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ON EDGE BIMAGIC LABELING OF BISTAR GRAPHS

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ABSTRACT. Bimagic labeling is introduced and analysed by J Basar Babujee and there after several types of graphs and some graphs derived from the standard graphs have been identified to be having bimagic labeling. In this paper, edge bimagic labeling of bistar graphs is obtained. The number of ways this labeling can be given to bistar graphs is also obtained.

1. INTRODUCTION

In this introduction, a brief history of magic and bimagic labelings is given. A brief literature about the bimagic labelings is also presented.

Labeling of graphs using integers started around the mid 60s of 20th Century. Rosa [7] is one among the few who introduced the idea of labeling of the elements of graphs with numbers. The magic labeling introduced by Rosa [4] was later termed edge magic labeling according to Ringel [6]. The concept edge magic total labelings was given by W. D. Wallis along with others [9]. The main idea of bimagic labeling discussed in this paper is first given by J. Baskar Babujee when he formally launched $(1, 1)$ vertex bimagic labeling [1]. $(1, 1)$ Edge bimagic labeling is first introduced by J Baskar Babujee & V.Vishnu Priya [2]. Further, it is observed that Vishnu Priya, & Baskar Babujee [8] obtained edge bimagic labeling for some trees. Murugesan and Amutha [5] investigated that bistar graphs have vertex bimagic total labeling. In this paper, apart from showing that the bistar graphs $B_{n,n}$ have $(1, 1)$ edge bimagic labeling or edge bimagic

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total labeling, the number of ways in which the labeling can be given is also obtained for the bistar graphs $B_{n,n}$. The two constants involved in this labeling are called bimagic constants. A survey of graph labeling given by Gallian [3] gives a detailed idea about the different type of edge magic labelings as well as edge bimagic labelings for different types of graphs. In general, labeling of graphs have useful applications in communication network, circuit design,etc, [2,8]. In particular, the applications of edge bimagic labeling of bistar graphs need to be explored.

2. PRELIMINARIES

Definition 2.1. *A Bistar graph is the graph which is obtained by drawing an edge between the apex vertices of two star graphs $K_{1,n}$ [2, 9].*

Definition 2.2. *An edge bimagic labeling, or $(1,1)$ edge bimagic labeling or edge bimagic total labeling f is defined to be a bijective function from $V \bigcup E$ of of graph G onto $\{1,2,...,s+q\}$ so that for every edge e of E given by $e=(u,v)\in E,\ f(u)+f(e)+f(v)=b1$ or $b2$, where V denotes vertex set and E denotes edge set of the graph G [2]. Further $s=|V|$ and $q=|E|$ represent the cardinality of V and E respectively.*

3. MAIN RESULTS

In this section, the orientation of elements (vertices & edges) of the bistar graph $B_{n,n}$ is presented for a clear understanding of the assigned labeling. Here n denotes the number of pendant vertices of $K_{1,n}$ which is a part of $B_{n,n}$. Orientation is followed by two results, the second of which is more fascinating.

3.1. **Orientation of vertices of the bistar graph $B_{n,n}$:** The apex vertex on one side of the bistar graph $B_{n,n}$ is considered to be the vertex v_1. The remaining vertices of the corresponding $K_{1,n}$ incident with this vertex v_1 are considered in the anticlockwise direction as $v_2, v_3, \dots, v_{n+1}$. The apex vertex on the other side of the bistar graph is considered as v_{2n+2}, the remaining vertices of the corresponding $K_{1,n}$ incident with this vertex v_{2n+2} are considered to be $v_{n+2}, v_{n+3}, v_{2n+1}$. The edges of $B_{n,n}$ are given by $e_p = v_1 v_{p+1},\ 1 \le p \le n$ and $e_p = v_{2n+2} v_{p+1},\ n+1 \le p \le 2n$. The edge connecting v_1 and v_{2n+2} is denoted by e_{2n+1}.

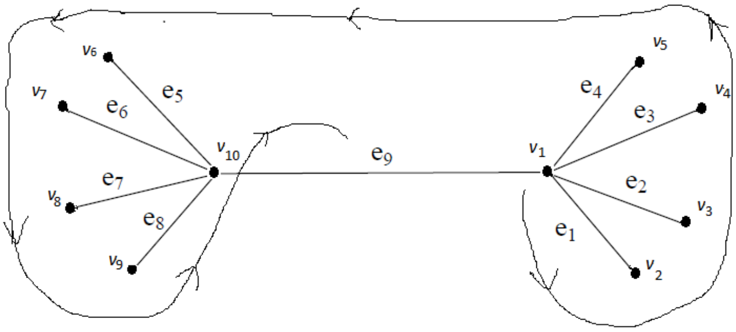

FIGURE 1. Orientation of vertices for the bistar graph $B_{4,4}$

Theorem 3.1. *Bistar graphs $B_{n,n}$ have a labeling which is edge bimagic total.*

Proof. Let the graph G given by $V \bigcup E$ be a bistar graph $B_{n,n}$, $n > 2$. Let the bijective function g be defined by $g\ : V \bigcup E \to \{1, 2, ..., 4n + 3\}$ as

$$g(v_p) = \begin{cases} 2n+1, & p = 1 \\ n-1+p, & 2 \leq p \leq n+1 \\ 2n+p, & n+2 \leq p \leq 2n+1 \\ 4n+3, & p = 2n+2 \end{cases}$$

and edges are given by

$$g(e_p) = \begin{cases} n+1-p, & 1 \leq p \leq n \\ 4n+2-p, & n+1 \leq p \leq 2n \\ 4n+2, & p = 2n+1 \end{cases}$$

Now, for any edge e_r we have

$$g(v_1) + g(e_r) + g(v_{r+1}) = (2n+1) + (n+1-r) + (n-1+(r+1)) = 4n+2,$$

when $1 \leq r \leq n$,

$$g(v_{2n+2}) + g(e_r) + g(v_{r+1}) = (4n+3) + (4n+2-r) + (2n+(r+1)) = 10n+6,$$

$n + 1 \leq r \leq 2n$ and

$$g(v_{2n+2}) + g(e_{2n+1}) + g(v_1) = (4n+3) + (4n+2) + (2n+1) = 10n+6.$$

It is observed that for any edge e_j, when the labels of that edge and the labels of vertices incident on that edge are added, two different sums are obtained. In view of this, we conclude that the bistar graphs have edge bimagic total labeling. □

One interesting thing about edge bimagic total labeling is that there is always scope to find more than one such labeling. In view of this, it is observed that there are fifteen more edge bimagic total labelings available for these bistar graphs.

Theorem 3.2. *There exists at least 16 edge bimagic total labelings for the bistar graphs $B_{n,n}$.*

Proof. One such labeling is already given in Theorem 3.1. So here fifteen more such labelings are presented with the same bimagic constants. These additional fifteen edge bimagic labelings are described by the bijective functions $f_1,\ f_2, \ldots, f_{15}$. For all these fifteen functions that are going to be defined, $f_m(v_1) = 2n+1$, $f_m(v_{2n+2}) = 4n+3$, and $f_m(e_{2n+1}) = 4n+2$, and corresponding edge sum is $f_m(v_1)+f_m(e_{2n+1})+f_m(v_{2n+2}) = (4n+3)+(4n+2)+(2n+1) = 10n+6$ for all $m = 1$ to 15. So we define only $f(v_p)$ for $2 \leq p \leq 2n+1$ and $f(e_p)$ for $1 \leq p \leq 2n$ for all these fifteen bijective functions in this proof.

(i) The bijective function f_1 is defined by $f_1 : V \bigcup E \rightarrow \{1, 2, ..., 4n+3\}$ as

$$f_1(v_p) = \begin{cases} 2n+2-p, & 2 \leq p \leq n+1 \\ 2n+p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_1(e_p) = \begin{cases} p, & 1 \leq p \leq n \\ 4n+2-p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_1(v_1) + f_1(e_r) + f_1(v_{r+1}) = (2n+1) + (r) + (2n+2-(r+1)) = 4n+2,$$

$1 \leq r \leq n$ and

$$f_1(v_{2n+2}) + f_1(e_r) + f_1(v_{r+1}) = (4n+3) + (4n+2-r) + (2n+(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n.$ It is observed that for any edge e_r, when the labels of that edge and the labels of vertices incident on that edge are added, two different sums are obtained. In view of this, it can be concluded that the bistar graphs have one more edge bimagic total labeling. A similar reasoning can be given at the end of evaluation of edge sums for all the bijective functions from f_2 to f_{15} and so such a reasoning is not repeated.

(ii) The bijective function f_2 is defined by $f_2 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_2(v_p) = \begin{cases} n-1+p, & 2 \le p \le n+1 \\ 5n+3-p, & n+2 \le p \le 2n+1 \end{cases},$$

and edges by

$$f_2(e_p) = \begin{cases} n+1-p, & 1 \le p \le n \\ n+1+p, & n+1 \le p \le 2n \end{cases}.$$

Now, for any edge e_r we have,

$f_2(v_1) + f_2(e_r) + f_2(v_{r+1}) = (2n+1) + (n+1-r) + (n-1+(r+1)) = 4n+2,$

$1 \le r \le n$, and

$f_2(v_{2n+2}) + f_2(e_r) + f_2(v_{r+1}) = (4n+3) + (n+1+r) + (5n+3-(r+1)) = 10n+6,$

$n+1 \le r \le 2n.$

(iii) The bijective function f_3 is defined by $f_3 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_3(v_p) = \begin{cases} 2n+2-p, & 2 \le p \le n+1 \\ 5n+3-p, & n+2 \le p \le 2n+1 \end{cases},$$

and edges by

$$f_3(e_p) = \begin{cases} p, & 1 \le p \le n \\ n+1+p, & n+1 \le p \le 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_3(v_1) + f_3(e_r) + f_3(v_{r+1}) = (2n+1) + (r) + (2n+2-(r+1)) = 4n+2,$$

$1 \le r \le n$, and

$f_3(v_{2n+2}) + f_3(e_r) + f_3(v_{r+1}) = (4n+3) + (n+1+r) + (5n+3-(r+1)) = 10n+6,$

$n+1 \le r \le 2n.$

(iv) The bijective function f_4 is defined by $f_4 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_4(v_p) = \begin{cases} p-1, & 2 \le p \le n+1 \\ 5n+3-p, & n+2 \le p \le 2n+1 \end{cases},$$

and edges by

$$f_4(e_p) = \begin{cases} 2n+1-p, & 1 \le p \le n \\ n+1+p, & n+1 \le p \le 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_4(v_1) + f_4(e_r) + f_4(v_{r+1}) = (2n+1) + (2n+1-r) + ((r+1)-1) = 4n+2,$$

$1 \leq r \leq n$ and

$$f_4(v_{2n+2}) + f_4(e_r) + f_4(v_{r+1}) = (4n+3) + (n+1+r) + (5n+3-(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n.$

(v) The bijective function f_5 is defined by $f_5 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_5(v_p) = \begin{cases} p-1, & 2 \leq p \leq n+1 \\ 2n+p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_5(e_p) = \begin{cases} 2n+1-p, & 1 \leq p \leq n \\ 4n+2-p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_5(v_1) + f_5(e_r) + f_5(v_{r+1}) = (2n+1) + (2n+1-r) + ((r+1)-1) = 4n+2,$$

$1 \leq r \leq n$, and

$$f_5(v_{2n+2}) + f_5(e_r) + f_5(v_{r+1}) = (4n+3) + (4n+2-r) + (2n+(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n.$

(vi) The bijective function f_6 is defined by $f_6 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_6(v_p) = \begin{cases} n+2-p, & 2 \leq p \leq n+1 \\ 5n+3-p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_6(e_p) = \begin{cases} n+p, & 1 \leq p \leq n \\ n+1+p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e$_r$ we have,

$$f_6(v_1) + f_6(e_r) + f_6(v_{r+1}) = (2n+1) + (n+r) + (n+2-(r+1)) = 4n+2,$$

$1 \leq r \leq n$ and

$$f_6(v_{2n+2}) + f_6(e_r) + f_6(v_{r+1}) = (4n+3) + (n+1+r) + (5n+3-(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n.$

(vii) The bijective function f_7 is defined by $f_7 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_7(v_p) = \begin{cases} n+2-p, & 2 \leq p \leq n+1 \\ 2n+p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_7(e_p) = \begin{cases} n+p, & 1 \leq p \leq n \\ 4n+2-p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_7(v_1) + f_7(e_r) + f_7(v_{r+1}) = (2n+1) + (n+r) + (n+2-(r+1)) = 4n+2,$$

$1 \leq r \leq n$, and

$$f_7(v_{2n+2}) + f_7(e_r) + f_7(v_{r+1}) = (4n+3) + (4n+2-r) + (2n+(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n$.

(viii) The bijective function f_8 is defined by $f_8 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_8(v_p) = \begin{cases} n+2-p, & 2 \leq p \leq n+1 \\ n+p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_8(e_p) = \begin{cases} n+p, & 1 \leq p \leq n \\ 5n+2-p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_8(v_1) + f_8(e_r) + f_8(v_{r+1}) = (2n+1) + (n+r) + (n+2-(r+1)) = 4n+2,$$

$1 \leq r \leq n$, and

$$f_8(v_{2n+2}) + f_8(e_r) + f_8(v_{r+1}) = (4n+3) + (5n+2-r) + (n+(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n$.

(ix) The bijective function f_9 is defined by $f_9 : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_9(v_p) = \begin{cases} n+2-p, & 2 \leq p \leq n+1 \\ 4n+3-p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_9(e_p) = \begin{cases} n+p, & 1 \leq p \leq n \\ 2n+1+p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_9(v_1) + f_9(e_r) + f_9(v_{r+1}) = (2n+1) + (n+r) + (n+2-(r+1)) = 4n+2,$$

$1 \leq r \leq n$, and

$$f_9(v_{2n+2}) + f_9(e_r) + f_9(v_{r+1}) = (4n+3) + (2n+1+r) + (4n+3-(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n$.

(x) The bijective function f_{10} is defined by $f_{10} : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_{10}(v_p) = \begin{cases} p-1, & 2 \leq p \leq n+1 \\ n+p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_{10}(e_p) = \begin{cases} 2n+1-p, & 1 \leq p \leq n \\ 5n+2-p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_{10}(v_1) + f_{10}(e_r) + f_{10}(v_{r+1}) = (2n+1) + (2n+1-r) + ((r+1)-1) = 4n+2,$$

$1 \leq r \leq n$, and

$$f_{10}(v_{2n+2}) + f_{10}(e_r) + f_{10}(v_{r+1}) = (4n+3) + (5n+2-r) + (n+(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n$.

(xi) The bijective function f_{11} is defined by $f_{11} : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_{11}(v_p) = \begin{cases} p-1, & 2 \leq p \leq n+1 \\ 4n+3-p, & n+2 \leq p \leq 2n+1 \end{cases},$$

and edges by

$$f_{11}(e_p) = \begin{cases} 2n+1-p, & 1 \leq p \leq n \\ 2n+1+p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_{11}(v_1) + f_{11}(e_r) + f_{11}(v_{r+1}) = (2n+1) + (2n+1-r) + ((r+1)-1) = 4n+2,$$

$1 \leq r \leq n$, and

$$f_{11}(v_{2n+2}) + f_{11}(e_r) + f_{11}(v_{r+1}) = (4n+3) + (2n+1+r) + (4n+3-(r+1)) = 10n+6,$$

$n+1 \leq r \leq 2n$.

(xii) The bijective function f_{12} is defined by $f_{12} : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_{12}(v_p) = \begin{cases} 2n+2-p, & 2 \le p \le n+1 \\ n+p, & n+2 \le p \le 2n+1 \end{cases},$$

and edges by

$$f_{12}(e_p) = \begin{cases} p, & 1 \le p \le n \\ 5n+2-p, & n+1 \le p \le 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_{12}(v_1) + f_{12}(e_r) + f_{12}(v_{r+1}) = (2n+1) + (r) + (2n+2-(r+1)) = 4n+2,$$

$1 \le r \le n$ and

$$f_{12}(v_{2n+2}) + f_{12}(e_r) + f_{12}(v_{r+1}) = (4n+3) + (5n+2-r) + (n+(r+1)) = 10n+6,$$

$n+1 \le r \le 2n$.

(xiii) The bijective function f_{13} is defined by $f_{13} : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_{13}(v_p) = \begin{cases} 2n+2-p, & 2 \le p \le n+1 \\ 4n+3-p, & n+2 \le p \le 2n+1 \end{cases},$$

and edges by

$$f_{13}(e_p) = \begin{cases} p, & 1 \le p \le n \\ 2n+1+p, & n+1 \le p \le 2n \end{cases}.$$

Now, for any edge e_r we have,

$$f_{13}(v_1) + f_{13}(e_r) + f_{13}(v_{r+1}) = (2n+1) + (r) + (2n+2-(r+1)) = 4n+2,$$

$1 \le r \le n$ and

$$f_{13}(v_{2n+2}) + f_{13}(e_r) + f_{13}(v_{r+1}) = (4n+3) + (2n+1+r) + (4n+3-(r+1)) = 10n+6,$$

$n+1 \le r \le 2n$.

(xiv)The bijective function f_{14} is defined by $f_{14} : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_{14}(v_p) = \begin{cases} n-1+p, & 2 \le p \le n+1 \\ 4n+3-p, & n+2 \le p \le 2n+1 \end{cases}.$$

and edges by

$$f_{14}(e_p) = \begin{cases} n+1-p, & 1 \le p \le n \\ 2n+1+p, & n+1 \le p \le 2n \end{cases}.$$

Now, for any edge e_r we have,

$f_{14}(v_1) + f_{14}(e_r) + f_{14}(v_{r+1}) = (2n+1) + (n+1-r) + (n-1+(r+1)) = 4n+2$,

$1 \leq r \leq n$ and

$f_{14}(v_{2n+2}) + f_{14}(e_r) + f_{14}(v_{r+1}) = (4n+3) + (2n+1+r) + (4n+3-(r+1)) = 10n+6$,

$n+1 \leq r \leq 2n$.

(xv) The bijective function f_{15} is defined by $f_{15} : V \bigcup E \to \{1, 2, \ldots, 4n+3\}$ as

$$f_{15}(v_p) = \begin{cases} n-1+p, & 2 \leq p \leq n+1 \\ n+p, & n+2 \leq p \leq 2n+1 \end{cases}.$$

and edges by

$$f_{15}(e_p) = \begin{cases} n+1-p, & 1 \leq p \leq n \\ 5n+2-p, & n+1 \leq p \leq 2n \end{cases}.$$

Now, for any edge e_r we have,

$f_{15}(v_1) + f_{15}(e_r) + f_{15}(v_{r+1}) = (2n+1) + (n+1-r) + (n-1+(r+1)) = 4n+2$,

$1 \leq r \leq n$ and

$f_{15}(v_{2n+2}) + f_{15}(e_r) + f_{15}(v_{r+1}) = (4n+3) + (5n+2-r) + (n+(r+1)) = 10n+6$,

$n+1 \leq r \leq 2n$.

This proves that there exists at least sixteen edge bimagic total labelings for bistar graphs $B_{n,n}$. □

4. Conclusions

The sixteen bijective functions $f, f_1, f_2, \ldots, f_{15}$ described in Theorem 3.2 and Theorem 3.3 infer that there exists 16 edge bimagic total labelings for the bistar graphs $B_{n,n}$. I strongly feel that there is a possibility of finding some more edge bimagic total labelings for the bistar graphs $B_{n,n}$. In view of this "at least 16" is included in the statement of Theorem 3.3. It also unfolds an open question of number of edge bimagic labelings that can be given to any graph if it already has an edge bimagic total labeling.

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