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SOME ASPECTS OF EXACT STATIC SPHERICALLY SYMMETRIC SOLUTION OF EINSTEIN'S FIELD EQUATIONS WITH SPECIFIED EQUATION OF STATE

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ABSTRACT. In this paper we have obtained some exact static spherical solution of Einstein's field equation with cosmological constant $\wedge = 0$ and equation of state $p = \rho$ (taking suitable choice of g_{11} and g_{44}). We have taken $e^{\beta} = lr^{n}, e^{\beta} = lr^{n-1},$ and $e^{\beta} = lr^{7/5}$ (where l is a constant), which help to investigate the value of e^{α}. Many previously known solutions are contained here in as a particular case. Various physical and geometrical properties have been studied.

1. INTRODUCTION

Mathematical analysis dealing with limits and related theories, such as differentiation, integration, measure, relativity and analytic functions. Analysis may be distinguished from geometry; however, it can be applied to any space of mathematical objects that has a specific distance between objects (a metric space). The vast majority of relativity, classical mechanics, and quantum mechanics is based on applied analysis and differential equations in particular. These important differential equations include Einstein's field equations, metric equations and the Newton's second law. Hence mathematical analysis is also a major factor in study of relativity and other branches of science.

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The present analysis deals with the exact solutions of the Einstein's field equations for the perfect fluid with variable gravitational and cosmological "constants" for a spatially homogeneous and anisotropic cosmological model. The Einstein's field equation has two parameters; the cosmological constants $\wedge$ and the gravitational constant G. Cosmological models with a cosmological constant are currently serious candidates to describe the dynamics of the Universe (Saha [16]). The rise of interest in the theory of General Relativity as a tool for studying the evolution and behaviour of various cosmological models has been rapid expensive. Since the early 1920's to the present, the Einstein's theory of relativity has been used extensively as a tool in the prediction and modelling of the cosmos. One reason for the prominence of modern relativity is its success in predicting the behaviour of large scale phenomena where gravitation plays a dominant role (Dicke [6], Feinstein [7]). Various researcher in theory of relativity have focused their mind to the study of solution of Einstein's field equation with cosmological constant $\wedge = 0$ and equation of state $p = \rho$. Solution of Einstein's field equation of state $p = \rho$ have been obtained by various authors e.g., Letelier [12], Letelier and Tabensky [13], Tabensky, R., et.al. [20] and Yadav [27]. Singh and Yadav [18] have also discussed the static fluid sphere with the equation of the state $p = \rho$. Further study in the line has been done by Yadav and Saini [26], which is more general than one due to Singh and Yadav [18]. Also in this case the relative mass m of a particle in the gravitational field related to its proper mass m_0 studied by Narlikar [14]. Schwarzschild considered the perfect fluid spheres with homogeneous density and isotropic pressure in general relativity and obtained the solutions of relativistic field equations. Tolman [22] developed a mathematical method for solving Einstein's field equations applied to static fluid spheres in such a manner as to provide explicit solutions in terms of known analytic functions. A number of new solutions were thus obtained and the properties of three of them were examined in detail.

No stationary in homogeneous solutions to Einstein's equations for an irrotational perfect fluid have featured equations of state $p = \rho$ (Letelier [12], Letelier and Tabensky [13] and Singh and Yadav [18]). Solutions to Einstein's equations with a simple equations of state have been found in various cases, e.g. for $\rho + 3p =$ constant (Whittaker [25]) for $\rho = 3p$ (Klein [9]); for $p = \rho +$ constant (Buchdahl and Land [4], Allnutt [1]) and for $\rho = (1 + a)\sqrt{p} + ap$ (Buchdahl [2]). But if one takes, e.g. polytrophic fluid sphere $\rho = ap^{1+\frac{1}{p}}$ (Klein [10], Tooper [23], Buchdahl [3]), one soon has to use numerical methods. Yadav and Saini [26] have also studied the static fluid sphere with equation of state $p = \rho$ (i.e. stiff matter). Davidson [5] has presented a solution a non stationary analog to the case when $p = \frac{1}{3}\rho$. Tolman [22], Thomas E Kiess [21], Karmer [11], Singh. et.al. [17], Raychaudhari [15], A.K. Singh et. al. [19], Walecka [24], Kandalkar [8], Yadav, et.al. [28], Singh [30], Fuloria [31] are some of the authors who have studied various aspects of interacting fields in the framework of Einstein's field equations for the perfect fluid with specified equation of state and general relativity.

In this paper we have obtained some exact static spherically symmetric solution of Einstein field equation for the static fluid sphere with cosmological constant $\wedge = 0$ and equation of state $p = \rho$. It has been obtained taking suitable choice of g_{11} and g_{44} (e.g.- $e^{\beta} = lr^{n}, e^{\beta} = lr^{n-1}, e^{\beta} = lr^{7/5}$ (where l is a constants). For different values of n we get many previously known solutions. To overcome the difficulty of infinite density at the centre, it is assumed that distribution has a core of radius r_0 and constant density ρ_0 which is surrounded by the fluid with the specified equation of state. Many previously known solutions are contained here in as a particular case. This paper has eight sections. First section is introduction. Second and third section deals about field equations and its solutions with different cases. Fourth section discusses about solution for the perfect fluid core. Finally, in fifth, sixth, seventh and eighth section we dealt about discussion, applications, future prospects and limitations of found results respectively.

2. Field Equations

We consider the static spherically symmetric metric given by

$$ds^2 = e^{\beta}dt^2 - e^{\alpha}dr^2 - r^2 d\theta^2 - r^2 \sin^2\theta d\varphi, \tag{2.1}$$

where α and β are functions of r only. Taking cosmological constant $\wedge$ into account, we obtain the field equations

$$R^i_j - \frac{1}{2}R\delta^i_j + \wedge\delta^i_j = -8\pi T^i_j. \tag{2.2}$$

For $\wedge = 0$, (2.2) gives

$$R^i_j - \frac{1}{2}R\delta^i_j = -8\pi T^i_j.$$

For the metric (2.1) in [22],

$$-8\pi T_1^1 = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

$$\begin{aligned} -8\pi T_2^2 &= -8\pi T_3^3 \\ &= e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} + \frac{\beta' - \alpha'}{2r}\right) \end{aligned}$$

$$-8\pi T_4^4 = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2},$$

where a prime denotes differentiation with respect to r.

Through the investigation, we set velocity of light C and gravitational constant G to be unity. A Zeldovich fluid can be regarded as a perfect fluid having the energy momentum tensor

$$T_j^i = (\rho + p)u^i u_j - \delta_j^i p.$$

Specified by the equation of state

$$\rho = p, \tag{2.3}$$

we use co-moving co-ordinates so that

$$u^1 = u^2 = u^3 = 0 \quad \text{and} \quad u^4 = e^{-\frac{\beta}{2}}.$$

The non-vanishing components of the energy momentum tensor are

$$T_1^1 = T_2^2 = T_3^3 = -p \quad \text{and} \quad T_4^4 = \rho.$$

We can then write the field equations:

$$8\pi p = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} \tag{2.4}$$

$$8\pi p = e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} + \frac{\beta' - \alpha'}{2r}\right)$$

$$8\pi p = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}. \tag{2.5}$$

3. Solution Of The Field Equations

Using equations (2.3), (2.4) and (2.5), we have

$$e^{-\alpha}\left(\frac{\beta'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2}=e^{-\alpha}\left(\frac{\alpha'}{r}-\frac{1}{r^2}\right)+\frac{1}{r^2}. \tag{3.1}$$

From (3.1) we see that if β is known, α can be obtained, so we choose

Case I:

$$e^{\beta}=lr^n \quad \text{(where } l \text{ is a constant)} \tag{3.2}$$

Using (3.2), equation (3.1) goes to the-

$$\frac{\mathrm{d}}{\mathrm{d}r}e^{-\alpha}+\frac{n+2}{r}e^{-\alpha}=\frac{2}{r}. \tag{3.3}$$

Put $(\tau=e^{-\alpha})$ in the equation (3.3) is reduced to

$$\frac{\mathrm{d}}{\mathrm{d}r}\tau+\frac{n+2}{r}\tau=\frac{2}{r}.$$

This is linear differential equation whose solution is given by

$$\tau=\frac{2}{n+2}+\frac{C}{r^{n+2}}.$$

Or,

$$e^{-\alpha}=\frac{2}{n+2}+\frac{C}{r^{n+2}}. \tag{3.4}$$

Case II:

$$e^{\beta}=lr^{n-1} \tag{3.5}$$

Using (3.5), in equation (3.1) we get

$$\frac{\mathrm{d}}{\mathrm{d}r}e^{-\alpha}+\frac{n+1}{r}e^{-\alpha}=\frac{2}{r}.$$

Put $(\tau=e^{-\alpha})$ in the equation (3.3) is reduced to

$$\frac{\mathrm{d}}{\mathrm{d}r}\tau+\frac{n+1}{r}\tau=\frac{2}{r}.$$

Solution of this is linear differential equation is given by

$$\tau=\frac{2}{n+1}+\frac{C}{r^{n+1}}.$$

Or,

$$e^{-\alpha}=\frac{2}{n+1}+\frac{C}{r^{n+1}}.$$

From Case I, we get a generalized value of $e^{-\alpha}$ in (3.4) and hence

$$e^{\alpha} = \frac{kr^k}{2r^k + kC} \tag{3.6}$$

(where $k = n + 2, n =$ power of r and $C=$ integral constant). Hence, using the equation (3.4), the metric (2.1) yields:

$$ds^2 = lr^n dt^2 - \left(\frac{2}{n+2} + \frac{C}{r^{n+2}}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta . d\varphi^2). \tag{3.7}$$

Absorbing the constant in the co-ordinates differentials dt and putting $C = 0$, the metric (3.7) goes to the form

$$ds^2 = r^n dt^2 - \frac{n+2}{2} dr^2 - r^2(d\theta^2 + \sin^2\theta . d\varphi^2). \tag{3.8}$$

Or,

$$ds^2 = r^n dt^2 - \frac{k}{2} dr^2 - r^2(d\theta^2 + \sin^2\theta . d\varphi^2). \tag{3.9}$$

The non-zero components of Reimann-Christoffel curvature tensor R_{hijk} for the metric [(3.8), (3.9)] are

$$\sin^2\theta R_{2424} = R_{3434} = \frac{n+2}{2} r^n \sin^2\theta = \frac{k}{2} r^n \sin^2\theta = R_{2323}.$$

We see that $R_{hijk} \to 0$ as $r \to \infty$. Hence, it follows that the space time is asymptotically homaloidal.

For the metric (3.8) the fluid velocity v' is given by

$$v^1 = v^2 = v^3 = 0; v^4 = r^{-\frac{n}{2}} = \frac{1}{r^{n/2}}.$$

The scalar of expansion $\Theta = v_i^j$ is identically zero (i.e. $\Theta = 0$). The non-vanishing components of the tensor of rotation ω_{ij} is defined by

$$\omega_{ij} = v_{i,j} - v_{j,i} \tag{3.10}$$

$$\omega_{14} = -\omega_{41} = -\frac{n}{2} r^{\frac{n}{2}-1} = -\frac{n}{2} r^{\frac{n-2}{2}}.$$

The components of the shear tensor σ_{ij} defined by

$$\sigma_{ij} = \frac{1}{2}(v_{ij} + v^{ij}) - \frac{1}{3} H_{ij}, \tag{3.11}$$

with the projection tensor

$$H_{ij} = g_{ij} - v_i v_j \tag{3.12}$$

$$\sigma_{14} = \sigma_{41} = \frac{n}{2} r^{\frac{n}{2}-1} = \frac{n}{2} r^{\frac{n-2}{2}}. \tag{3.13}$$

(Particular Case)

If we choose

$$e^{\beta} = l r^{7/5}, \tag{3.14}$$

using (3.14) in equation (3.1) we get:

$$\frac{\mathrm{d}e^{-\alpha}}{\mathrm{d}r} + \frac{17}{5r} e^{-\alpha} = \frac{2}{r}. \tag{3.15}$$

Substituting $\tau = e^{-\alpha}$, the equation (3.15) reduced to,

$$\frac{\mathrm{d}\tau}{\mathrm{d}r} + \frac{17}{5r}\tau = \frac{2}{r},$$

which is a linear differential equation whose solution is given by:

$$\tau = \frac{10}{17} + \frac{C}{r^{17/5}}.$$

Or,

$$e^{-\alpha} = \frac{10}{17} + \frac{C}{r^{17/5}},$$

where C is integration constant.

Hence the metric (2.1) yields

$$ds^2 = l r^{7/5} dt^2 - \left(\frac{10}{17} + \frac{C}{r^{17/5}}\right)^{-1} dr^2 - r^2(d\theta^2 + r^2 \sin^2\theta d\varphi). \tag{3.16}$$

Absorbing the constant l in the co-ordinate differential dt and put $C = 0$ the metric (3.16) goes to the form

$$ds^2 = l r^{7/5} dt^2 - \frac{17}{10} dr^2 - r^2(d\theta^2 + r^2 \sin^2\theta d\varphi). \tag{3.17}$$

The non-zero component of Reimann-christoffel curvature tensor R_{hijk} for the metric (3.17) is

$$\sin^2\theta R_{2424} = R_{3434} = \frac{17}{10} r^{7/5} \ \sin^2\theta = R_{2323}.$$

For the metric [(3.17)] the fluid velocity v' is given by

$$v^1 = v^2 = v^3 = 0, v^4 = r^{-\frac{7}{10}} = \frac{1}{r^{7/10}}.$$

In the usual notation, we have the rotation and shear tensor same as equations (3.10), (3.11), (3.12) and (3.13) which gives results for metric (3.17) as:

$$\Theta = 0, \omega_{14} = -\omega_{41} = \frac{-7}{10} r^{-\frac{3}{10}} = \frac{-7}{10 r^{3/10}}$$

and

$$\sigma_{14} = \sigma_{41} = \frac{7}{10} r^{-\frac{3}{10}} = \frac{7}{10 r^{3/10}}.$$

4. Solution for the perfect fluid core

Pressure and density for the metric (3.7), (3.8), (3.17) are

$$8\pi p = 8\pi\rho = \frac{n+1}{r^2}\left[\frac{2}{n+2} + \frac{C}{r^{n+2}}\right] - \frac{1}{r^2}. \tag{4.1}$$

If we consider $C = 0$, then equation (4.1) reduces to

$$8\pi p = 8\pi\rho = \frac{n+1}{r^2}\left[\frac{2}{n+2}\right] - \frac{1}{r^2} \tag{4.2}$$

$$8\pi p = 8\pi\rho = \frac{18}{13 r^2}. \tag{4.3}$$

It follows from ((4.1)-(4.3)) that the density of the distribution tends to infinity as r tends to zero. In order to get rid of singularity at $r = 0$ in the density we visualize that the distribution has a core of radius r_0 and constant ρ_0. The field inside the core is given by Schwarzchild internal solution

$$e^{-\alpha} = 1 - \frac{r^2}{R^2} \tag{4.4}$$

$$e^{\beta} = \left[\overline{L} - \overline{M}\left(1 - \frac{r^2}{R^2}\right)\right]^2 \tag{4.5}$$

$$8\pi p = \frac{1}{R^2}\left[\frac{3\overline{M}\left(1 - \frac{r^2}{R^2}\right) - \overline{L}}{\overline{L} - \overline{M}\left(1 - \frac{r^2}{R^2}\right)^{\frac{1}{2}}}\right], \tag{4.6}$$

where $\overline{L}, \overline{M}$ are constants and $R^2 = \frac{3}{8\pi\rho}$.

The continuity condition for the metric (3.7) and [(4.4)-(4.6)] at the boundary gives

$$R^2 = \frac{r_0^2}{\left(\frac{n}{n+2} - \frac{C}{r_0^{n+2}}\right)}$$

$$\overline{L} = r_0^{n/2} + \frac{nR^2}{2r_0^{2-\frac{n}{2}}}\left(1 - \frac{r_0^2}{R^2}\right)$$

$$\overline{M} = \frac{nR^2}{2r_0^{2-\frac{n}{2}}}\left(1 - \frac{r_0^2}{R^2}\right)^{1/2}$$

$$C = r_0^{n+2}\left(\frac{n}{n+2} - \frac{r_0^2}{R^2}\right)$$

and the density of the core

$$\rho_0 = \frac{3}{8\pi r^2}\left(\frac{n}{n+2} - \frac{C}{r_0^{n+2}}\right),$$

which complete the solution for the perfect fluid of radius r_0 surrounded by considered fluid. The energy condition $T_{ij}u^iu_j > 0$ and the Hawking and Penrose condition

$$(T_{ij} - \frac{1}{2}g_{ij}T)u^iu_j > 0.$$

Both reduces to $\rho > 0$, which is obviously satisfied.

For different value of n, solution obtained above in case I and case II provide many previously known solutions. For $n = 2$ and by suitable adjustment of constant we get the solution due to Singh and Yadav [18] and Yadav and Saini [26]. Also, for $n = 3$, we get solution due to Yadav *et. al.* [27].

5. DISCUSSION

In this chapter we have obtained some exact static spherical solution of Einstein's field equation with cosmological constant $\wedge = 0$ and equation of state $p = \rho$. We have shown that when cosmological constant $\wedge = 0$, then in the absence of electromagnetic field pressure and density become equal and conversely if pressure and density are equal there is no electromagnetic field. Our assumption is $e^\beta = lr^n, e^\beta = lr^{n-1}, e^\beta = lr^{7/5}$, which investigate a generalized value of e^α. It describe several important cases, e.g.- relativistic model, fluid velocity, rotation, shear tensor, scalar of expansion. It also helpful to investigates solution for the perfect fluid core.

6. Applications

(i) In this paper we get a generalized value of e^{α} and metric. Now we can easily obtained the metric for any given value of n, where n is the power of r.
(ii) It helpful to investigates solution for the perfect fluid core.
(iii) To study of staler body and radiation.
(iv) To study of casual limit for ideal gas has also for $\rho = p$ and relativistic model, fluid velocity, rotation, shear tensor, scalar of expansion.

7. Future prospects

The investigation on this topic can be further taken up in different directions:

- This topic has been a proliferation of works on higher dimensional space times both in localized and cosmological domains.
- It is important in a natural way to make a search for exact solutions of theories of gravitation for different types of distributions of matter and for different type of symmetries of space time.
- This also helpful to provide the idea about study of physical situation at the early stages of the formation of the universe.

8. Limitation

If we consider $n = -2$ for equations (3.6), then the value of $e^{(-\alpha)}$ is undefined and e^{α} become a constant value.

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