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ON A NEW CLASS OF SEMI GENERALIZED CLOSED SETS IN STRONG GENERALIZED TOPOLOGICAL SPACES

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ABSTRACT. This paper deals with the concepts of semi generalized closed sets in strong generalized topological spaces such as $sg_\mu^{\star\star}$-closed set, $sg_\mu^{\star\star}$-open set, $g_\mu^{\star\star}$-closed set, $g_\mu^{\star\star}$-open set and studied some of its basic properties included with $sg_\mu^{\star\star}$-continuous maps, $sg_\mu^{\star\star}$-irresolute maps and $T_{\frac{1}{2}}$-space in strong generalized topological spaces.

1. INTRODUCTION

In the year 1963 N. Levin introduced semi open sets and semi continuity in topological spaces tried to generalize topology by replacing open sets with semi open sets [3]. After him, similar works have been done by many topologists. In 1997, A.Csaszar generalized these new open sets by introducing the concepts of γ-open sets [4]. The concept of generalized topology was devised by him in 2002 [1]. Min has introduced and studied various types of continuous functions and almost continuous functions in generalized topological space [5]. In 2005, A.Csaszar introduced generalized open sets in generalized topologies [3].

Further he developed and studied the notions of $g\alpha$-open sets, gs-open sets, gp-open and $g\beta$-open sets in generalized topological spaces. The concepts of a topological space are often generalized by replacing open sets with other kind

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of subsets. A set X with a generalized topology μ on it, is called a generalized topological space and is denoted by (X, μ) or simply X for short. A generalized topology is named strong if $X \in \mu$. The usual concepts defined in topological spaces can be used again in generalized topological spaces. Moreover we introduce the notions of semi generalized closed $(sg_\mu^{\star\star}C(X))$ sets.

Further defined the notion of $sg_\mu^{\star\star}$-open sets and investigated some of its properties. Finally introduce the concept of continuous, irresolute and $T_{\frac{1}{2}}$ space by using these notions.

2. PRELIMINARIES

Throughout this paper we introduced represent the strong generalized topological spaces (X, μ), (Y, σ) and (Z, η) as X, Y and Z respectively.

Definition 2.1. *[2] A subset of a strong generalized topological spaces is called a semi-open set $SO_\mu(X)$: If $A \subseteq cl_\mu(int_\mu(A))$, and an semi-closed set $SC_\mu(X)$ if $int_\mu(cl_\mu(A)) \subseteq A$ in (X, μ).*

Definition 2.2. *[7] A subset of a strong generalized topological spaces is called a semi-μ-open if $A \subseteq cl_\mu(int_\mu(A))$ and semi-μ-closed if $int_\mu(cl_\mu(A)) \subseteq A$.*

Definition 2.3. *A subset A of a spaces is called a*

(i) *g_μ-closed set [4] if $cl_\mu(A) \subseteq U$ whenever $A \subseteq U$ & U is open in (X, μ).*
(ii) *sg_μ-closed set [4] if $scl_\mu(A) \subseteq U$ whenever $A \subseteq U$ & U is semi open in (X, μ).*
(iii) *$g_\mu s$-closed set [2] if $scl_\mu(A) \subseteq U$ whenever $A \subseteq U$ & U is open in (X, μ).*
(iv) *$T_{\frac{1}{2}}$ space [8] if every g_μ-closed set is closed.*
(v) *semi regular [6] if it is both semi open and semi closed.*

Definition 2.4. *[7] A subset A of a Strong Generalized Topological spaces (X, μ) is called*

(i) *$g_\mu^\star$-closed set if $cl_\mu(A) \subseteq U$ whenever $A \subseteq U$ & U i s g_μ-open in (X, μ).*
(ii) *$g_\mu^{\star\star}$-closed set if $cl_\mu(A) \subseteq U$ whenever $A \subseteq U$ & U is $g_\mu^\star$-open in (X, μ).*

Definition 2.5. *[1] A map $f : (X, \mu) \to (Y, \sigma)$ is called a sg_μ-continuous if $f^{-1}(V)$ is a sg_μ- closed set of (X, μ) and closed set V of (Y, σ).*

Definition 2.6. *A subset A of an ideal nano space $(U, \mathcal{N}, I)$ is called a nano $I_{\pi g}$-closed (written in short as $I_{n\pi g}$-closed) if $A \subseteq H$, $H \in n\pi$-open $\Longrightarrow A_n^\star \subseteq H$.*

3. SEMI $g_\mu^{\star\star}$-CLOSED SET

In this section we introduce a new class of closed set $sg_\mu^{\star\star}$-closed set.

Definition 3.1. *Let A be subset of a strong generalized topological spaces (X, μ) is called a semi $g_\mu^{\star\star}$-closed set ($sg_\mu^{\star\star}$-closed set) if $scl_\mu(A) \subseteq U$ whenever $A \subseteq U$ & U is $g_\mu^{\star\star}$-open.*

The set of all $sg_\mu^{\star\star}$-closed sets denoted by $sg_\mu^{\star\star}C(X)$.

Remark 3.1. *The concepts of g_μ-closed set and $sg_\mu^{\star\star}$-closed set are independent.*

Example 1. *Let $X = \{a, b, c, d\}$, $\mu = \{\phi, X, \{a\}, \{a, b\}\}$. Then g_μ-closed sets are $\phi, X, \{c\}, \{a, c\}, \{b, c\}$ and $sg_\mu^{\star\star}$-closed sets are $\phi, X, \{b\}, \{c\}, \{b, c\}$. Thus {a, c} is g_μ-closed set but not $sg_\mu^{\star\star}$-closed set. Also {b} is $sg_\mu^{\star\star}$-closed set but not g_μ-closed set.*

Theorem 3.1. *Let A be subset of a strong generalized topological spaces (X, μ) and if A is a sg_μ-closed set in (X, μ), then A is $sg_\mu^{\star\star}$-closed.*

Proof. Let A be a sg_μ-closed set in (X, μ) and $A \subseteq U$ Where U is $sg_\mu^{\star\star}$-open. Since every $sg_\mu^{\star\star}$-open set is semi open and A is sg_μ-closed $scl_\mu(A) \subseteq U$. Hence A is $sg_\mu^{\star\star}$-closed set in (X, μ). □

The reverse implication does not hold.

Example 2. *Let $X = \{a, b, c\}$, $\mu = \{\phi, X, \{b\}, \{c\}, \{a, c\}, \{b, c\}\}$. Here the set $\{a, b\}$ is $sg_\mu^{\star\star}$-closed but not sg_μ-closed.*

Theorem 3.2. *Let A be a subset of a strong generalized topological space (X, μ) and if A is $sg_\mu^{\star\star}$-closed set in (X, μ), then A is $g_\mu s$-closed.*

Proof. Let A be a $sg_\mu^{\star\star}$-closed set in (X, μ) and $A \subseteq U$ where U is open. Since every open set is $sg_\mu^{\star\star}$-open and A is $sg_\mu^{\star\star}$-closed, $scl_\mu(A) \subseteq U$. Hence A is $g_\mu s$-closed in (X, μ). □

The reverse implication does not hold.

Example 3. *Let $X = \{a, b, c\}$, $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Then {c}, {a, c}, {b, c} are $g_\mu s$-closed sets but not $sg_\mu^{\star\star}$-closed.*

Remark 3.2. *These relations are shown in the diagram.*

$$\begin{tikzcd}
\textbf{\textit{closed}} \arrow[d] & & \\
g_\mu^{\star\star}\textbf{\textit{-closed}} \arrow[rr] & & \textbf{\textit{semi closed}} \arrow[d] \\
sg_\mu\textbf{\textit{-closed}} \arrow[d] & & g_\mu^{\star}s\textbf{\textit{-closed}} \arrow[ll] \\
sg_\mu^{\star\star}\textbf{\textit{-closed}} \arrow[r] & g_\mu s\textbf{\textit{-closed}} &
\end{tikzcd}$$

The converse of each statement is not true.

Theorem 3.3. *Let A be a $sg_\mu^{\star\star}$-closed set in strong generalized topological space (X,μ), then A is sg_μ-closed if (X,μ) is $T_{\frac{1}{2}}$ space.*

Proof. Let A be a $sg_\mu^{\star\star}$-closed set in (X,μ) and $A \subseteq U$ where U is semi open. Since (X,μ) is $T_{\frac{1}{2}}$ space and A is $sg_\mu^{\star\star}$-closed, every semi open set is $g_\mu^{\star\star}$-open and hence $scl_\mu(A) \subseteq U$. Therefore A is sg_μ-closed set in (X,μ). □

Theorem 3.4. *Let A be a $g_\mu s$-closed set in strong generalized topological space (X,μ), then A is $sg_\mu^{\star\star}$-closed if (X,μ) is $g_\mu^{\star\star}$-$T_{\frac{1}{2}}$ space.*

Proof. Let A be a $g_\mu s$-closed set in (X,μ) and $A \subseteq U$ where U is $g_\mu^{\star\star}$-open, since (X,μ) is $g_\mu^{\star\star}$-$T_{\frac{1}{2}}$ space and A is $g_\mu s$-closed, every $g_\mu^{\star\star}$-open set is open and hence $scl_\mu(A) \subseteq U$. Therefore A is $sg_\mu^{\star\star}$-closed set in (X,μ). □

Remark 3.3.

(i) *Union of two $sg_\mu^{\star\star}$-closed sets need not be $sg_\mu^{\star\star}$-closed.*
(ii) *Intersection of $sg_\mu^{\star\star}$-closed sets need not be $sg_\mu^{\star\star}$-closed.*

Example 4. *Let $X = \{a,b,c,d\}$, $\mu = \{\phi, X, \{a\}, \{a,b\}, \{a,c\}, \{a,b,c\}\}$. Then the sets ϕ, X, {c}, {d}, {b, c}, {b, d}, {b, c, d} are $sg_\mu^{\star\star}$-closed.*

(i) *the union of {c} and {d} are not in $sg_\mu^{\star\star}$-closed.*
(ii) *the intersection of {a, b, c} and {a, b, d} are not in $sg_\mu^{\star\star}$-closed.*

Theorem 3.5. *If a set A is $sg_\mu^{\star\star}$-closed in (X,μ), then $scl_\mu(A) - A$ contains no non empty $g_\mu^{\star\star}$-closed set.*

Proof. Let F be a $g_\mu^{\star\star}$-closed set such that $F \subseteq scl_\mu(A) - A$. Since $X - F$ is $g_\mu^{\star\star}$-open and $A \subseteq X - F$, from the Definition of $sg_\mu^{\star\star}$-closed set. It follows that $scl_\mu(A) \subseteq X - F$. Thus $F \subseteq X - scl_\mu(A)$. This implies that $F \subseteq scl_\mu(A) \cap X - scl_\mu(A) = \phi$. □

Corollary 3.1. *Let A be a $sg_\mu^{\star\star}$-closed set in (X,μ). Then A is semi closed $\Longleftrightarrow$ $scl_\mu(A) - A$ is $g_\mu^{\star\star}$-closed.*

Proof. Necessity. Let A be semi closed in (X,μ), Then $scl_\mu(A) \subseteq A$. This implies that $scl_\mu(A) - A = \phi$. Therefore $scl_\mu(A) - A$ is $g_\mu^{\star\star}$-closed.

Sufficiency. Suppose $scl_\mu(A) - A$ is $g_\mu^{\star\star}$-closed. Then by Theorem 3.5, $scl_\mu(A) - A$ contains does not contain any nonempty $g_\mu^{\star\star}$-closed set and hence $scl_\mu(A) - A = \phi$. This implies that $scl_\mu(A) = A$. Therefore A is semi closed in (X,μ). ☐

Theorem 3.6. *Let A be a $sg_\mu^{\star\star}$-closed set in (X,μ). If A is $g_\mu^{\star\star}$-open, then $scl_\mu(A) - A = \phi$.*

Proof. Let A be a $g_\mu^{\star\star}$-open (X,μ). Since A is $sg_\mu^{\star\star}$-closed and $scl_\mu \subseteq A$. This implies that $scl_\mu(A) - A = \phi$. ☐

Corollary 3.2. *Let A be a $sg_\mu^{\star\star}$-closed set in (X,μ). If A is $g_\mu^{\star\star}$-open, then A is semi regular.*

Proof. Let A be a $g_\mu^{\star\star}$-open in (X,μ). since A is $sg_\mu^{\star\star}$-closed and $scl_\mu(A) \subseteq A$. This implies that $scl_\mu(A) - A = \phi$. ☐

Theorem 3.7. *Suppose $B \subseteq A \subseteq X$, B is $sg_\mu^{\star\star}$-closed set relative to A and that A is $sg_\mu^{\star\star}$-closed subset of (X,μ). Then B is $sg_\mu^{\star\star}$-closed relative to (X,μ).*

Proof. Let $B \subseteq U$ and suppose that U is $g_\mu^{\star\star}$-open in (X,μ). Then $B \subseteq A \cap U$ and hence $scl_\mu(B) \subseteq A \cap U$ it follows that $A \cap scl_\mu(B) \subseteq A \cap U$ and $A \subseteq U \subseteq (X - scl_\mu(B))$, since A is $sg_\mu^{\star\star}$-closed in (X,μ) and $A \nsubseteq (X - scl_\mu(B))$, we have $scl_\mu(A) \subseteq U$. Therefore $scl_\mu(B) \subseteq scl_\mu(A) \subseteq U$. Hence B is a $sg_\mu^{\star\star}$-closed set relative to (X,μ). ☐

Theorem 3.8. *Let $A \subseteq Y \subseteq X$ and suppose that A is $sg_\mu^{\star\star}$-closed in (X,μ). Then A is $sg_\mu^{\star\star}$-closed relative to (Y,σ).*

Proof. Let $A \subseteq Y \cap U$ and suppose that U is $g_\mu^{\star\star}$-open in (X,μ), then $A \subseteq U$ and hence $scl_\mu(A) \subseteq U$. It follows $Y \cap scl_\mu(A) \subseteq Y \cap U$. Therefore $scl_\mu(A) \subseteq Y \cap U$ and $Y \cap U$ is $g_\mu^{\star\star}$-open in (Y,σ). Hence A is $sg_\mu^{\star\star}$-closed relative to (Y,σ). ☐

Theorem 3.9. *If A is a $sg_\mu^{\star\star}$-closed set in (X,μ) and $A \subseteq B \subseteq scl_\mu(A)$, then B is $sg_\mu^{\star\star}$-closed in (X,μ).*

Proof. Let $B \subseteq U$ where U is $g_\mu^{\star\star}$-open. Since A is $sg_\mu^{\star\star}$-closed in (X, μ) and $A \subseteq U$ it follows that $scl_\mu(A) \subseteq U$. By hypothesis $B \subseteq scl_\mu(A)$ and hence $scl_\mu(B) \subseteq scl_\mu(A)$.

Consequently $scl_\mu(B) \subseteq U$ and B becomes $sg_\mu^{\star\star}$-closed in (X, μ). □

Theorem 3.10. *If $G_\mu^{\star\star}O(X) = G_\mu^{\star\star}C(X)$, then every subset of is $sg_\mu^{\star\star}$-closed in (X, μ).*

Proof. Let A be a subset of (X, μ) such that $A \subseteq U$ where $U \in G_\mu^{\star\star}O(X)$. Then $U \in G_\mu^{\star\star}C(X)$. Since every $g_\mu^{\star\star}$-closed set in semi closed and $A \subseteq U$ it follows that $scl_\mu(A) \subseteq scl_\mu(U) = U$. Hence A is $sg_\mu^{\star\star}$-closed in (X, μ). □

Theorem 3.11. *For each $x \in X$, $\{x\}$ is $g_\mu^{\star\star}$-closed or its complement $X - \{x\}$ is $sg_\mu^{\star\star}$-closed in a space (X, μ).*

Proof. Suppose that $\{x\}$ is not $g_\mu^{\star\star}$-closed in (X, μ). Since $X - \{c\}$ is not $g_\mu^{\star\star}$-open, the space (X, μ) itself is only $g_\mu^{\star\star}$-open containing $X - \{x\}$. Therefore $scl_\mu(X - \{x\}) \subseteq X$ holds and so $X - \{x\}$ is $sg_\mu^{\star\star}$-closed in (X, μ). □

4. Semi $g_\mu^{\star\star}$-open set

In this section, we introduce a new class of open set called $sg_\mu^{\star\star}$-open set and study some of their properties.

Definition 4.1. *A subset A of a topological space is called semi $g_\mu^{\star\star}$-open ($sg_\mu^{\star\star}$-open) $\iff$ $X - A$ is $sg_\mu^{\star\star}$-closed.*

The set of all $sg_\mu^{\star\star}$-open sets denoted by $SG_\mu^{\star\star}O(X)$.

Remark 4.1.

(i) *Union of two $sg_\mu^{\star\star}$-open sets need not be $sg_\mu^{\star\star}$-open.*

(ii) *Intersection of two $sg_\mu^{\star\star}$-open sets need not be $sg_\mu^{\star\star}$-open.*

Example 5. *Let $X = \{a, b, c\}$, $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$. Then the sets ϕ, X, {a}, {b}, {c}, {a, c}, {b, c} are $sg_\mu^{\star\star}$-open. Here the sets {a} and {b} are $sg_\mu^{\star\star}$-open set but the union of these sets are not $sg_\mu^{\star\star}$-open.*

Example 6. *Let $X = \{a, b, c, d\}$, $\mu = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}$. The $sg_\mu^{\star\star}$-open sets are ϕ, X, {a}, {b}, {a, b}, {a, c}, {a, d}, {b, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}. Here the sets {a, c, d} and {b, c, d} are $sg_\mu^{\star\star}$-open sets both the intersection of these sets are not $sg_\mu^{\star\star}$-open.*

Theorem 4.1. *A set A is $sg_\mu^{\star\star}$-open in (X,μ) $\iff$ $F \subseteq sint_\lambda(A)$ whenever F is $g_\mu^{\star\star}$-closed and $F \subseteq A$.*

Proof. Necessity. Let A be $sg_\mu^{\star\star}$-open and suppose $F \subseteq A$ where F is $g_\mu^{\star\star}$-closed. $X - A$ is $sg_\mu^{\star\star}$-closed. $X - A$ is $sg_\mu^{\star\star}$-closed. Also $X - A$ is contained in the $g_\mu^{\star\star}$-open set $X - F$. This implies that $scl_\lambda(X - A) = X - sint_\lambda(A)$. Hence $X - sint_\lambda(A) \subseteq X - F$. Therefore $F \subseteq sint_\lambda(A)$.

Sufficiency: If F is $g_\mu^{\star\star}$-closed set with $F \subseteq sint_\lambda(A)$ whenever $F \subseteq A$, it follows that $X - A \subseteq X - F$. Thus $X - sint_\lambda(A) \subseteq X - F$. Hence $X - A$ is $sg_\mu^{\star\star}$-closed and A becomes $sg_\mu^{\star\star}$-open. □

Theorem 4.2. *If $sint_\lambda(A) \subseteq B \subseteq A$ and A is $sg_\mu^{\star\star}$-open in (X,μ), then B is $sg_\mu^{\star\star}$-open.*

Proof. By hypothesis $X - A \subseteq X - B \subseteq X - sint_\lambda(A)$. Thus $X - A \subseteq X - B \subseteq X - (X - scl_\lambda(X - A)) = scl(X - A)$. By hypothesis. Now $X - A$ is $sg_\mu^{\star\star}$-closed and hence $X - B$ is $sg_\mu^{\star\star}$-closed. Hence B is $sg_\mu^{\star\star}$-open in (X,μ). □

5. $sg_\mu^{\star\star}$-CONTINUOUS MAP AND $sg_\mu^{\star\star}$-IRRESOLUTE MAP

Definition 5.1. *A map $f : (X,\mu) \to (Y,\sigma)$ is called a*

(i) *$sg_\mu^{\star\star}$-continuous if $f^{-1}(V)$ is $sg_\mu^{\star\star}$-closed in $(X,\mu$ for every closed set V of (Y,σ).*

(ii) *$sg_\mu^{\star\star}$-irresolute if $f^{-1}(V)$ is $sg_\mu^{\star\star}$-closed in (X,μ) for every closed set V of (Y,σ).*

Theorem 5.1. *Let $f : (X,\mu) \to (Y,\sigma)$ be sg_μ-continuous. Then f is $sg_\mu^{\star\star}$- continuous.*

Proof. Let V be closed set in (Y,σ), Then $f^{-1}(V)$ is sg_μ-closed in (X,μ), since f is sg_μ-continuous. Every $sg_\mu^{\star\star}$-closed set is sg_μ-closed. Therefore $f^{-1}(V)$ is closed in (X,μ). Hence f is sg_μ-continuous. The converse need not be true as seen from the following example. □

Example 7. *Let $X = \{a,b,c,d\}$, $\mu = \{\phi, X, \{b\}, \{a,b\}\}$ and $Y = \{p,q,r,s\}$, $\sigma = \{\phi, Y, \{p,s\}, \{q,r\}\}$. Define a map $f : (X,\mu) \to (Y,\sigma)$ by f(a) = r, f(b)= s, f(c) = p, f(d) = q. Then f is $sg_\mu^{\star\star}$-continuous. $f^{-1}(\{p,s\}) = \{b,c\}$ is not sg_μ-closed in (X,τ) for the closed set of (Y,σ). So f is not sg_μ-continuous.*

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