ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9401–9415 ISSN: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.11.45

EXISTENCE OF HOPF-BIFURCATION IN A MATHEMATICAL MODEL ON DEPLETION OF GRASSLAND UNDER THE CATASTROPHIC EFFECT OF INVASIVE SPECIES IN KAZIRANGA NATIONAL PARK (KNP), ASSAM, INDIA

MUNMI SAIKIA

ABSTRACT. The grasslands of Kaziranga National Park, Assam, India, have been threatened by certain species of plants such as Mikenia, Mimosa, Simul (Bombax ceiba) etc. These plants are found to be invasive and without efforts to control their growth the grasslands along with their wildlife could face catastrophic situation. Controlled annual burning, manual weed removal, stubbing are some of the control measures the park has been employing in this regard. Taking the cumulative densities of all the invasive species of plants which affect the grasslands as bad biomass and employing effort to control their growth for healthy grassland density, this paper attempts to study the long term effect of bad biomass on grassland biomass using non-linear mathematical model. For this, the feasible equilibria of the model was obtained and their local stability discussed. Through qualitative analysis it was observed that grassland density could be maintained at equilibrium with proper effort. It was also observed that reducing the effort could adversely affect the dynamics of the system. When the effort was reduced beyond a certain value, Hopf bifurcation was observed. These findings were validated through numerical simulation.

Key words and phrases. Grassland biomass, Depreciation, Stability, Hopf-bifurcation.

²⁰²⁰ Mathematics Subject Classification. 93D20, 93C10, 34D20, 93A30, 93C15, 74H60, 70K05.

1. INTRODUCTION

Kaziranga National Park (KNP) is located in the floodplains of the Brahmaputra river within the Nagaon and Golaghat district of Assam, India. Conservation of the park started in 1908 when it was declared as Reserve Forest. Today, this park is a thriving site within the Indo-Burma biodiversity hotspot. It is famous for the Greater one-horned rhinoceros (Rhinoceros unicornis) as it harbors 85% of the world's total population of this species. The rhino thrives in the grasslands within the park. Apart from rhino, the grasslands have to support a significant number of three other large herbivores — the Asian elephant, the wild Asian water buffalo and the eastern swamp deer (Cervus duvauceli ranjitsinhi) and other smaller herbivores like Hog Deer (Axis porcinus), Sambar (Cervus unicolor), Barking Deer (Muntiacus muntjak),Wild Boar (Sus scrofa), Hog Badger (Arctonyx collaris), Wild Buffalo (Bubalus bubalis) etc. (Source: PCCF Wildlife, Assam). Thus, the grassland management is one of the prime objectives at the park.

The grasslands of Kaziranga National Park have been threatened by some proliferating species of weed like Mikenia, Mimosa and the tree species Simul (Bombax ceiba). Mimosa is a straggling herb seen climbing to the top of several meters high elephant grasses. The quick growing herb not only destroy the grasses, it hampers the free movement of wild animals. Mimosin, a harmful toxin released by Mimosa is known to affect herbivore population particularly ruminants. The large and small tea gardens on the high ground beyond southern boundary of the park have been using Mimosa for rehabilitating the degraded soil and to curb grasses like Imperata cylindrica and other tenacious grasses since late sixties. The runoff water from these gardens carry the seeds and flow to the park through numerous channels and rivers. Kushwaha et al. [7] found a considerable loss of short grasses in KNP. Lahkar et al. [8] marked the slow and steady intrusion of invasive species such as Mimosa in grassland habitat as one of the major threats which directly reduces flora for rhino and other herbivores. Medhi & Saha [9] observed considerable reduction of grassland areas and small water bodies. In order to mitigate such invasion, the KNP authority carries out management effort in the form of manual removal of weed, annual controlled burning of grassland, uprooting of tree saplings, stubbing etc. which are important tools for maintaining the seral stage of the grasslands.

Studies have shown that invasive species decrease species diversity, cause economic loss, and reduce forest health and productivity. These invasive plants will not stop spreading unless controlled. However, longer the management is postponed, the more the invasive plants will spread and the more costly they will be to eradicate. Using mathematical modelling as an essential tool a diverse range of various ecological situations can be studied (Pathak [11]). Some mathematical models by Agarwal & Mishra [1]; Dubey & Narayanan [5]; Kumar & Agarwal [6]; Patra et al. [12] and Sundar et al. [15] have tried to study the effect of various factors such as pollution, deforestation, urbanization and industrialization on the forestry biomass . Agarwal et al. [2] studied the effect of depletion of forestry biomass in a habitat due to pressure of industrialization on the survival of forestry biomass dependent wildlife species. They have shown that, under some conditions, the forest biomass density decreases due to an increase in industrialization pressure which leads to decrease in the density of wildlife species and it may even lead to extinction if the industrialization continues without control. Ramdhani et al. [14] found that if the crowding by industrialization increases, then biomass density of forestry resources decreases and hence it is necessary to control industrialization to protect the forestry resources stability. Anderson et al. [3] in their study found that, invasive plant species have various negative impacts on the ecosystems they invade. Misra & Lata [10] studied the effect of time delay on conservation of forestry biomass by proposing a non-linear mathematical model. They found in their analysis that the density of forestry biomass may be conserved if the technological effort is applied within the appropriate time. Rai [13] presented a model for the wetland part of Keolado National Park which provides clear perspectives on future management strategies and policy decisions.

Given this brief literature survey, this paper proposes a mathematical model on depletion of grassland incorporating the effect of management effort. The paper is organized as follows: In section 2 the mathematical model is proposed and the boundedness of its solutions is proved. Section 3 is devoted to study the existence of equilibrium points and the nature of their local stability is discussed in section 4. In section 5, existence of Hopf bifurcation around the interior equilibrium point is analyzed. In section 6, numerical simulation of the system is discussed to illustrate the theoretical results. Finally conclusion is drawn in section 7.

2. MATHEMATICAL MODELLING

Some assumptions and factors of importance for the proposed model are listed below:

- (1) Bad biomass is the cumulative densities of Mikenia, Mimosa, Bombax Ceiba and Eichhornia that depletes the grassland biomass.
- (2) Densities of grassland biomass and bad biomass are governed by logistic type equations.
- (3) Effect of invasive species is catastrophic as invasive plants will not stop spreading unless controlled. It can lead to decline of grassland and eventually the whole wildlife grazing population.
- (4) There exists inter-specific competition between bad biomass and grassland biomass for abiotic factors such as soil, water, light etc.
- (5) The effort to control increases proportionally with the density of the existing bad biomass.

Based on these assumptions and in conformity with the reality in KNP, the system of non-linear differential equations that governs the dynamics of the problem is given below:

(2.1)

$$\frac{dG}{dt} = r_1 G \left(1 - \frac{G}{K} \right) - c_1 G B - c_2 G^2 B$$

$$\frac{dB}{dt} = r_2 B \left(1 - \frac{B}{L} \right) - eBE - aGB$$

$$\frac{dE}{dt} = \delta_1 B - \delta_2 E,$$

with initial conditions $G(0) \ge 0$, $B(0) \ge 0$, $E(0) \ge 0$, $0 \le \delta_1$, $\delta_2 \le 1$, where, G is the density of grassland biomass, B is the density of bad biomass and E is the measure of effort applied for conservation of grassland biomass. The constants r_1 and K are intrinsic growth rate and carrying capacity of grassland biomass. r_2 and L are intrinsic growth rate and carrying capacity of bad biomass respectively. The constants c_1 and c_2 are the depletion rate coefficients of intrinsic growth rate and carrying capacity of grassland biomass respectively. The constants e and a are effort coefficient needed to control the biomass [10]. The constants e and a are effort coefficient of bad biomass and grassland biomass respectively. δ_1 and δ_2 respectively correspond to the growth rate and depletion rate coefficients of effort.

Boundedness:

Theorem 2.1. The set $\Omega = \{(G, B, E) \in R_3^+: 0 < W = G + B + E \leq \frac{\nu}{\eta}\}$ is a region of attraction for all solutions initiating in the interior of the positive orthant, where η is a constant such that $0 < \eta < \delta_2$, $\nu = \frac{K(r_1+\eta)^2}{4r_1} + \frac{L(r_2+\delta_1+\eta)^2}{4r_2}$.

Proof. Let us consider a time dependent function W(t) = G(t) + B(t) + E(t)and $\eta > 0$ be a constant. Then

(2.2)
$$\frac{dW}{dt} + \eta W = (r_1 + \eta)G - \frac{r_1G^2}{K} + (r_2 + \delta_1 + \eta)B - \frac{r_2B^2}{L} - (c_1 + a)GB - c_2G^2B - eBE - (\delta_2 - \eta)E.$$

Now we choose η such that $0 < \eta < \delta_2$ Then, the equation (2.2) can be written as

$$\frac{dW}{dt} + \eta W \le (r_1 + \eta)G - \frac{r_1G^2}{K} + (r_2 + \delta_1 + \eta)B - \frac{r_2B^2}{L}$$
$$= \frac{K(r_1 + \eta)^2}{4r_1} + \frac{L(r_2 + \delta_1 + \eta)^2}{4r_2} = \nu.$$

By using differential inequality ([4]), we obtain $0 < W(G(t), B(t), E(t)) \le \frac{\nu}{\eta}(1 - e^{-\eta t}) + (G(0), B(0), E(0))e^{-\eta t}$ which gives $0 < W(t) \le \frac{\nu}{\eta}$ as $t \to \infty$. \Box

3. Equilibrium Analysis

Equating the system (2.1) the following equilibrium points are obtained.

i. $P_0(0, 0, 0)$, ii. $P_1(K, 0, 0)$ iii. $P_2(\bar{G}, \bar{B}, 0)$ iv. $P_3(G^*, B^*, E^*)$

Here, the trivial equilibrium point P_0 and the axial equilibrium point P_1 always exist. The existence of P_2 i.e. in the absence of effort to control the bad biomass and the coexisting equilibrium $P_3(G^*, B^*, E^*)$ is discussed below.

3.1. Existence of $P_2(\bar{G}, \bar{B}, 0)$. In the absence of control effort E, \bar{G} and \bar{B} can be obtained by solving the following equations:

(3.1)
$$r_1 \left(1 - \frac{G}{K}\right) - c_1 B - c_2 G B = 0$$

(3.2)
$$r_2(1-\frac{B}{L}) - aG = 0.$$

Equation (3.2) gives,

(3.3)
$$B = \frac{L}{r_2}(r_2 - aG),$$

which upon substitution in equation (3.1) yields a quadratic equation in G given as,

$$(3.4) B_1 G^2 + B_2 G + B_3 = 0$$

where,

$$B_{1} = \frac{aLc_{2}}{r_{2}}$$

$$B_{2} = \frac{ac_{1}L}{r_{2}} - \frac{r_{1}}{K} - c_{2}L$$

$$B_{3} = r_{1} - Lc_{1}.$$

Now, solving for the roots of the equation (3.4) gives $\bar{G} = \frac{-B_2 + \sqrt{B_2^2 - 4B_1B_3}}{2B_1}$, which is a positive real root if the following condition holds

 $r_1 < Lc_1$.

Knowing the value of \overline{G} , the value of \overline{B} is evaluated from equation (3.3). It may be noted that for \overline{B} to be positive, the following must hold:

 $r_2 > a\bar{G}.$

3.2. Existence of $P_3(G^*, B^*, E^*)$. To see the existence of $P_3(G^*, B^*, E^*)$, we note that G^* , B^* and E^* are the positive solutions of the following equations.

(3.5)
$$r_1\left(1-\frac{G}{K}\right) - c_1B - c_2B = 0$$

$$(3.6) r_2\left(1-\frac{B}{L}\right) - eE - aG = 0$$

$$\delta_1 B - \delta_2 E = 0.$$

Equation (3.6) implies

$$(3.8) B = \frac{L}{r_2} \Big(r_2 - eE - aG \Big)$$

using this in equations (3.5) & (3.7), we get

(3.9)
$$f(G, E) = r_1 \left(1 - \frac{G}{K}\right) - \frac{c_1 L}{r_2} \left(r_2 - eE - aG\right) - \frac{c_2 GL}{r_2} \left(r_2 - eE - aG\right) = 0$$

and

(3.10)
$$g(G, E) = \frac{\delta_1 L}{r_2} \left(r_2 - eE - aG \right) - \delta_2 E = 0.$$

From equation (3.10) we note the following:

i. If G = 0, then $E = \frac{\delta_1 r_2}{e\delta_1 + \frac{r_2\delta_2}{L}} = E_1$ where E_1 is a solution of g(0, E) = 0. ii. If E = 0, then $G = \frac{r_2}{a} = G_1$ where G_1 is a solution of g(G, 0) = 0. iii. $\frac{dG}{dE} = -\frac{\partial g}{\partial E} / \frac{\partial g}{\partial G} < 0$, as $\frac{\partial G}{\partial E} = -\frac{e\delta_1 L}{r_2} - \delta_2 < 0$ and $\frac{\partial g}{\partial G} = -\frac{a\delta_1 L}{r_2} < 0$.

Equation (3.9) implies that,

- i. If G = 0, then $E = \frac{r_2(Lc_1-r_1)}{ec_1L} = E_2$ where E_2 is a solution of f(0, E) = 0. Here, if $Lc_1 > r_1$, then E > 0.
- ii. If E = 0, we get $G = G_2$ (say), where G_2 is the positive root of f(G, 0) = 0 which yields $B_1G^2 + B_2G + B_3 = 0$, where

$$B_{1} = \frac{Lc_{2}a}{r_{2}}$$
$$B_{2} = \frac{ac_{1}L}{r_{2}} - \frac{r_{1}}{K} - Lc_{2}$$
$$B_{3} = r_{1} - c_{1}L.$$

Thus, G_2 is positive if $r_1 < c_1 L$. iii. From equation (3.9),

$$\begin{split} \frac{\partial f}{\partial E} &= \frac{Le(c_1 + c_2 G)}{r_2} > 0, \\ \frac{\partial f}{\partial G} &= -\frac{r_1}{K} + \frac{ac_1 L}{r_2} - \frac{c_2 L}{r_2} \Big(r_2 - eE - aG \Big) + \frac{ac_2 GL}{r_2} \quad \text{and} \\ \frac{dG}{dE} &= -\frac{\partial f}{\partial E} \Big/ \frac{\partial f}{\partial G} > 0 \text{ if } \frac{\partial f}{\partial E} \text{ and } \frac{\partial f}{\partial G} \text{ have opposite signs} \end{split}$$

If we also assume that , $G_2 > G_1$, then the two isoclines (3.9) and (3.10) intersect at a unique point (G^* , E^*), and B^* can be calculated from equation (3.8). It may be noted that B^* is positive if the following inequality holds:

$$r_2 > eE^* + aG^*.$$

4. STABILITY ANALYSIS

In this section the local stability of the proposed system is analyzed. Evaluating the Jacobian at each equilibrium point and using the eigenvalue method & Routh Hurwitz criteria the following results are obtained.

- i. P_0 is unstable.
- ii. P_1 is stable if $aK < r_2$ otherwise unstable.
- iii. P_2 is locally asymptotically stable iff

$$\frac{r_1 r_2 GB}{KL} + \frac{r_2 c_2}{L} \bar{G} \bar{B}^2 + a c_2 \bar{G}^2 \bar{B} > a \bar{B} c_1 \bar{G}$$

otherwise unstable.

iv. To study the stability of $P_3(G^*, B^*, E^*)$ it is noted that the characteristic equation of the Jacobian matrix evaluated at P_3 is

(4.1)
$$\lambda^3 + C_1 \lambda^2 + C_2 \lambda + C_3 = 0$$

where,

$$C_{1} = \delta_{2} + \frac{r_{2}B^{*}}{L} + \frac{G^{*}r_{1}}{K} + c_{2}G^{*}B^{*}$$

$$C_{2} = \frac{\delta_{2}r_{2}B^{*}}{L} + e\delta_{1}B^{*} + \left(\delta_{2} + \frac{r_{2}B^{*}}{L}\right)\left(\frac{G^{*}r_{1}}{K} + c_{2}G^{*}B^{*}\right)$$

$$+ aB^{*}(c_{1}G^{*} - c_{2}G^{*2})$$

$$C_{3} = \left(\frac{G^{*}r_{1}}{K} + c_{2}G^{*}B^{*}\right)\left(\frac{\delta_{2}r_{2}B^{*}}{L} + e\delta_{1}B^{*}\right)$$

$$+ \delta_{2}aB^{*}(c_{1}G^{*} - c_{2}G^{*2}).$$

By Routh Hurwitz criteria it implies that, all the solutions of (4.1) have negative real parts iff,

$$C_i > 0$$
, $i = 1, 3$ and $C_1 C_2 > C_3$.

Clearly, $C_1 > 0$, $C_3 > 0$ iff $G^* < \frac{c_1}{c_2}$. It is also easy to to verify that $C_1C_2 > C_3$ holds true for $G^* < \frac{c_1}{c_2}$.

Thus, we state the following theorem.

Theorem 4.1. The positive equilibrium point is locally asymptotically stable if and only if

$$G^* < \frac{c_1}{c_2}.$$

5. EXISTENCE OF HOPF-BIFURCATION

The aim of this section is to investigate the Hopf-bifurcation of (2.1) around the interior equilibrium point P_3 .

Choosing the depreciation rate of effort δ_2 as the bifurcation parameter, the necessary and sufficient conditions for Hopf-bifurcation to occur at $\delta_2 = \delta_2^*$ (critical value) are found to be:

(5.1)
1.
$$C_1(\delta_2^*) > 0 \text{ and } C_3(\delta_2^*) > 0$$

2. $f(\delta_2^*) \equiv C_1(\delta_2^*)C_2(\delta_2^*) - C_3(\delta_2^*) = 0$
3. $Re\left[\frac{d\lambda_j}{d\delta_2}\right]_{\delta_2 = \delta_2^*} \neq 0, \quad j = 1, 2, 3,$

where λ_j is the eigenvalue of the variational matrix associated with $P_3(G^*, B^*, E^*)$.

At the critical value $\delta_2 = \delta_2^*$, the characteristic equation (4.1) can be written as

$$(\lambda^2 + C_2)(\lambda + C_1) = 0.$$

This has three roots i.e. $\lambda_{1,2} = \pm i\omega$, $\lambda_3 = \mu$ where

$$\begin{split} \mu &= -C_1 \\ &= -\left\{\delta_2 + \frac{r_2 B^*}{L} + \frac{G^* r_1}{K} + c_2 G^* B^*\right\} \\ \omega &= \sqrt{C_2} = \sqrt{\left\{\frac{\delta_2 r_2 B^*}{L} + e\delta_1 B^* + \left(\delta_2 + \frac{r_2 B^*}{L}\right) \left(\frac{G^* r_1}{K} + c_2 G^* B^*\right) \right.} \\ &+ aB^* (c_1 G^* - c_2 G^{*2}) \right\}. \end{split}$$

Thus at $\delta_2 = \delta_2^*$, the characteristic equation (4.1) has purely imaginary roots while the third root is negative. To show the transversality condition, let at any point δ_2 in ϵ -neighborhood of δ_2^* , $\lambda_{1,2} = b_1(\delta_2) \pm ib_2(\delta_2)$.

Substituting this in characteristic equation (4.1) and operating real and imaginary parts, and taking its derivative, we have,

$$R(\delta_2)b'_1(\delta_2) - S(\delta_2)b'_2(\delta_2) + L(\delta_2) = 0$$

$$S(\delta_2)b'_1(\delta_2) + R(\delta_2)b'_2(\delta_2) + N(\delta_2) = 0$$

where

$$R(\delta_2) = 3\{b_1^2(\delta_2) - b_2^2(\delta_2)\} + 2C_1b_1(\delta_2) + C_2$$

$$S(\delta_2) = 6b_1(\delta_2)b_2(\delta_2) + 2C_1b_2(\delta_2)$$

$$L(\delta_2) = C_1'(b_1^2(\delta_2) - b_2^2(\delta_2) + C_2'b_1(\delta_2) + C_3'$$

$$N(\delta_2) = 2C_1'b_1(\delta_2)b_2(\delta_2) + C_2'b_2(\delta_2).$$

Hence, $Re\left[\frac{d\lambda_j}{d\delta_2}\right]_{\delta_2=\delta_2^*} = -\left[\frac{LR+SN}{R^2+S^2}\right]_{\delta_2=\delta_2^*} \neq 0.$ Therefore the transversality condition holds and Hopf-bifurcation occurs at

Therefore the transversality condition holds and Hopf-bifurcation occurs at $\delta_2 = \delta_2^*$ around the interior equilibrium $P_3(G^*, B^*, E^*)$. Thus we can write the following theorem:

Theorem 5.1. There is a simple Hopf-bifurcation at equilibrium point P_3 under conditions (5.1), at some critical value of the parameter δ_2 , given by the equation $f(\delta_2^*) = 0$.

6. NUMERICAL SIMULATION

The units of the parameters are given in Table (1)

Parameter	Unit
G, K	ton
B, E, L	m^{-2}
$r_1, r_2, \delta_1, \delta_2$	$y ear^{-1}$
c_1, e	$m^{-2}year^{-1}$
c_2	$ton^{-1}m^{-2}year^{-1}$
a	$ton^{-1}year^{-1}$

TABLE 1. Parameters and their unit

Here, the ecologically healthy situation will be defined first. The depreciation rate in the effort is the key parameter that influences the dynamics of the system and eventually the ecological health of the park. The park is said to be ecologically healthy if it maintains higher density of the grassland biomass compared to the bad biomass and vice versa.



FIGURE 1. Graph trajectory of G, B, E showing ecologically unhealthy situation of the park for (a) $\delta_2 = 0.14$ (b) $\delta_2 = 0.2$

Ecologically unhealthy situation of the park: Consider the following set of value of parameters.

$$r_1 = 2, \ K = 100, \ c_1 = 0.07, \ c_2 = 0.02, \ r_2 = 3$$

 $L = 1000, \ e = 0.4, \ a = 0.01, \ \delta_1 = 0.1$

with initial conditions (1, 1, 1).

For the above set of values of parameters, we observe in Fig. (1) that, as the depreciation rate δ_2 increases the bad biomass also increases w.r.t. *t* and settles at higher equilibrium level than that of grassland biomass and hence it shows an ecologically unhealthy situation of the park.

Hopf Bifurcation: For the choice of the following values of parameters:

$$r_1 = 2, K = 100, c_1 = 0.07, c_2 = 0.02, r_2 = 3,$$

 $L = 1000, e = 0.4, a = 0.01, \delta_1 = 0.1,$

Occurence of Hopf Oscillations has been investigated. The critical value of depreciation parameter δ_2 (i.e. $\delta_2^* = 0.105$) at which stability loss occurs has been calculated and existence of limit cycle is observed.



FIGURE 2. (a) Hopf Oscillations for $\delta_2 = 0.105$, (b) limit cycle

Local stability of the interior equilibrium point and stable limit cycle: For the following set of values of parameters

$$r_1 = 2, K = 100, c_1 = 0.07, c_2 = 0.02, r_2 = 3,$$

 $L = 1000, e = 0.4, a = 0.01, \delta_1 = 0.291, \delta_2 = 0.2$

with initial conditions (1, 1, 1). There exist an unique equilibrium point $P_3(G^*,$ B^*, E^*) where $G^* = 14.0850$, $B^* = 4.8884$ and $E^* = 7.1112$. It may further be observed that the condition of local stability of the Theorem (4.1) is satisfied. Also, the stable limit cycle (Fig. 3(b)) around the interior equilibrium point $P_3(14.0850, 4.8884, 7.1112)$ is observed.

Ecologically healthy situation of the park: For the same set of value of parameters taking different values of δ_2 , the behavior of the grassland biomass and the bad biomass w.r.t time t is shown in Fig. (4(a)-(b)). It is observed that when the growth rate of effort, δ_1 , is greater than the depreciation rate coefficient, δ_2 , the grassland biomass settles at higher equilibrium level than that of bad biomass, and hence it is showing an ecologically healthy situation of the park.



FIGURE 3. (a) Local stability of the interior equilibrium (b) 3D plot of the system (2.1)



FIGURE 4. Behavior of grassland biomass, bad biomass and effort for (a) $\delta_2=0.24$ (b) $\delta_2=0.29$

7. CONCLUSION

In this paper a mathematical model of KNP for the depletion of grassland biomass under the catastrophic effect of invasive species was proposed and analyzed. It explains the effect of bad biomass growth on the growth of grassland biomass using logistic growth model for both the biomasses. Eigenvalue method and Routh Hurwitz criteria were applied to analyze the existence of equilibrium points and local stability. It was observed that the management effort could play a vital role in controlling the undesirable growth of bad biomass. Also, it was found that the bad biomass increases and grassland decreases if the effort applied fell below a critical value. The results were verified through numerical simulation. This model suggest that the control of bad biomass in an efficient manner is important to maintain the grassland biomass of the park in healthy seral stage.

REFERENCES

- [1] M. AGARWAL, N. MISHRA: A model for a forested grassland: Effect of overgrazing and pollution, Indian J. of pure appl. Math., **32**(4) (2000), 613-628.
- [2] M. AGARWAL, T. FATIMA, H. I. FREEDMAN: Depletion of forestry resource biomass due to industrialization pressure: A ratio dependent mathematical model, J. Biol. Dynamics, 4 (2000), 381-396.
- [3] M. ANDERSON, M. K. CROSBY: An assessment of invasive plants on ShorterUniversity's campus, Mathematical and Computational Forestry & Natural-Resources Sciences(MCFNS), 10(1) (2018), 24-29.
- [4] G. BIRKOFF, G. C. ROTA: Ordinary differential equations, Ginn, 1982.
- [5] B. DUBEY, A. S. NARAYANAN: Modelling effects of industrialization, population and pollution on a renewable resource, Nonlinear Analysis: Real World Applications, 11 (2010), 2833-2848.
- [6] A. KUMAR, M. AGARWAL: Degradation and subsequent regeneration of forestry biomass using genetic resource, International Journal of Mathematics And its Applications, 5(1C) (2017), 291-300.
- [7] S. P. S. KUSHWAHA, P. S. ROY, A. AZEEM, P. BORUAH, P. LAHAN: Land area change and rhino habitat suitability analysis in Kaziranga National Park, Assam, Tiger paper, 27(2) (2000), 9-17.
- [8] B. P. LAHKAR, B. K. TALUKDAR, P. SARMA: Invasive species in grassland habitat: an ecological threat to the great one-horned rhino (Rhinoceros unicornis), Pachyderm, 49 (2011), 33-39.

- [9] A. MEDHI, A. K. SAHA: Land Cover Change and Rhino Habitat Mapping of Kaziranga National Park, Assam. Climate change and biodiversity, Advances in Geographical and Environmental Sciences, Springer, Tokyo, **10** (2014), 125-138.
- [10] A. K. MISRA, K. LATA: Modeling the effects of time delay on the conservation of forestry biomass, Chaos, Solitons & Fractals, 46 (2013), 1-11.
- [11] R. PATHAK: Depletion of forest resources and wildlife population with habitat complexity: A mathematical model, Open Journal of Ecology, 8 (2018), 579-589.
- [12] A. PATRA, J. TUSHAR, B. DUBEY: Modeling and simulation of a wetland park: Anapplication to Keoladeo National Park, India, Journal of Mathematics and computersin simulation, 134 (2017), 54-78.
- [13] V. RAI: Modeling a wetland system: The case of Keolado National Park (KNP), India, Ecological modeling, 210 (2008), 247-252.
- [14] V. RAMDHANI, JAHARUDDIN, E. H. NUGRAHANI: Dynamical system of modelling the the depletion of forestry resources due to crowding by industrialisation, Applied Mathematical Sciences, 9(82) (2015), 4067-4079.
- [15] S. SUNDAR, N. SWAROOP, R. NARESH: Modeling the effect of population and population augmented industrialization on forestry resources, European Journal of Engineering Research and Science, 2(1) (2017), 65-70.

DEPARTMENT OF MATHEMATICS THE ASSAM ROYAL GLOBAL UNIVERSITY GUWAHATI, ASSAM, INDIA, 781035. *Email address*: munmi.saikia@rgi.edu.in