Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9129–9136
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.18

INTERVAL VALUED ANTI FUZZY NEAR ALGEBRA OVER INTERVAL VALUED ANTI FUZZY FIELD

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ABSTRACT. The notion of an interval valued anti fuzzy field and interval valued anti fuzzy near-algebra over an interval valued anti fuzzy field is introduced. Using this notion we have described some basic properties with suitable examples.

1. INTRODUCTION

Zadeh [12] initiated the notion of a fuzzy set in 1965. Kim, Jun and Yon [7] have studied the concept of an anti fuzzy sub near-rings and anti fuzzy ideals of nearrings. Chandrasekhara Rao and Swaminathan [4] introduced the concept of an anti homomorphism in fuzzy ideals of near-rings. Brown [3] introduced the concept of near-algebras. In [9] Srinivas and Narasimha swamy have introduced the concept of fuzzy near-algebra over fuzzy field. Srinivas, et. al. [10], [1] discussed anti fuzzy near-algebra over an anti fuzzy field and also an anti homomorphism and anti fuzzy ideal of near-algebras. Some basic information about the interval valued fuzzy set given in [6]. Biswas [2] defined interval-valued fuzzy subgroups of the same nature of Rosenfeld fuzzy subgroups. Davvaz [5] introduced fuzzy ideals of near-rings with interval valued membership functions.

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2020 *Mathematics Subject Classification.* 16Y30, 03E72.

Key words and phrases. Near-algebra, Anti fuzzy field, Anti fuzzy near algebra, interval valued number.

Thillaigovindan et al. [11] have studied interval valued fuzzy ideals and anti fuzzy ideals of near-rings. Interval valued fuzzy near algebra over interval valued fuzzy field is introduced by Narasimha Swamy, et. al. in [8].

In this paper we introduce the concept of an interval valued anti fuzzy field and interval valued anti fuzzy near-algebra over interval valued anti fuzzy field.

2. Preliminaries

For the sake of continuity we recall some basic definitions.

An interval valued number $\widetilde{p}$ on $[0,1]$ is a closed subinterval of $[0,1]$, that is $\widetilde{p} = [p^-, p^+]$ such that $0 \leq p^- \leq p^+ \leq 1$ where p^- and p^+ are lower and upper limits of $\widetilde{p}$ respectively. The set of all closed sub intervals of $[0,1]$ is denoted by D[0, 1]. In this notation $\widetilde{0} = [0^-, 0^+]$ and $\widetilde{1} = [1^-, 1^+]$. We also identify the interval $[p,p]$ by the number $p \in [0,1]$. For any two interval numbers $\widetilde{p} = [p^-, p^+]$ and $\widetilde{q} = [q^-, q^+]$ on $[0,1]$, we define

(i) $\widetilde{p} \leq \widetilde{q} \Leftrightarrow p^- \leq q^-$ and $p^+ \leq q^+$,

(ii) $\widetilde{p} = \widetilde{q} \Leftrightarrow p^- = q^-$ and $p^+ = q^+$,

(iii)$\widetilde{p} < \widetilde{q} \Leftrightarrow \widetilde{p} \leq \widetilde{q}$ and $\widetilde{p} \neq \widetilde{q}$,

(iv) $k\overline{p} = [kp^-, kp^+]$, for $0 \leq k \leq 1$.

For any interval valued numbers $\widetilde{p_i} = [p_i^-, p_i^+], \widetilde{q_i} = [q_i^-, q_i^+] \in D[0,1], i \in I$ an index set we define

$\max^i\{\widetilde{p_i}, \widetilde{q_i}\} = [\max^i\{p_i^-, q_i^-\}, \max^i\{p_i^+, q_i^+\}]$,

$\min^i\{\widetilde{p_i}, \widetilde{q_i}\} = [\min^i\{p_i^-, q_i^-\}, \min^i\{p_i^+, q_i^+\}]$.

$\inf^i \widetilde{a_i} = [\overset{i}{\inf_{i\in I}} a_i^-, \overset{i}{\inf_{i\in I}} a_i^+]$ and

$\sup^i \widetilde{a_i} = [\overset{i}{\sup_{i\in I}} a_i^-, \overset{i}{\sup_{i\in I}} a_i^+]$.

Definition 2.1. *Let Z be a non-empty set. A mapping $\widetilde{\mu} : Z \to D[0,1]$ is called an interval valued fuzzy subset of Z. For all $x \in Z, \widetilde{\mu} = [\mu^-, \mu^+]$, μ^- and μ^+ are fuzzy subsets of Z such that $\mu^- \leq \mu^+$. Thus $\widetilde{\mu}(x)$ is an interval (a closed subset of $[0,1]$) not a number from the interval [0, 1] as in the case of fuzzy set. Let $\widetilde{\mu}, \widetilde{\nu}$ be an interval valued fuzzy subset of Z. Then the following are holds:*

(i) $\widetilde{\mu} \leq \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) \leq \widetilde{\nu}(x)$,

(ii) $\widetilde{\mu} = \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) = \widetilde{\nu}(x)$,

(iii) $\widetilde{\mu} = \widetilde{\nu} \Leftrightarrow \widetilde{\mu}(x) = \widetilde{\nu}(x)$,

(iv) $(\widetilde{\mu} \cup \widetilde{\nu})(x) \Leftrightarrow \max^i\{\widetilde{\mu}(x), \widetilde{\nu}(x)\}$,

(v) $(\widetilde{\mu} \cap \widetilde{\nu})(x) \Leftrightarrow \min^i\{\widetilde{\mu}(x), \widetilde{\nu}(x)\}$,

(vi) $\bigcap_{i\in I} \widetilde{\mu}_i(x) = \inf^i\{\widetilde{\mu}_i(x) : i \in I\}$,

(vii) $\bigcup_{i\in I} \widetilde{\mu}_i(x) = \sup^i\{\widetilde{\mu}_i(x) : i \in I\}$.

Here $\inf^i\{\widetilde{\mu}_i(x)/i \in \Lambda\} = \left[\inf_{i\in\Lambda}\{\mu_i^-(x)\}, \inf_{i\in\Lambda}\{\mu_i^+(x)\}\right]$ *is called interval valued infimum norm and* $\sup^i\{\widetilde{\mu}_i(x)/i \in \Lambda\} = \left[\sup_{i\in\Lambda}\{\mu_i^-(x)\}, \sup_{i\in\Lambda}\{\mu_i^+(x)\}\right]$ *is called interval valued supremum norm*

Definition 2.2. *A mapping* $\min^i : D[0,1]\times D[0,1] \to D[0,1]$ *defined by* $\min^i(\overline{a}, \overline{b}) = [\min(a^-, b^-), \min(a^+, b^+)]$ *for all* $\overline{a}, \overline{b} \in [0,1]$ *is called an interval min-norm. A mapping* $\max^i : D[0,1] \times D[0,1] \to D[0,1]$ *defined by* $\max^i(\overline{a}, \overline{b}) = [\max(a^-, b^-), \max(a^+, b^+)]$ *for all* $\overline{a}, \overline{b} \in [0,1]$ *is called interval max-norm. Let* $\min^i$ *and* $\max^i$ *be the interval valued min-norm and interval valued max-norm on* $D[0,1]$ *respectively. Then the following are true.*

(1) $\overset{i}{\min}\{\bar{a}, \bar{a}\} = \bar{a}$ *and* $\overset{i}{\max}\{\bar{a}, \bar{a}\} = \bar{a}\ \forall\ \bar{a} \in D[0,1]$.

(2) $\overset{i}{\min}\{\bar{a}, \bar{b}\} = \overset{i}{\min}\{\bar{b}, \bar{a}\}$ *and* $\overset{i}{\max}\{\bar{a}, \bar{b}\} = \overset{i}{\max}\{\bar{b}, \bar{a}\}\ \forall\ \bar{a}, \bar{b} \in D[0,1]$.

(3) *If* $\forall\ \bar{a}, \bar{b}, \bar{c} \in D[0,1],\ \bar{a} \geq \bar{b},$ *then* $\overset{i}{\min}\{\bar{a}, \bar{c}\} \geq \overset{i}{\min}\{\bar{b}, \bar{c}\}$ *and* $\overset{i}{\max}\{\bar{a}, \bar{c}\} \leq \overset{i}{\max}\{\bar{b}, \bar{c}\}$.

Definition 2.3. *A* right near-algebra Y *over a field* Z *is a linear space* Y *over* Z *on which a multiplication is defined such that*

(i) Y *forms a semigroup under multiplication,*

(ii) *multiplication is right distributive over addition(i.e.* $(a+b)c = ac + bc$) *for every* $a, b, c \in Y$*, and*

(iii) $\lambda(ab) = (\lambda a)b$ *for every* $a, b \in Y$ *and* $\lambda \in Z$.

3. Interval Valued Anti Fuzzy Field

Definition 3.1. *Let* $\tilde{\Psi}$ *be an interval valued anti fuzzy subset of a field* Z*. Then* $\tilde{\Psi}$ *is called an Interval Valued Anti Fuzzy Field of* Z *if the following conditions hold:*

(i) $\tilde{\Psi}(l+d) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d)) = \tilde{\Psi}(l) \vee \tilde{\Psi}(d)$ *for every* $l, d \in Z$*,*

(ii) $\tilde{\Psi}(-l) \leq \tilde{\Psi}(l)$ *for every* $r \in Z$*,*

(iii) $\tilde{\Psi}(ld) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d)) = \tilde{\Psi}(l) \vee \tilde{\Psi}(d)$ *for every* $l, d \in Z$*,*
(iv) $\tilde{\Psi}(l^{-1}) \leq \tilde{\Psi}(l)$ *for every* $l(\neq 0) \in Z$*.*

Theorem 3.1. *If* $\tilde{\Psi}$ *is an interval valued anti fuzzy field of* Z*, then* (i) $\tilde{\Psi}(0) \leq \tilde{\Psi}(l)$ *for every* $l \in Z$*,* (ii) $\tilde{\Psi}(1) \leq \tilde{\Psi}(l)$ *for every* $l(\neq 0) \in Z$ *and* (iii) $\tilde{\Psi}(0) \leq \tilde{\Psi}(1)$*.*

Proof.
(i) $\forall\, l \in Z$, $\tilde{\Psi}(0) = \tilde{\Psi}(l - l) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(-l)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(l)) = \tilde{\Psi}(l)$.
(ii) For any $l(\neq 0) \in Z$, $\tilde{\Psi}(1) = \tilde{\Psi}(ll^{-1}) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(l^{-1})) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(l)) = \tilde{\Psi}(l)$.
(iii) Particularly for $l = 1$ in (i), we get $\tilde{\Psi}(0) \leq \tilde{\Psi}(1)$. □

Note: If $\tilde{\Psi}$ is an interval valued anti fuzzy field of Z, then $\tilde{\Psi}(0) \leq \tilde{\Psi}(1) \leq \tilde{\Psi}(l)$ for every $l(\neq 0) \in Z$.

Theorem 3.2. $\tilde{\Psi}$ *is an interval valued anti fuzzy field of* Z *iff the following two conditions holds:*
(i) $\tilde{\Psi}(l - d) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d))$ *for every* $l, d \in Z$*;*
(ii) $\tilde{\Psi}(ld^{-1}) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d))$ *for every* $l, d(\neq 0) \in Z$*.*

Proof. Suppose that $\tilde{\Psi}$ is an interval valued anti fuzzy field of Z. Then
(i) $\tilde{\Psi}(l - d) = \tilde{\Psi}(l + (-d)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(-d)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d))$ for every $l, d \in Z$,
(ii) $\tilde{\Psi}(ld^{-1}) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d^{-1})) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d))$ for every $l, d(\neq 0) \in Z$.
Conversely, suppose that the two conditions of the hypothesis hold. Now

$$\tilde{\Psi}(-l) = \tilde{\Psi}(0 - l) \leq \max(\tilde{\Psi}(0), \tilde{\Psi}(l)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(l)) = \tilde{\Psi}(l),$$
$$\tilde{\Psi}(l + d) = \tilde{\Psi}(l - (-d)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(-d)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d)),$$
$$\tilde{\Psi}(l^{-1}) = \tilde{\Psi}(1l^{-1}) \leq \max(\tilde{\Psi}(1), \tilde{\Psi}(l)) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(l)) = \tilde{\Psi}(l) (\text{for } l \neq 0)$$

and

$$\tilde{\Psi}(ld) = \tilde{\Psi}(l(d^{-1})^{-1}) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d^{-1}) \leq \max(\tilde{\Psi}(l), \tilde{\Psi}(d)).$$

Thus Ψ is an interval valued anti fuzzy field of Z. □

Theorem 3.3. *Let* Z *and* Z' *be two fields. Let* $\Phi : Z \to Z'$ *be an onto homomorphism. If* $\tilde{\Psi}$ *is an interval valued anti fuzzy field of* Z *and* $\tilde{\Psi'}$ *is an interval valued anti fuzzy field of* Z'*, then* $\Phi^{-1}(\tilde{\Psi'})$ *is an interval valued anti fuzzy field of* Z *and* $\Phi(\tilde{\Psi})$ *is an interval valued anti fuzzy field of* Z'*.*

4. Interval Valued Anti Fuzzy Near-Algebras

Definition 4.1. *Let L be a near algebra over a field Z. An interval valued anti fuzzy subset $\tilde{\vartheta}$ of L is called an Interval Valued Anti Fuzzy Near Algebra of L over an interval valued anti fuzzy field $\tilde{\Psi}$ of Z if it satisfies the following four conditions:*
(i) $\tilde{\vartheta}(a+d) \leq \max(\tilde{\vartheta}(a), \tilde{\vartheta}(d)) = \tilde{\vartheta}(a) \vee \tilde{\vartheta}(d)$,
(ii) $\tilde{\vartheta}(\lambda a) \leq \max(\tilde{\Psi}(\lambda), \tilde{\vartheta}(a)) = \tilde{\Psi}(\lambda) \vee \tilde{\vartheta}(a)$,
(iii) $\tilde{\vartheta}(ad) \leq \max(\tilde{\vartheta}(a), \tilde{\vartheta}(d)) = \tilde{\vartheta}(a) \vee \tilde{\vartheta}(d)$,
(iv) $\tilde{\Psi}(1) \leq \tilde{\vartheta}(a)$ *for all* $a, d \in L$ *and* $\lambda \in Z$, 1 *is the unity in* Z.

5. Examples and Results

Let $Z = Z_2 = \{0,1\}_{+_2, \times_2}$ be a field. And let $\tilde{\Psi} : Z \to D[0,1]$ be an interval valued anti fuzzy subset of Z defined by $\tilde{\Psi}(0) = [0, 0.1]$, $\tilde{\Psi}(1) = [0.1, 0.2]$, $\tilde{\Psi}(a - d) \leq \max(\tilde{\Psi}(a), \tilde{\Psi}(d))$ and $\tilde{\Psi}(ad^{-1}) \leq (\tilde{\Psi}(a), \tilde{\Psi}(d))$(particularly for $d \neq 0$). Then $\tilde{\Psi}$ is an interval valued anti fuzzy field of Z.

Let $L = \{0, p, q, r\}$ be a set with two binary operations "+" and "·" whose composition tables are as follows

$+$	0	p	q	r
0	0	p	q	r
p	p	0	r	q
q	q	r	0	p
r	r	q	p	0

$\cdot$	0	p	q	r
0	0	0	0	0
p	0	q	0	q
q	0	0	0	0
r	0	q	0	q

Scalar multiplication on L is defined by $0 \cdot x = 0, 1 \cdot x = x$ for every $x \in L$, where $0, 1 \in Z$. Clearly L is a near-algebra over the field Z.

Let $\tilde{\vartheta} : L \to D[0,1]$ be an interval valued anti fuzzy subset of L defined by $\tilde{\vartheta}(0) = [0.5, 0.6]$ and $\tilde{\vartheta}(p) = \tilde{\vartheta}(q) = \tilde{\vartheta}(r) = [0.7, 0.8]$. Then $\tilde{\vartheta}$ is an interval valued anti fuzzy near algebra of L over an interval valued anti fuzzy field $\tilde{\Psi}$ of Z.

Theorem 5.1. *If $\tilde{\vartheta}$ is an interval valued anti fuzzy near algebra of L over interval valued anti fuzzy field $\tilde{\Psi}$ of Z, then $\tilde{\Psi}(0) \leq \tilde{\vartheta}(a)$ and $\tilde{\vartheta}(0) \leq \tilde{\vartheta}(a)$ for every $a \in L$.*

Proof. By the definition of an interval valued anti fuzzy field and interval valued anti fuzzy near algebra, we have $\tilde{\Psi}(0) \leq \tilde{\Psi}(1)$, $\tilde{\Psi}(1) \leq \tilde{\vartheta}(a)$. This implies that $\tilde{\Psi}(0) \leq \tilde{\vartheta}(a)$ for every $a \in L$. Now

$$\begin{aligned}
\tilde{\vartheta}(0) &= \tilde{\vartheta}(a-a) \\
&= \tilde{\vartheta}(1a-1a) \\
&\leq \max(\max(\tilde{\Psi}(1),\tilde{\vartheta}(a)),\max(\tilde{\Psi}(-1),\tilde{\vartheta}(a))) \\
&\leq \max(\max(\tilde{\vartheta}(a),\tilde{\vartheta}(a)),\max(\tilde{\Psi}(1),\tilde{\vartheta}(a))) \\
&\leq \max(\tilde{\vartheta}(a),\max(\tilde{\vartheta}(a),\tilde{\vartheta}(a))) \\
&= \tilde{\vartheta}(a).
\end{aligned}$$

□

Theorem 5.2. *$\tilde{\vartheta}$ is an interval valued anti fuzzy near algebra of L over an interval valued anti fuzzy field$\tilde{\Psi}$ of Z iff the following conditions hold:*
$(i)\tilde{\vartheta}(\lambda l+\mu d)\leq \max(\max(\tilde{\Psi}(\lambda),\tilde{\vartheta}(l)),\max(\tilde{\Psi}(\mu),\tilde{\vartheta}(d)))$,
$(ii)\tilde{\vartheta}(ld)\leq \max(\tilde{\vartheta}(l),\tilde{\vartheta}(d))$ and $(iii)\tilde{\Psi}(1)\leq\tilde{\vartheta}(l)$
for each $l,d\in L$; $\lambda,\mu\in Z$.

Proof. Suppose that $\tilde{\vartheta}$ is an interval valued anti fuzzy near algebra of L over interval valued anti fuzzy field $\tilde{\Psi}$ of Z. Then for every $\lambda,\mu\in Z$ and $l,d\in L$, we have $\tilde{\vartheta}(\lambda l+\mu d)\leq\max(\tilde{\vartheta}(\lambda l),\tilde{\vartheta}(\mu d))\leq\max(\max(\tilde{\Psi}(\lambda),\tilde{\vartheta}(l)),\max(\tilde{\Psi}(\mu),\tilde{\vartheta}(d)))$. Since $\tilde{\vartheta}$ is an interval valued anti fuzzy near algebra of L, then the remaining two conditions hold directly. Conversely, suppose that the three conditions of the hypothesis hold. Then

$$\begin{aligned}
\tilde{\vartheta}(l+d) &= \tilde{\vartheta}(1l+1d) \\
&\leq \max(\max(\tilde{\Psi}(1),\tilde{\vartheta}(l)),\max(\tilde{\Psi}(1),\tilde{\vartheta}(d))) \\
&\leq \max(\max(\tilde{\vartheta}(l),\tilde{\vartheta}(l)),\max(\tilde{\vartheta}(d),\tilde{\vartheta}(d))) \\
&\leq \max(\tilde{\vartheta}(l),\tilde{\vartheta}(d)), \\
\tilde{\vartheta}(\lambda l) &= \tilde{\vartheta}(\lambda l+0l) \\
&\leq \max(\max(\tilde{\Psi}(\lambda),\tilde{\vartheta}(l)),\max(\tilde{\Psi}(0),\tilde{\vartheta}(l))) \\
&\leq \max(\max(\tilde{\Psi}(\lambda),\tilde{\vartheta}(l)),\max(\tilde{\vartheta}(l),\tilde{\vartheta}(l))) \\
&\leq \max(\max(\tilde{\Psi}(\lambda),\tilde{\vartheta}(l)),\tilde{\vartheta}(l)) \\
&= \max(\tilde{\Psi}(\lambda),\tilde{\vartheta}(l)).
\end{aligned}$$

Also we have $\tilde{\vartheta}(ld)\leq\max(\tilde{\vartheta}(l),\tilde{\vartheta}(d))$ and $\tilde{\Psi}(1)\leq\tilde{\vartheta}(l)$.

Hence $\tilde{\vartheta}$ is an interval valued anti fuzzy near algebra of L over interval valued anti fuzzy field $\tilde{\Psi}$ of Z. □

Theorem 5.3. *Intersection of a family of interval valued anti fuzzy near algebras is an interval valued anti fuzzy near algebra.*

Definition 5.1. *For any family of interval valued anti fuzzy sets* $\{\tilde{\vartheta}_i : i \in \Lambda\}$ *of a Near Algebra* L*, the union of* $\{\tilde{\vartheta}_i : i \in \Lambda\}$ *is defined by* $(\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(a) = \sup\{\tilde{\vartheta}_i(a) : i \in \Lambda\}$.

Theorem 5.4. *If* $\{\tilde{\vartheta}_i : i \in \Lambda\}$ *is a collection of interval valued anti fuzzy near algebras of a near algebra* L*, then so is* $\bigvee_{i\in\Lambda} \vartheta_i$.

Proof. Let $\{\tilde{\vartheta}_i : i \in \Lambda\}$ be a family of interval valued anti fuzzy near algebras of L over interval valued anti fuzzy field $\tilde{\Psi}$ of Z. Let $a, d \in L$ and $\lambda \in Z$. Then

$$
\begin{aligned}
(\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(a+d) &= \sup\{\tilde{\vartheta}_i(a+d) : i \in \Lambda\} \\
&\leq \sup\{\max(\tilde{\vartheta}_i(a), \tilde{\vartheta}_i(d)) : i \in \Lambda\} \\
&= \max\{\sup(\tilde{\vartheta}_i(a) : i \in \Lambda), \sup(\tilde{\vartheta}_i(d) : i \in \Lambda)\} \\
&= \max\{(\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(a), (\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(d)\}, \\
(\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(\lambda a) &= \sup\{\tilde{\vartheta}_i(\lambda a) : i \in \Lambda\} \\
&\leq \sup\{\max(\tilde{\Psi}(\lambda), \tilde{\vartheta}_i(a)) : i \in \Lambda\} \\
&= \max\{sup(\tilde{\Psi}(\lambda), \tilde{\vartheta}_i(a)) : i \in \Lambda\} \\
&= \max\{\tilde{\Psi}(\lambda), \sup(\tilde{\vartheta}_i(a) : i \in \Lambda)\} \\
&= \max\{\tilde{\Psi}(\lambda), (\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(a)\}, \\
(\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(ad) &= \sup\{\tilde{\vartheta}_i(ad) : i \in \Lambda\} \\
&\leq \sup\{\max(\tilde{\vartheta}_i(a), \tilde{\vartheta}_i(d)) : i \in \Lambda\} \\
&= \max\{\sup(\tilde{\vartheta}_i(a) : i \in \Lambda), \sup(\tilde{\vartheta}_i(d) : i \in \Lambda)\} \\
&= \max\{(\bigvee_{i\in\Lambda} \tilde{B}_i)(a), (\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(d)\}.
\end{aligned}
$$

Since each $\tilde{\vartheta}_i$ is an interval valued anti fuzzy near algebra of L, then for each i we have $\tilde{\Psi}(1) \leq \tilde{\vartheta}_i(a)$ for every $a \in L$. This implies that $\tilde{\Psi}(1) \leq (\bigvee_{i\in\Lambda} \tilde{\vartheta}_i)(a)$.

Thus $\bigvee_{i\in\Lambda} \tilde{\vartheta}_i$ is an interval valued anti fuzzy near algebra of L over the interval valued anti fuzzy field $\tilde{\Psi}$ of Z. □

ACKNOWLEDGMENT

The authors very much grateful to Prof. T. Srinivas for his valuable comments and suggestions for improving this paper.

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