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FUZZY TOTALLY SOMEWHAT IRRESOLUTE MAPPINGS

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ABSTRACT. This article is devoted to introduce the concept of fuzzy totally somewhat irresolute mapping and fuzzy totally somewhat irresolute semiopen. Besides, some interesting properties of those mappings are given.

1. INTRODUCTION

Zadeh introduced the concept of fuzzy sets in his classical paper in [10]. Using fuzzy sets, Chang [3] first introduced the concept of fuzzy topology in 1968. The notions of fuzzy totally somewhat continuous mapping and fuzzy totally somewhat semicontinuous mapping were investigated by Vadivel and Swaminathan in [9]. In this paper we introduce the fuzzy totally somewhat irresolute and fuzzy totally somewhat irresolute semiopen mappings and since some examples and relationships between these new classes with other classes of fuzzy functions are obtained. In Section 3, the compositions of the these new mappings are given. In Section 4, fuzzy totally completely irresolute open mapping and fuzzy totally irresolte mapping are studied with some examples. Finally, in Section 5, some properties of these mappings are studied.

The known definitions which are used in this paper are seen in [1], [2], [6], [8] and [9].

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2. FUZZY TOTALLY SOMEWHAT IRRESOLUTE MAPPINGS

In this section, we introduce fuzzy totally somewhat irresolute mappings which are stronger than a fuzzy totally irresolute mappings. And we characterize a totally somewhat fuzzy irresolute mappings.

Definition 2.1. *A mapping $f : F \to \Omega$ is called fuzzy totally somewhat irresolute if there exists a fuzzy semiclopen set $\alpha \neq 0_F$ on F such that $\alpha \leq f^{-1}(\beta) \neq 0_F$ for any fuzzy semiopen set $\beta \neq 0_\Omega$ on Ω.*

It is clear that every fuzzy totally irresolute mapping is a fuzzy totally somewhat irresolute mapping. And every fuzzy totally somewhat irresolute mapping is a fuzzy totally somewhat semicontinuos mapping. But the converses are not true in general as the following examples show.

Example 1. *Let $K_1(x)$, $K_2(x)$ and $K_3(x)$ be fuzzy sets on $I = [0, 1]$ defined as follows:*

$$K_1(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ 2x - 1, & \frac{1}{2} \leq x \leq 1 \end{cases}$$

$$K_2(x) = \begin{cases} 1, & 0 \leq x \leq \frac{1}{2} \\ -4x + 2, & \frac{1}{2} \leq x \leq \frac{3}{4} \\ 0, & \frac{3}{4} \leq x \leq 1 \end{cases}$$

$$K_3(x) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{4} \\ \frac{1}{3}(4x - 1), & \frac{1}{4} \leq x \leq 1 \end{cases}$$

Let $\mathcal{J}_1 = \{0, K_1, K_2, K_1 \vee K_2, 1\}$ and $\mathcal{J}_2 = \{0, K_1 \vee K_2, 1\}$ be a fuzzy topologies on I. Let $f : (I, \mathcal{J}_1) \to (I, \mathcal{J}_2)$ be defined by $f(x) = x$ for each $x \in I$. It is observed that, for the fuzzy semiclopen set K_1 on $(I, \mathcal{J}_1)$, $K_1 \leq f^{-1}(K_1 \vee K_2) = K_1 \vee K_2$. Therefore, f is fuzzy totally somewhat irresolute mapping. But for a fuzzy semiopen set $K_1 \vee K_2$ on $(I, \mathcal{J}_2)$, $f^{-1}(K_1 \vee K_2) = K_1 \vee K_2$ which is fuzzy semiopen set but not fuzzy semiclosed set on $(I, \mathcal{J}_1)$. Hence f is not fuzzy totally irresolute mapping.

Example 2. *As described in Example 1, consider the fuzzy topology $\mathcal{J}_3 = \{0, K_1, K_2',$ $1\}$ and a mapping $f : (I, \mathcal{J}_1) \to (I, \mathcal{J}_3)$ defined by $f(x) = \frac{x}{2}$ for each $x \in I$. It is observed that, for the fuzzy semiopen sets K_1 and K_2' on $(I, \mathcal{J}_3)$, $f^{-1}(K_1) = 0$, $K_1 \leq f^{-1}(K_2') = K_1$. Therefore, f is fuzzy totally somewhat semicontinuous mapping. But for a fuzzy semiopen set K_3 on $(I, \mathcal{J}_1)$, $f^{-1}(K_3) = M(x)$ for each $x \in I$. Hence* *ther is no non-zero fuzzy semiclopen set smaller than* $f^{-1}(K_3) = M(x) = K_3 f(\frac{x}{2}) = \begin{cases} 0, & 0 \leq x \leq \frac{1}{2} \\ \frac{1}{3}(2x-1), & \frac{1}{2} \leq x \leq 1 \end{cases}$ *on* $(I, \mathcal{J}_1)$*. Hence* f *is not fuzzy totally somewhat irresolute mapping.*

Theorem 2.1. *Let* F_1 *be product related to* F_2 *and let* Ω_1 *be product related to* Ω_2*. If* $f_1 : F_1 \to \Omega_1$ *and* $f_2 : F_2 \to \Omega_2$ *are fuzzy totally somewhat irresolute mappings, then the product* $f_1 \times f_2 : F_1 \times F_2 \to \Omega_1 \times \Omega_2$ *is also fuzzy totally somewhat irresolute mapping.*

Proof. Let $\lambda = \vee_{i,j}(\alpha_i \times \beta_j)$ be a fuzzy semiopen set on $\Omega_1 \times \Omega_2$ where $\alpha_i \neq 0_{\Omega_1}$ and $\beta_j \neq 0_{\Omega_2}$ are fuzzy semiopen sets on Ω_1 and Ω_2 respectively. Then $(f_1 \times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\alpha_i) \times f_2^{-1}(\beta_j))$. Since f_1 is fuzzy totally somewhat irresolute, there exists a fuzzy semiclopen set $\sigma_i \neq 0_{F_1}$ such that $\sigma_i \leq f_1^{-1}(\alpha_i) \neq 0_{F_1}$. And, since f_2 is fuzzy totally somewhat irresolute, there exists a fuzzy semiclopen set $\gamma_j \neq 0_{F_2}$ such that $\gamma_j \leq f_2^{-1}(\beta_j) \neq 0_{F_2}$. Now $\sigma_i \times \gamma_j \leq f_1^{-1}(\alpha_i) \times f_2^{-1}(\beta_j) = (f_1 \times f_2)^{-1}(\alpha_i \times \beta_j)$ and $\sigma_i \times \gamma_j \neq 0_{F_1} \times 0_{F_2}$ is a fuzzy semiclopen set on $F_1 \times F_2$. Hence $\bigvee_{i,j}(\sigma_i \times \gamma_j) \neq 0_{F_1} \times 0_{F_2}$ is a fuzzy semiclopen set on $F_1 \times F_2$ such that $\bigvee_{i,j}(\sigma_i \times \gamma_j) \leq \bigvee_{i,j}(f_1^{-1}(\alpha_i) \times f_2^{-1}(\alpha_i)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\alpha_i \times \beta_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{F_1 \times F_2}$. Therefore, $f_1 \times f_2$ is fuzzy totally somewhat irresolute. □

Lemma 2.1. *(Azad [2]) Let* $g : F \to F \times \Omega$ *be the graph of a mapping* $f : F \to \Omega$*. If* α *is a fuzzy set in* F *and* β *is a fuzzy set in* Ω*, then* $g^{-1}(\alpha \times \beta) = \alpha \wedge f^{-1}(\beta)$*.*

Theorem 2.2. *Let* $f : F \to \Omega$ *be a mapping. If the graph* $g : F \to F \times \Omega$ *of* f *is fuzzy totally somewhat irresolute, then* f *is also fuzzy totally somewhat irresolute.*

Proof. Let $\beta \neq 0_\Omega$ be a fuzzy semiopen set on Ω. Then $f^{-1}(\beta) = 1 \wedge f^{-1}(\beta) = g^{-1}(1 \times \beta)$. Since g is fuzzy totally somewhat irresolute and $(1 \times \beta)$ is a fuzzy semiopen set on $F \times \Omega$, there exists a fuzzy semiclopen set $\alpha \neq 0_F$ on F such that $\alpha \leq g^{-1}(1 \times \beta) = f^{-1}(\beta) \neq 0_F$. Therefore, f is fuzzy totally somewhat irresolute. □

3. Compositions of fuzzy totally somewhat irresolute mappings

In this section the composition of fuzzy totally somewhat irresolute mappings with other fuzzy mappings are studied.

Theorem 3.1.

(i) *If $f : F \to \Omega$ is fuzzy totally somewhat irresolute and $g : \Omega \to Z$ is fuzzy semicontinuous, then $g \circ f : F \to Z$ is fuzzy totally somewhat semicontinuous.*
(ii) *If $f : F \to \Omega$ is fuzzy totally somewhat irresolute and $g : \Omega \to Z$ is fuzzy irresolute, then $g \circ f : F \to Z$ is fuzzy totally somewhat irresolute.*

Proof. Obvious. □

Theorem 3.2. *If $f : F \to \Omega$ is fuzzy totally irresolute and $g : \Omega \to Z$ is fuzzy totally somewhat irresolute, then $g \circ f : F \to Z$ is fuzzy totally irresolute.*

Proof. Straightforward. □

4. Fuzzy totally somewhat irresolute semiopen mappings

In this section, we introduce a fuzzy totally somewhat irresolute semiopen mappings and we discuss some of their properties.

Definition 4.1. *A mapping $f : F \to \Omega$ is called fuzzy totally irresolute semiopen if $f(\alpha)$ is a fuzzy semiclopen set on Ω for any fuzzy semiopen set α on F.*

Definition 4.2. *A mapping $f : F \to \Omega$ is called fuzzy totally somewhat irresolute semiopen if there exists a fuzzy semiclopen set $\beta \neq 0_\Omega$ on Ω such that $\beta \leq f(\alpha) \neq 0_\Omega$ for any fuzzy semiopen set $\alpha \neq 0_F$ on F.*

Theorem 4.1. *Let $f : F \to \Omega$ be a bijection. Then the following are equivalent.*

(1) *f is fuzzy totally somewhat irresolute semiopen.*
(2) *If α is a fuzzy semiclosed set on F such that $f(\alpha) \neq 1_\Omega$, then there exists a fuzzy semiclopen set $\beta \neq 1_\Omega$ on Ω such that $f(\alpha) < \beta$.*

Proof.

(1)⇒(2): Let α be a fuzzy semiclosed set on F such that $f(\alpha) \neq 1_\Omega$. Since f is bijective and α^c is a fuzzy semiopen set on F, $f(\alpha^c) = (f(\alpha))^c \neq 0_\Omega$. And, since f is fuzzy totally somewhat irresolute semiopen mapping, there exists a fuzzy semiclopen set $\sigma \neq 0_\Omega$ on Ω such that $\sigma < f(\alpha^c) = (f(\alpha))^c$. Consequently, $f(\alpha) < \sigma^c = \beta \neq 1_\Omega$ and β is a fuzzy semiclopen set on Ω .

(2)⇒(1): Let α be a fuzzy semiopen set on F such that $f(\alpha) \neq 0_\Omega$. Then α^c is a fuzzy semiclosed set on F and $f(\alpha^c) \neq 1_\Omega$. Hence there exists a fuzzy semiclopen set $\beta \neq 1_\Omega$ on Ω such that $f(\alpha^c) < \beta$. Since f is bijective, $f(\alpha^c) = (f(\alpha))^c < \beta$. Hence $\beta^c < f(\alpha)$ and $\beta^c \neq 0_F$ is a fuzzy semiclopen set on Ω. Therefore, f is fuzzy totally somewhat irresolute semiopen mapping.

□

Definition 4.3. *A fuzzy topological space (F, T) is said to be fuzzy semiconnected (Ghosh [4]) if there do not exist fuzzy semiopen sets λ and α such that $\lambda + \alpha = 1, \lambda \neq 0, \alpha \neq 0$.*

Proposition 4.1. *If f is fuzzy totally somewhat fuzzy irresolute mapping from a fuzzy semiconnected space F into any fuzzy topological space Ω, then Ω is indiscrete fuzzy topological space.*

Proof. If possible suppose Ω is not indiscrete. Then Ω has a proper ($\neq 0$ and $\neq 1$) fuzzy open set λ (say). Then by hypothesis on $f, f^{-1}(\lambda)$ is a proper fuzzy semiclopen set of F, which is a contradiction to the assumption that F is fuzzy semiconnected. Hence the proposition. □

Definition 4.4. *(Ghosh [4]) Let (F, T) be any fuzzy topological space. (F, T) is called fuzzy semi-T_2 $\Leftrightarrow$ For any pair of distinct fuzzy points x_t and y_s there exist fuzzy semiopen sets λ and α such that $x_t \in \lambda,\ y_s \in \alpha$ and $sCl\lambda \leq 1 - sCl\alpha$.*

Proposition 4.2. *Let $f : (F, T) \to (\Omega, S)$ be an injective fuzzy totally somewhat irresolute mapping. If Ω is fuzzy semi T_0, then F is fuzzy semi-T_2.*

Proof. Let x_t and y_s be any two distinct fuzzy points of F. Then $f(x_t) \neq f(y_s)$. That is $(f(x))_t \neq (f(y))_s$. Since Ω is fuzzy semi T_0, there exists a fuzzy semiopen set say $\lambda \neq 0$ in Ω such that $f(x_t) \in \lambda$ and $f(y_s) \notin \lambda$. This means $x_t \in f^{-1}(\lambda)$ and $y_s \notin f^{-1}(\lambda)$. Since f is fuzzy totally somewhat irresolute, there exists fuzzy semi-clopen set $\alpha \neq 0$ in F such that $\alpha \leq f^{-1}(\lambda)$ is fuzzy semi-clopen set of F.

Also $x_t \in f^{-1}(\lambda)$ and $y_s \in 1-f^{-1}(\lambda)$. Now put $\alpha = 1-f^{-1}(\lambda)$. Then $f^{-1}(\lambda) = sCl(\lambda)$ and $sCl(1-f^{-1}(\lambda)) = sCl\alpha = 1-f^{-1}(\lambda)$ and $sCl(\lambda) = f^{-1}(\lambda) = 1-sCl(\alpha) \leq 1-sCl(\alpha)$. Hence the Proposition. □

Proposition 4.3. *Let (F, T) be any fuzzy semiconnected space. Then every fuzzy totally somewhat irresolute mapping from a space F onto any fuzzy semi-T_0-space Ω is constant.*

Proof. Given that (F, T) is fuzzy semiconnected. Suppose $f : F \to \Omega$ be any fuzzy totally somewhat irresolute mapping and we assume that Ω is a fuzzy semi-T_0 space. Then by Proposition 4.1, Ω should be an indiscrete space. But an indiscrete fuzzy topological space containing two or more points cannot be fuzzy semi-T_0. Therefore, Ω must be singleton and this proves that f must be a constant function. □

5. Some preservation results

In this section by means of fuzzy totally somewhat irresolute and fuzzy totally somewhat semicontinuous mapping preservation of some fuzzy topological structures are discussed.

Definition 5.1. *A fuzzy topological space (F, T) is called*

(i) *fuzzy semi-compact (Malakar [5]) if every fuzzy semiopen cover has a finite subcover.*
(ii) *fuzzy s-closed (Sinha and Malakar [7]) if every fuzzy semiclopen cover has a finite subcover.*

Theorem 5.1. *Every surjective fuzzy totally somewhat irresolute image of a fuzzy s-closed space is fuzzy semi-compact.*

Proof. Let $f : F \to \Omega$ be a fuzzy totally somewhat irresolute mapping of a fuzzy s-closed space (F, T_1) onto a fuzzy space (Ω, T_2). Let $\{\beta_a : a \in A\}$ be any fuzzy semiopen cover of Ω. Since f is fuzzy totally somewhat irresolute, $\left\{f^{-1}(\beta_a) : a \in A\right\}$ is a fuzzy semiclopen cover of F. Since F is a fuzzy s-closed space, then there exists a finite subfamily $\left\{f^{-1}(\beta_{a_i}) : i = 1, ..., n\right\}$ of $\left\{f^{-1}(\beta)\right\}$ which covers F. It implies that $\{\beta_{a_i} : i = 1, ..., n\}$ is a finite subcover of $\{\beta_a : a \in A\}$ which covers Ω. Hence $f(F) = \Omega$ is fuzzy semi-compact. □

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