we introduce the following notation:

$$Z(\theta) = Q(\theta)e^{-b_1 h\theta}, \; \frac{dZ(\theta)}{d\theta} = \frac{dQ(\theta)}{d\theta}e^{-b_1 h\theta} - Q(\theta)e^{-b_1 h\theta}b_1 h. \tag{3.2}$$

Using (3.2), we write (3.1) as follows:

$$\begin{aligned} &\frac{1}{h}\frac{dQ(\theta)}{d\theta}e^{-b_1 h\theta} - Q(\theta)e^{-b_1 h\theta}b_1 = \\ =&\frac{A\varphi^6(\theta-1)(1+B\varphi^6(\theta-1))^2}{((1+B\varphi^6(\theta-1))^2+C\varphi^6(\theta-1))((1+B\varphi^6(\theta-1))^2+D\varphi^6(\theta-1))} - \\ &-b_1 Q(\theta)e^{-b_1 h\theta} \end{aligned}$$

and simplify as follows:

$$\begin{aligned} &\frac{dQ(\theta)}{d\theta} = \\ &e^{b_1 h\theta}h\frac{A\varphi^6(\theta-1)(1+B\varphi^6(\theta-1))^2}{((1+B\varphi^6(\theta-1))^2+C\varphi^6(\theta-1))((1+B\varphi^6(\theta-1))^2+D\varphi^6(\theta-1))}. \end{aligned}$$

Integrate the resulting expression.

$$\begin{aligned} &Q(\theta) = Q(1) + \\ &h\int_1^\theta e^{b_1 h\tau}\frac{A\varphi^6(\tau-1)(1+B\varphi^6(\tau-1))^2}{((1+B\varphi^6(\tau-1))^2+C\varphi^6(\tau-1))((1+B\varphi^6(\tau-1))^2+D\varphi^6(\tau-1))}d\tau. \end{aligned}$$

Using the entry (3.2), we can write the following expression:

$$Z(\theta)e^{b_1h\theta} = Z(1)e^{b_1h}+$$
$$h\int_1^\theta e^{b_1h\tau}\frac{A\varphi^6(\tau-1)(1+B\varphi^6(\tau-1))^2}{((1+B\varphi^6(\tau-1))^2+C\varphi^6(\tau-1))((1+B\varphi^6(\tau-1))^2+D\varphi^6(\tau-1))}d\tau.$$

Simplifying the expression, we obtain the following equation:

$$Z(\theta) = Z(1)e^{b_1h(1-\theta)}+$$
$$h\int_1^\theta e^{b_1h(\tau-\theta)}\frac{A\varphi^6(\tau-1)(1+B\varphi^6(\tau-1))^2}{((1+B\varphi^6(\tau-1))^2+C\varphi^6(\tau-1))((1+B\varphi^6(\tau-1))^2+D\varphi^6(\tau-1))}d\tau.$$

Given that the function Z (1) is continuous at $\theta = 1$ point, the following is appropriate:

(3.3)

$$Z(\theta) = \varphi(1)e^{b_1h(1-\theta)}+$$
$$h\int_1^\theta e^{b_1h(\tau-\theta)}\frac{A\varphi^6(\tau-1)(1+B\varphi^6(\tau-1))^2}{((1+B\varphi^6(\tau-1))^2+C\varphi^6(\tau-1))((1+B\varphi^6(\tau-1))^2+D\varphi^6(\tau-1))}d\tau.$$

The found equation (3.3) is a solution of equation (2.1) in the interval $(1, 2]$ and is the initial function for the interval $(2, 3]$. Thus, continuing the process of sequential integration, we can find continuous solutions of equation (2.1) for $\theta > 1$.

4. Uniqueness Of The Solutions

Let's check the uniqueness of the solutions. If equation (2.1) has a single solution $Z_0(\theta) = \varphi(\theta)$ in the interval $[0, 1]$, suppose that in section $(1, 2]$, equation (2.1) has 2 solutions, $Z_1(\theta)$ and $Z_2(\theta)$:

$$\frac{1}{h}\frac{dZ_1(\theta)}{d\theta}$$
$$=\frac{A\varphi^6(\theta-1)(1+B\varphi^6(\theta-1))^2}{((1+B\varphi^6(\theta-1))^2+C\varphi^6(\theta-1))((1+B\varphi^6(\theta-1))^2+D\varphi^6(\theta-1))}-$$
$$-\,b_1Z_1(\theta);$$

$$\begin{aligned}&\frac{1}{h}\frac{dZ_2(\theta)}{d\theta}\\&=\frac{A\varphi^6(\theta-1)(1+B\varphi^6(\theta-1))^2}{((1+B\varphi^6(\theta-1))^2+C\varphi^6(\theta-1))((1+B\varphi^6(\theta-1))^2+D\varphi^6(\theta-1))}-\\&-b_1Z_2(\theta).\end{aligned}$$

Let's check the differences of the solutions

$$\begin{aligned}\frac{1}{h}\frac{dZ_1(\theta)-dZ_2(\theta)}{d\theta}&=b_1(Z_2(\theta)-Z_1(\theta)),\\ \frac{dZ_1(\theta)-dZ_2(\theta)}{Z_1(\theta)-Z_2(\theta)}&=-b_1h\ d\theta,\\ \ln(Z_1(\theta)-Z_2(\theta))-\ln(Z_1(1)-Z_2(1))&=b_1h(1-\theta),\\ Z_1(\theta)-Z_2(\theta)&=(Z_1(1)-Z_2(1))\,e^{b_1h(1-\theta)},\end{aligned}$$

where given that $Z_1(1)=Z_2(1)$ is a continuous solutions and uniqueness of solution at this point, then $Z_1(\theta)=Z_2(\theta)$ is appropriate in the interval $\theta\in(1,2]$. It follows that the solutions of equation (2.1) in the interval $(1,2]$ are unique. Continuing the process of sequential integration in this way, we see that the solutions of equation (2.1) in the case $\theta>1$ are unique. Therefore, when the initial continuous function is given on the interval $[0,1]$, we can say that equation (2.1) of the regulatorika of cardiac activity have unique and continuous solutions for $\theta>1$.

5. Limitations Of Solutions

Let's check that solutions are limited or not. For non-negative values of the parameters of equation (2.1), that is,$A>0,\ B>0,\ C>0,\ D>0,\ b_1>0$, for very large values of $Z(\theta)$ and $Z(\theta-1)$, the first term of the right-side of equation (2.1) should be as follows:

$$\lim_{Z(\theta-1)\to\infty}\frac{AZ^6(\theta-1)(1+BZ^6(\theta-1))^2}{((1+BZ^6(\theta-1))^2+CZ^6(\theta-1))((1+BZ^6(\theta-1))^2+DZ^6(\theta-1))}=0$$

In this case, equation (2.1) can be written as follows:

$$\frac{1}{h}\frac{dZ(\theta)}{d\theta}=-b_1Z(\theta). \tag{5.1}$$

It is seen that the solution

$$Z(\theta) = \varphi(1) + e^{-b_1 h\theta} \tag{5.2}$$

of equation (5.1) is limited. It follows that the solutions of equation (2.1) are also limited.

From a biological point of view, the activity of living systems is studied in the first quadrant of the phase space. Only in this case, the description of biological processes makes sense. Therefore, the solutions of equation (2.1) are also required to be non-negative. From the fact that in solution (5.2) the initial function $\varphi(1) > 0$ given in the interval $\theta \in [0, 1]$ is in positive form, it is clear that the solutions of equation (2.1) are non-negative for the case $\theta > 1$.

Thus, the existence, continuity, uniqueness, limitation, and nonnegative properties of solutions of equation (2.1) make it possible to express and study the activity of a biological object. As a continuation of the qualitative analysis of the solutions of equation (2.1) of regulatory mechanisms of cardiac activity, we must analyze the conditions for the existense of its equilibrium points, trivial and functional attractors, check the stability of the functional attraction and the emergence of periodic oscillation solutions around it (topic for our future paper).

6. Conclusion

Summarizing the above considerations, it can be noted that the studied equation (2.1) can be applied in the study of the properties of the considered biological system and it has been mathematically proven. Using this mathematical model allows us to study the regulatory mechanisms of cardiac activity, the causes of heart disease and choose the tactics of treatment. The adequacy of the mathematical model can be demonstrated by comparing the results of the qualitative analysis with the results of the quantitative analysis given in the works [13-14].

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