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SOME INEQUALITIES OF INTUITIONISTIC FUZZY MATRICES

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ABSTRACT. In this paper, some inequalities of an intuitionistic fuzzy matrix (IFM) are discussed under the Cartesian product representation of an intuitionistic fuzzy matrix, in terms of its membership and non-membership parts.

1. INTRODUCTION

In [5], Kim and Roush have developed the theory of fuzzy matrices, under max min composition analogous to that of Boolean matrices. Regular fuzzy matrices play an important role in estimation and inverse problem in fuzzy relational equations [12] and in fuzzy optimization problem [13]. For more details on Fuzzy matrix and its applications discussed in [6]. The min max composition of fuzzy matrices have been studied by Ragab and Emam [11]. Atanassov has introduced and developed the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets [1,2].The intuitionistic fuzzy matrices as a generalization of the results on fuzzy matrices have been discussed in [9]. Basic properties of intuitionistic fuzzy matrices under Cartesian product representation has discussed in [3]. The generalized inverse and regularity of intuitionistic fuzzy matrices was discussed in [4,7]. In [8], Muthuraji and Lalitha have discussed some new operations of intuitionistic fuzzy matrices with respect to algebraic sum and algebraic product. In [10], Sriram and Boobalan have developed several properties of intuitionistic fuzzy matrices.In this paper, an intuitionistic fuzzy matrix as

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the Cartesian product of its membership matrix and its non-membership matrix are represented. By using this representation , some inequalities of an intuitionistic fuzzy matrix are discussed.

2. Preliminaries

In this section, some definitions of intuitionistic fuzzy matrices that are needed in the sequel are presented.

Let $(IF)_{mxn}$ be the set of all intuitionistic fuzzy matrices of order mxn, F^M_{mxn} be the set of all fuzzy matrices of order mxn, under the maxmin composition and F^N_{mxn} be the set of all fuzzy matrices of order mxn, under the minmax composition.

Definition 2.1. *[7] For $A = (a_{ij}) \in (IF)_{mxn}$. Let $A = a_{ij} = (< a_{ij\mu}, a_{ij\nu} >) \in (IF)_{mxn}$. We define $A_\mu = (a_{ij\mu}) \in F^M_{mxn}$ as the membership part of A and $A_\nu = (< a_{ij\nu}) \in F^N_{mxn}$ as the non membership part of A. Thus A is written as the Cartesian product of A_μ and A_ν , $A =< A_\mu, A_\nu >$ with $A_\mu \in F^M_{mxn}$, $A_\nu \in F^N_{mxn}$.*

Definition 2.2. *For $A, B \in (IF)_{mxn}$, if $A =< A_\mu, A_\nu >$ and $B =< B_\mu, B_\nu >$ then $A + B =< A_\mu + B_\mu, A_\nu + B_\nu >$.*

Definition 2.3. *For $A \in (IF)_{mxp}, B \in (IF)_{pxn}$, if $A =< A_\mu, A_\nu >$ and $B =< B_\mu, B_\nu >$ then $AB =< A_\mu B_\mu, A_\nu B_\nu >$,where $A_\mu B_\mu$ is the maxmin composition in F^M_{mxn} and $A_\nu B_\nu$ is the minmax composition in F^N_{mxn}.*

Definition 2.4. *Let $A =< A_\mu, A_\nu >$ and $B =< B_\mu, B_\nu >$ are two IFMs of order mxn. Then $A \leq B \iff A_\mu \leq B_\mu$ and $A_\nu \geq B_\nu$.*

Definition 2.5. *Let $A =< A_\mu, A_\nu >$ and $B =< B_\mu, B_\nu >$ are said to be comparable if either $A \leq B \iff A_\mu \leq B_\mu$ and $A_\nu \geq B_\nu$ (or) $B \leq A \iff B_\mu \leq A_\mu$ and $B_\nu \geq A_\nu$.*

3. Basic Properties of IFMs

In this section, first represent $A \in (IF)_{mxn}$ as Cartesian product of fuzzy matrices then derive some basic properties of IFMs by using Cartesian product representation.

Theorem 3.1. *Let A, B and $C \in (IF)_{mxn}$ .If $A \leq C$ and $B \leq C$, then $A + B \leq C$.*

Proof. Let $A =< A_\mu, A_\nu >$, $B =< B_\mu, B_\nu >$and $C =< C_\mu, C_\nu >$.Since $A \leq C$, by Definition 2.4, it follows that $A_\mu \leq C_\mu$ and $A_\nu \geq C_\nu$ and $B \leq C$, by Definition 2.4, it follows that $B_\mu \leq C_\mu$ and $B_\nu \geq C_\nu$ .If $A_\mu \leq C_\mu \iff a_{ij\mu} \leq c_{ij\mu}$ for all i and j and $B_\mu \leq C_\mu \iff b_{ij\mu} \leq c_{ij\mu}$ for all i and j $\iff max(a_{ij\mu}, b_{ij\mu}) \leq c_{ij\mu}$. Therefore

$$A_\mu + B_\mu \leq C_\mu. \tag{3.1}$$

If $A_\nu \geq C_\nu \iff a_{ij\nu} \geq c_{ij\nu}$ for all i and j and $B_\nu \geq C_\nu \iff b_{ij\nu} \geq c_{ij\nu}$ for all i and j $\iff min(a_{ij\nu}, b_{ij\nu}) \geq c_{ij\nu}$. Therefore

$$A_\nu + B_\nu \geq C_\nu. \tag{3.2}$$

From (3.1) and (3.2), $A + B \leq$ C. Hence the proof. □

Theorem 3.2. *Let* A, B *and* $C \in (IF)_{mxn}$.*If* $A \leq B$*, then* $A + C \leq B + C$.

Proof. Let $A =< A_\mu, A_\nu >$, $B =< B_\mu, B_\nu >$ and $C =< C_\mu, C_\nu >$. If $A \leq B$ then $A_\mu \leq B_\mu$ and $A_\nu \geq B_\nu \implies A_\mu + C_\mu \leq B_\mu + C_\mu$ and $A_\nu + C_\nu \geq B_\nu + C_\nu$ $\implies A + C \leq B + C$. Hence the proof. □

Theorem 3.3. *Let* A, B *and* $C \in (IF)_{mxn}$. *If* $C \leq A$ *and* $C \leq B$, *then* $C \leq A \wedge B$.

Proof. Let $A =< A_\mu, A_\nu >$, $B =< B_\mu, B_\nu >$and $C =< C_\mu, C_\nu >$. If $C \leq A$ then $C_\mu \leq A_\mu$ and $C_\nu \geq A_\nu$ and $C \leq B$ then $C_\mu \leq B_\mu$ and $C_\nu \geq B_\nu \implies C_\mu \leq min < A_\mu, B_\mu >$ and $C_\nu \geq max < A_\nu, B_\nu > \implies < C_\mu, C_\nu > \leq (min < A_\mu, B_\mu >, max < A_\nu, B_\nu >) \implies C \leq A \wedge B$. Hence the proof. □

Theorem 3.4. *Let* A, B *and* $C \in (IF)_{mxn}$. *If* $A \leq B$ *and* $A \leq C$ *and* $B \wedge C = 0$, *then* $A = 0$.

Proof. Let $A =< A_\mu, A_\nu >$, $B =< B_\mu, B_\nu >$and $C =< C_\mu, C_\nu >$. By Definition 2.4, if $A \leq B$ then $A_\mu \leq B_\mu$ and $A_\nu \geq B_\nu$ and $A \leq C$ then $A_\mu \leq C_\mu$ and $A_\nu \geq C_\nu \implies B \wedge C = (min < B_\mu, C_\mu >, max < B_\nu, C_\nu >) =< 0, 1 > \implies min < B_\mu, C_\mu >= 0, max < B_\nu, C_\nu >= 1 \implies A_\mu = 0, A_\nu = 1 \implies < A_\mu, A_\nu >=< 0, 1 > \implies A = 0$. Hence the proof. □

Proposition 3.1. *Let* A, B *and* $C \in (IF)_{mxn}$. *If* $A \leq Band$ $B \wedge C = 0$, *then* $A \wedge C = 0$.

Proof. This can be proved along the same lines as that of Theorem 3.4. □

Theorem 3.5. *Let* $A, B \in (IF)_{mxn}$.*If* $A \leq B \iff B^C \leq A^C$.

Proof. Let $A^C =< A_\nu, A_\mu >$ and $B^C =< B_\nu, B_\mu >$. By Definition 2.4, if $A \leq B$ then $A_\mu \leq B_\mu$ and $A_\nu \geq B_\nu \implies B_\nu \leq A_\nu$ and $B_\mu \geq A_\mu \implies < B_\nu, B_\mu > \leq < A_\nu, A_\mu > \implies B^C \leq A^C$. $\square$

4. Conclusion

The aim of this paper is derive some inequalities of an intuitionistic fuzzy matrix under the Cartesian product representation of intuitionistic fuzzy matrix.

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