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A NEW APPROACH TO SOLVE ASSIGNMENT PROBLEM USING CONGRUENCE MODULO AND ITS CODING IN MATLAB

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ABSTRACT. AP is the fundamental application of TP studied in the area of Operations research. In this paper authors have proposed a new algorithm using congruence modulo approach to solve AP which is also demonstrated by illustrating numerical examples. Obtained results show that this is one of the efficient and simple method for obtaining an optimal solution of AP, then some methods commonly used in the literature. Proposed algorithm is also coded in MATLAB which makes it user friendly.

ABBREVIATIONS

Linear Programming Problem (LPP), Assignment Problem (AP), Transportation Problem (TP), Bottleneck Assignment Problem (BAP).

1. INTRODUCTION

Assignment Problem (AP) is one of the fundamental application in the theory of optimization, which is about the best way of assigning a set of m elements to another set of m elements in which, each element of the first set has to be assigned to exactly one element of the second set and vice-versa. AP appears in some decision making situations when a set of jobs has to be assigned to a

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set of machines or set of workers. More precisely, it is a special class of LPP in which number of activities are assigned to equal number of resources with the objective of completing task at a minimum cost or maximum profit.

Algorithm for the classic assignment problem was first studied by Easter Field. Hungarian method was one of the best methods available in literature for finding optimal solution of AP, published by Kuhn [1]. In the honor of two Hungarian mathematicians Dénes Kőnig and Jenő Egerváry who actually worked for this method, he gave the name "Hungarian method". Approximation method for large scale assignment problem was proposed by Kurtzberg [2]. Many researchers have studied several versions of classical AP including but not limited to Semi-Assignment problem [3], the bottleneck assignment (BAP) [4], the fractional AP [5]. A. Ahmed et al [6] and A. Khandelwal [7] proposed innovative methods for finding solution of Assignment problem. K. Ghadle et al. [8, 9] has given a new approach to solve balanced and unbalanced AP.

In this research article, we have proposed a new method for solving AP using congruence modulo approach [10]. Numerical example is solved to illustrate proposed method. It is found that this method provides better solution in less iterations. The algorithm is also coded in MATLAB to make it user friendly.

2. Preliminary

Congruence Modulo: If two numbers b and c have the property that their difference is integrally divisible by a number m i.e. $(b - c)/m$ is an integer, then we say that b is congruent to c mod m. It means c is the remainder when b is divided by m. In the proposed algorithm, this concept of congruence modulo is used.

3. Mathematical formulation of assignment problem

Let AP of m resources to m activities so as to minimize the overall cost or time in such a way that each resource can be associated with one and only one job. Its corresponding cost matrix is given by Fig. 1.

This cost matrix is same as that of TP except that availability at each of resources and requirement at each of destination is unity.

Let $X_{ij} = 1$ if i resource is assigned to j activity and $X_{ij} = 0$ otherwise.

TABLE 1

	Activity				
	a_1	a_2	...	a_m	Available
r_1	c_{11}	c_{12}	...	c_{1m}	1
r_2	c_{21}	c_{22}	...	c_{2m}	1
.	.	.		.	.
.	.	.	...	.	.
.	.	.		.	.
r_m	c_{m1}	c_{m2}	...	c_{mm}	1
Required	1	1	...	1	

Then the mathematical formulation of AP is: minimize $Z = \Sigma_{i=1}^{m}\Sigma_{j=1}^{m}C_{ij}X_{ij}$ subject to constraints, $\Sigma_{i=1}^{m}X_{ij} = 1$ and $\Sigma_{j=1}^{m}X_{ij} = 1$.

4. PROPOSED GHADLE-MUNOT ALGORITHM FOR AP

Step I: Formulate the given problem.

Step II: Calculate difference between the smallest cost and next highest cost available in each row/column, call it as penalty. If there are two smallest costs, then the penalty is zero.

Step III: Select maximum entry among that penalty, say P. Form allocation table by calculating congruence mod P of all table entries and represent it in the form $(r_s)^{(q_s)}$ where (r_s) is remainder and (q_s) is quotient mod P.

Step IV: For minimization (maximization) problem, select the row/column, which has the largest (smallest) penalty (in particular, P for the first time) and make allocation in the cell having the least (highest) cost in the selected row/column by looking at quotient $0, 1, 2, \ldots$. If quotient of two entries happens to be same then compare remainder and select minimum (maximum) among that. If two or more equal penalties exist, select one where a row/column contains minimum (maximum) unit cost.

Step V: Delete the row/column, which has been assigned.

Step VI: Repeat step IV until the entire assignment has been made.

5. Numerical example with illustration

Example 1. *Consider the following assignment problem. Assign the five jobs to five machines so as to minimize the cost.*

Table 2

	A	*B*	*C*	*D*	*E*
i	*12*	*8*	*7*	*15*	*4*
ii	*7*	*9*	*1*	*14*	*10*
iii	*9*	*6*	*12*	*6*	*7*
iv	*7*	*6*	*14*	*6*	*10*
v	*9*	*6*	*12*	*10*	*6*

Step I: *Formulate the given problem.*

Step II:

Table 3

Penalty		0	0	6	0	2
		A	B	C	D	E
3	i	12	8	7	15	4
6	ii	7	9	1	14	10
0	iii	9	6	12	6	7
0	iv	7	6	14	6	10
0	v	9	6	12	10	6

Step III: *Select maximum entry among that penalty, say P. Form allocation table by calculating congruence mod P of all table entries and represent it in the form* $(r_s)^{(q_s)}$ *where* r_s *is remainder and* q_s *is quotient mod P. Here the maximum penalty is 6. So, we calculate the table congruence modulo 6, given in Fig.3.*

Table 4

Penalty		0	0	6	0	2
		A	B	C	D	E
3	i	0^2	2^1	1^1	3^2	4^0
6	ii	1^1	3^1	1^0	2^2	4^1
0	iii	3^1	0^1	0^2	0^1	7
0	iv	1^1	0^1	2^2	0^1	4^1
0	v	3^1	0^1	0^2	4^1	0^1

Step IV: *For minimization (maximization) problem, select the row/column, which has the largest (smallest) penalty (in particular, P for first time) and make allocation in the cell having the least (highest) cost in the selected row/column by looking at quotient 0,1,2,... . If quotient of two entries happens to be same then compare remainder and select minimum (maximum) among that. If two or more equal penalties exist, select one where a row/column contains minimum (maximum) unit cost.*

TABLE 5

Penalty		0	0	6	0	2
		A	B	C	D	E
3	i	0^2	2^1	1^1	3^2	4^0
6	ii	1^1	3^1	$[1^0]$	2^2	4^1
0	iii	3^1	0^1	0^2	0^1	7
0	iv	1^1	0^1	2^2	0^1	4^1
0	v	3^1	0^1	0^2	4^1	0^1

Here, first assignment is made at second row and third column as per Step IV.

Step V: *Delete the row/column, which has been assigned.*

TABLE 6

Penalty		0	0	6	0	2
		A	B	C	D	E
3	i	0^2	2^1	1^1	3^2	4^0
6	ii	1^1	3^1	$[1^0]$	2^2	4^1
0	iii	3^1	0^1	0^2	0^1	7
0	iv	1^1	0^1	2^2	0^1	4^1
0	v	3^1	0^1	0^2	4^1	0^1

Here, delete second row and third column.

Step VI: *Repeat Step IV until the entire assignment has been made.*

TABLE 7

Penalty		0	0	6	0	2
		A	B	C	D	E
3	i	0^2	2^1	1^1	3^2	$[4^0]$
6	ii	1^1	3^1	$[1^0]$	2^2	4^1
0	iii	3^1	0^1	0^2	$[0^1]$	7
0	iv	$[1^1]$	0^1	2^2	0^1	4^1
0	v	3^1	$[0^1]$	0^2	4^1	0^1

After the assignment, Minimum Cost = 4+1+6+7+6 =24.

6. CONCLUSION

In this research article, a new method is introduced to solve AP. This approach of congruence modulo can be used for maximizing as well as minimizing objective function of AP, also it requires less iterations compared to other methods available in the literature. A numerical example has been illustrated to demonstrate the algorithm. This algorithm gives better solution also it is user friendly due to its coding in MATLAB, which gives answer in 0.012664 seconds only. Further, this concept can be extended to semi-assignment problem, BAP etc.

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