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SOMEWHAT PAIRWISE FUZZY δ-IRRESOLUTE CONTINUOUS MAPPINGS

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ABSTRACT. In the present article, the concepts of somewhat pairwise fuzzy δ-irresolute continuous mapping and somewhat pairwise fuzzy irresolute δ-open mapping have been introduced. Besides, some interesting properties of those mappings are discussed.

1. INTRODUCTION

The concepts of fuzzy sets were introduced by Zadeh [6]. Chang [1] studied the notion of fuzzy topology in 1968. Petricevic [2] introduced the concept of fuzzy δ-open sets and fuzzy δ-closed sets in fuzzy topological spaces. The notion of fuzzy δ-continuous functions fuzzy topological spaces was introduced by Supriti Saha [3]. The concepts of somewhat fuzzy δ-continuous functions and somewhat fuzzy δ-open functions are introduced and studied by Thangaraj and Dinakaran in [5] and consequently the concepts of somewhat fuzzy δ-irresolute continuous mappings and somewhat fuzzy irresolute δ-open mappings were introduced by Swaminathan and Balasubramaniyan in [4]. The purpose of this paper is to introduce and study the concepts of somewhat pairwise fuzzy δ-irresolute continuous mappings and somewhat pairwise fuzzy irresolute δ-open mappings on a fuzzy bitopological spaces and study some of their properties.

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2. Somewhat pairwise fuzzy δ-irresolute continuous mappings

In this section we introduce a somewhat pairwise fuzzy δ-irresolute continuous mapping.

Definition 2.1. *Let $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ be a function from fts (X, γ_1, γ_2) to another fts (Y, η_1, η_2). Then:*

(i) *f is called pairwise fuzzy δ-continuous if $f^{-1}(\beta)$ is a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set on (X, γ_1, γ_2) for any η_1-fuzzy open or η_2-fuzzy open set β on (Y, η_1, η_2).*

(ii) *f is called pairwise fuzzy δ-irresolute continuous if $f^{-1}(\beta)$ is a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set on (X, γ_1, γ_2) for any η_1-fuzzy δ-open or η_2-fuzzy δ-open set β on (Y, η_1, η_2).*

Definition 2.2. *Let $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ be a function from fts (X, γ_1, γ_2) to another fts (Y, η_1, η_2). Then:*

(i) *f is called somewhat pairwise fuzzy δ-continuous if there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\alpha \neq 0_X$ on (X, γ_1, γ_2) such that $\alpha \leq f^{-1}(\beta) \neq 0_X$ for any η_1-fuzzy open or η_2-fuzzy open set $\beta \neq 0_Y$ on (Y, η_1, η_2) .*

(ii) *f is called somewhat pairwise fuzzy δ-irresolute continuous if there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\alpha \neq 0_X$ on (X, γ_1, γ_2) such that $\alpha \leq f^{-1}(\beta) \neq 0_X$ for any η_1-fuzzy δ-open or η_2-fuzzy δ-open set $\beta \neq 0_Y$ on (Y, η_1, η_2).*

From the definitions above, it is clear that every pairwise fuzzy δ-irresolute continuous mapping is a somewhat pairwise fuzzy δ-irresolute continuous mapping. And every somewhat pairwise fuzzy δ-irresolute continuous mapping is a pairwise fuzzy δ-continuous mapping. Also, every pairwise fuzzy δ-continuous mapping is a somewhat pairwise fuzzy δ-continuous mapping from the above definition. But the converses are not true in general as the following examples show.

Example 1. *Let α_1, α_2 and α_3 be fuzzy sets on $X = \{a_1, b_1, c_1\}$ and let β_1, β_2 and β_3 be fuzzy sets on $Y = \{x_1, y_1, z_1\}$. Then $\alpha_1 = \frac{0.3}{a_1} + \frac{0.3}{b_1} + \frac{0.3}{c_1}$, $\alpha_2 = \frac{0.7}{a_1} + \frac{0.7}{b_1} + \frac{0.7}{c_1}$, $\alpha_3 = \frac{0.5}{a_1} + \frac{0.5}{b_1} + \frac{0.5}{c_1}$ and $\beta_1 = \frac{0.2}{x_1} + \frac{0.3}{y_1} + \frac{0.2}{z_1}$, $\beta_2 = \frac{0.8}{x_1} + \frac{0.7}{y_1} + \frac{0.8}{z_1}$, $\beta_3 = \frac{0.5}{x_1} + \frac{0.5}{y_1} + \frac{0.5}{z_1}$ are defined as follows: consider $\gamma_1 = \{0_X, a_1, 1_X\}$, $\gamma_2 = \{0_X, \alpha_1, \alpha_2, 1_X\}$, $\eta_1 = \{0_Y, \beta_1, 1_Y\}$, $\eta_2 = \{0_Y, \beta_1, \beta_2, \beta_3, 1_Y\}$. Then (X, γ_1, γ_2) and (Y, η_1, η_2) are fuzzy* *bitopologies and* $f : (X,\gamma_1,\gamma_2) \to (Y,\eta_1,\eta_2)$ *defined by* $f(a_1) = y_1$, $f(b_1) = y_1$, $f(c_1) = y_1$. *Then we have* $f^{-1}(\beta_1) = \alpha_1$, $\alpha_1 \le f^{-1}(\beta_2) = \alpha_2$ *and* $\alpha_1 \le f^{-1}(\beta_3) = \alpha_3$. *Since* α_1 *is a* γ_1*-fuzzy* δ*-open set on* (X,γ_1,γ_2), f *is somewhat pairwise fuzzy* δ*-irresolute continuous. But* $f^{-1}(\beta_3) = \alpha_3$ *is not a* γ_1*-fuzzy* δ*-open or* γ_2*-fuzzy* δ*-open sets on* (X,γ_1,γ_2). *Hence* f *is not a pairwise fuzzy* δ*-irresolute continuous mapping.*

Example 2. *Let* α_1 *and* α_2 *be fuzzy sets on* $X = \{a_1,b_1,c_1\}$ *and let* β_1, β_2 *and* β_3 *be fuzzy sets on* $Y = \{x_1,y_1,z_1\}$. *Then* $\alpha_1 = \frac{0.4}{a_1} + \frac{0.4}{b_1} + \frac{0.4}{c_1}$, $\alpha_2 = \frac{0.5}{a_1} + \frac{0.5}{b_1} + \frac{0.5}{c_1}$ *and* $\beta_1 = \frac{0.4}{x_1}+\frac{0.0}{y_1}+\frac{0.4}{z_1}$, $\beta_2 = \frac{0.5}{x_1}+\frac{0.0}{y_1}+\frac{0.5}{z_1}$, $\beta_3 = \frac{0.5}{x_1}+\frac{0.5}{y_1}+\frac{0.5}{z_1}$ *are defined as follows: Consider* $\gamma_1 = \{0_X,\alpha_1,1_X\}$, $\gamma_2 = \{0_X,\alpha_2,1_X\}$ *and* $\eta_1 = \{0_Y,\beta_1,1_Y\}$, $\eta_2 = \{0_Y,\beta_2,1_Y\}$. *Then* (X,γ_1,γ_2) *and* (Y,η_1,η_2) *are fuzzy bitopologies and* $f : (X,\gamma_1,\gamma_2) \to (Y,\eta_1,\eta_2)$ *defined by* $f(a_1) = y_1$, $f(b_1) = y_1$, $f(c_1) = y_1$. *Then we have* $f^{-1}(\beta_1) = 0_X$, $f^{-1}(\beta_2) = 0_X$ *and* $f^{-1}(\beta_3) = \alpha_2$ *are* γ_2*-fuzzy* δ*-open sets on* (X,γ_1,γ_2), f *is pairwise fuzzy* δ*-continuous. But for an* η_1*-fuzzy* δ*-open set* $\beta_2 \ne 0_Y$ *on* (Y,η_1,η_2), $f^{-1}(\beta_2) = 0_X$. *Hence* f *is not a somewhat pairwise fuzzy* δ*-irresolute continuous mapping.*

Example 3. *In Example 1, for an* η_2*-fuzzy* δ*-open sets on* (Y,η_1,η_2), $f^{-1}(\beta_1) = \alpha_1$, $\alpha_1 \le f^{-1}(\beta_2) = \alpha_2$ *and* $\alpha_1 \le f^{-1}(\beta_3) = \alpha_3$. *Since* α_1 *is a* γ_1*-fuzzy* δ*-open set on* (X,γ_1,γ_2), f *is somewhat pairwise fuzzy* δ*-continuous. But* $f^{-1}(\sigma_3) = \alpha_3$ *is not a* γ_1*-fuzzy or* γ_2*-fuzzy* δ*-open set on* (X,γ_1,γ_2). *Hence* f *is not a pairwise fuzzy* δ*-continuous mapping.*

Definition 2.3. *A fuzzy set* α *on a fuzzy bitopological space* (X,γ_1,γ_2) *is called pairwise* δ*-dense fuzzy set if there exists no* γ_1*-fuzzy* δ*-closed or* γ_2*-fuzzy* δ*-closed set* β *in* (X,γ_1,γ_2) *such that* $\alpha < \beta < 1$.

Theorem 2.1. *Let* $f : (X,\gamma_1,\gamma_2) \to (Y,\eta_1,\eta_2)$ *be a mapping. Then the following are equivalent:*

(1) f *is somewhat pairwise fuzzy* δ*-irresolute continuous.*
(2) *If* β *is an* η_1*-fuzzy* δ*-closed or* η_2*-fuzzy* δ*-closed set of* (Y,η_1,η_2) *such that* $f^{-1}(\beta) \ne 1_X$, *then there exists a* γ_1*-fuzzy* δ*-closed or* γ_2*-fuzzy* δ*-closed set* $\alpha \ne 1_X$ *of* (X,γ_1,γ_2) *such that* $f^{-1}(\beta) \le \alpha$.
(3) *If* α *is a pairwise* δ*-dense fuzzy set on* (X,γ_1,γ_2), *then* $f(\alpha)$ *is a pairwise* δ*-dense fuzzy set on* (Y,η_1,η_2).

Proof.

(1)$\Rightarrow$ (2): Let β be an η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set on (Y,η_1,η_2) such that $f^{-1}(\beta) \ne 1_X$. Then β^c is an η_1-fuzzy δ-open or η_2-fuzzy δ-open set on (Y,η_1,η_2) and $f^{-1}(\beta^c) = (f^{-1}(\beta))^c \neq 0_X$. Since f is somewhat pairwise fuzzy δ-irresolute continuous, there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\lambda \neq 0_X$ on (X,γ_1,γ_2) such that $\lambda \leq f^{-1}(\beta^c)$. Let $\alpha = \lambda^c$. Then $\alpha \neq 1_X$ is a γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set such that $f^{-1}(\beta) = 1 - f^{-1}(\beta^c) \leq 1 - \lambda = \lambda^c = \alpha$.

(2) $\Rightarrow$ (3): Let α be a pairwise δ-dense fuzzy set on (X,γ_1,γ_2) and suppose $f(\alpha)$ is not pairwise δ-dense fuzzy set on (Y,η_1,η_2). Then there exists an η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set β on (Y,η_1,η_2) such that $f(\alpha) < \beta < 1$. Since $\beta < 1$ and $f^{-1}(\beta) \neq 1_X$, there exists a γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set $\delta \neq 1_X$ such that $\alpha \leq f^{-1}(f(\alpha)) < f^{-1}(\beta) \leq \delta$. This contradicts the assumption that α is a pairwise δ-dense fuzzy set on (X,γ_1,γ_2). Hence $f(\alpha)$ is a pairwise δ-dense fuzzy set on (Y,η_1,η_2).

(3) $\Rightarrow$ (1): Let $\beta \neq 0_Y$ be an η_1-fuzzy δ-open or η_2-fuzzy δ-open set on (Y,η_1,η_2) and let $f^{-1}(\beta) \neq 0_X$. Suppose that there exists no γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\alpha \neq 0_X$ on (X,γ_1,γ_2) such that $\alpha \leq f^{-1}(\beta)$. Then $(f^{-1}(\beta))^c$ is a γ_1-fuzzy set or γ_2-fuzzy set on (X,γ_1,γ_2) such that there is no γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set δ on (X,γ_1,γ_2) with $(f^{-1}(\beta))^c < \delta < 1$. In fact, if there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set δ^c such that $\delta^c \leq f^{-1}(\beta)$, then it is a contradiction. So $(f^{-1}(\beta))^c$ is a pairwise δ-dense fuzzy set on (X,γ_1,γ_2). Then $f((f^{-1}(\beta))^c)$ is a pairwise δ-dense fuzzy set on (Y,η_1,η_2). But $f((f^{-1}(\beta))^c) = f(f^{-1}(\beta^c)) \neq \beta^c < 1$. This is a contradiction to the fact that $f((f^{-1}(\beta))^c)$ is pairwise δ-dense fuzzy set on (Y,η_1,η_2). Hence there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\alpha \neq 0_X$ on (X,γ_1,γ_2) such that $\alpha \leq f^{-1}(\beta)$. Consequently, f is somewhat pairwise fuzzy δ-irresolute continuous. $\square$

Theorem 2.2. *Let $(X_1,\gamma_1,\gamma_2),(X_2,\omega_1,\omega_2),(Y_1,\eta_1,\eta_2),(Y_2,\sigma_1,\sigma_2)$ be fuzzy bitopological spaces. Let (X_1,γ_1,γ_2) be product related to (X_2,ω_1,ω_2) and let (Y_1,η_1,η_2) be product related to (Y_2,σ_1,σ_2). If $f_1 : (X_1,\gamma_1,\gamma_2) \to (Y_1,\eta_1,\eta_2)$ and $f_2 : (X_2,\omega_1,\omega_2) \to (Y_2,\sigma_1,\sigma_2)$ is a somewhat pairwise fuzzy δ-irresolute continuous mappings, then the product $f_1 \times f_2 : (X_1,\gamma_1,\gamma_2) \times (X_2,\omega_1,\omega_2) \to (Y_1,\eta_1,\eta_2) \times (Y_2,\sigma_1,\sigma_2)$ is also somewhat pairwise fuzzy δ-irresolute continuous.*

Proof. Let $\lambda = \bigvee_{i,j}(\alpha_i \times \beta_j)$ be η_i-fuzzy δ-open or σ_j-fuzzy δ-open set on $(Y_1,\eta_1,\eta_2)\times(Y_2,\sigma_1,\sigma_2)$ where $\alpha_i \neq 0_{Y_1}$ is η_i-fuzzy δ-open set and $\beta_j \neq 0_{Y_2}$ is σ_j-fuzzy δ-open set on (Y_1,η_1,η_2) and (Y_2,σ_1,σ_2) respectively. Then $(f_1\times f_2)^{-1}(\lambda) = \bigvee_{i,j}(f_1^{-1}(\alpha_i)\times f_2^{-1}(\beta_j))$.

Since f_1 is somewhat pairwise fuzzy δ-irresolute continuous, there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\delta_i \neq 0_{X_1}$ such that $\delta_i \leq f_1^{-1}(\alpha_i) \neq 0_{X_1}$. And, since f_2 is somewhat pairwise fuzzy δ-irresolute continuous, there exists a ω_1-fuzzy δ-open or ω_2-fuzzy δ-open set $\alpha_j \neq 0_{X_2}$ such that $\alpha_j \leq f_2^{-1}(\beta_j) \neq 0_{X_2}$. Now $\delta_i \times \alpha_j \leq f_1^{-1}(\alpha_i) \times f_2^{-1}(\beta_j) = (f_1 \times f_2)^{-1}(\alpha_i \times \beta_j)$ and $\delta_i \times \alpha_j \neq 0_{X_1 \times X_2}$ is a δ_i-fuzzy δ-open or β_j-fuzzy δ-open set on $(X_1, \gamma_1, \gamma_2) \times (X_2, \omega_1, \omega_2)$. Hence $\bigvee_{i,j}(\delta_i \times \alpha_j) \neq 0_{X_1 \times X_2}$ is a γ_i-fuzzy δ-open or ω_j-fuzzy δ-open set on $(X_1, \gamma_1, \gamma_2) \times (X_2, \omega_1, \omega_2)$ such that $\bigvee_{i,j}(\delta_i \times \alpha_j) \leq \bigvee_{i,j}(f_1^{-1}(\alpha_i) \times f_2^{-1}(\beta_j)) = (f_1 \times f_2)^{-1}(\bigvee_{i,j}(\alpha_i \times \beta_j)) = (f_1 \times f_2)^{-1}(\lambda) \neq 0_{X_1 \times X_2}$. Therefore, $f_1 \times f_2$ is somewhat pairwise fuzzy δ-irresolute continuous. □

Theorem 2.3. *Let $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ be a mapping. If the graph $g : (X, \gamma_1, \gamma_2) \to (X, \gamma_1, \gamma_2) \times (Y, \eta_1, \eta_2)$ of f is a somewhat pairwise fuzzy δ-irresolute continuous mapping, then f is also somewhat pairwise fuzzy δ-irresolute continuous.*

Proof. Let β be an η_1-fuzzy δ-open or η_2-fuzzy δ-open set on (Y, η_1, η_2). Then $f^{-1}(\beta) = 1 \wedge f^{-1}(\beta) = g^{-1}(1 \times \beta)$. Since g is somewhat pairwise fuzzy δ-irresolute continuous and $1 \times \beta$ is a γ_i-fuzzy δ-open or η_j-fuzzy δ-open set on $(X, \gamma_1, \gamma_2) \times (Y, \eta_1, \eta_2)$, there exists a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\alpha \neq 0_X$ on (X, γ_1, γ_2) such that $\alpha \leq g^{-1}(1 \times \beta) = f^{-1}(\beta) \neq 0_X$. Therefore, f is somewhat pairwise fuzzy δ-irresolute continuous. □

3. Somewhat pairwise fuzzy irresolute δ-open mappings

In this section, we introduce a somewhat pairwise fuzzy irresolute δ-open mapping and we characterize a somewhat pairwise fuzzy irresolute δ-open mapping.

Definition 3.1. *A mapping $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ is called pairwise fuzzy δ-open if $f(\alpha)$ is an η_1-fuzzy δ-open or η_2-fuzzy δ-open set on (Y, η_1, η_2) for any γ_1-fuzzy open or γ_2-fuzzy open set α on (X, γ_1, γ_2).*

Definition 3.2. *A mapping $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy δ-open if there exists an η_1-fuzzy δ-open or η_2-fuzzy δ-open set $\beta \neq 0_Y$ on (Y, η_1, η_2) such that $\beta \leq f(\alpha) \neq 0_Y$ for any γ_1-fuzzy open or γ_2-fuzzy open set α on (X, γ_1, γ_2).*

Definition 3.3. *A mapping $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ is called somewhat pairwise fuzzy irresolute δ-open if there exists an η_1-fuzzy δ-open or η_2-fuzzy δ-open set $\beta \neq 0_Y$ on (Y, η_1, η_2) such that $\beta \leq f(\alpha) \neq 0_Y$ for any γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set $\alpha \neq 0_X$ on (X, γ_1, γ_2).*

Theorem 3.1. *Let $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ be a bijection. Then the following are equivalent:*

(1) *f is somewhat pairwise fuzzy irresolute δ-open.*
(2) *If α is a γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set on (X, γ_1, γ_2) such that $f(\alpha) \neq 1_Y$, then there exists an η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set $\beta \neq 1_Y$ on (Y, η_1, η_2) such that $f(\alpha) < \beta$.*

Proof.

(1) $\Rightarrow$ (2): Let α be a γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set on (X, γ_1, γ_2) such that $f(\alpha) \neq 1_Y$. Since f is bijective and α^c is a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set on (X, γ_1, γ_2), $f(\alpha^c) = (f(\alpha))^c \neq 0_Y$. And, since f is somewhat pairwise fuzzy irresolute δ-open mapping, there exists an η_1-fuzzy δ-open or η_2-fuzzy δ-open set $\delta \neq 0_Y$ on (Y, η_1, η_2) such that $\delta < f(\alpha^c) = (f(\alpha))^c$. Consequently, $f(\alpha) < \delta^c = \beta \neq 1_Y$ and β is an η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set on (Y, η_1, η_2).

(2) $\Rightarrow$ (1): Let α be a γ_1-fuzzy δ-open or γ_2-fuzzy δ-open set on (X, γ_1, γ_2) such that $f(\alpha) \neq 0_Y$. Then α^c is a γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set on (X, γ_1, γ_2) and $f(\alpha^c) \neq 1_Y$. Hence there exists an η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set $\beta \neq 1_Y$ on (Y, η_1, η_2) such that $f(\alpha^c) < \beta$. Since f is bijective, $f(\alpha^c) = (f(\alpha))^c < \beta$. Hence $\beta^c < f(\alpha)$ and $\beta^c \neq 0_X$ is an η_1-fuzzy δ-open or η_2-fuzzy δ-open set on (Y, η_1, η_2). Therefore, f is somewhat pairwise fuzzy irresolute δ-open. □

Theorem 3.2. *Let $f : (X, \gamma_1, \gamma_2) \to (Y, \eta_1, \eta_2)$ be a surjection. Then the following are equivalent:*

(1) *f is somewhat pairwise fuzzy irresolute δ-open.*
(2) *If β is a pairwise δ-dense fuzzy set on (Y, η_1, η_2), then $f^{-1}(\beta)$ is a pairwise δ-dense fuzzy set on (X, γ_1, γ_2).*

Proof. (1) $\Rightarrow$ (2): Let β be a pairwise δ-dense fuzzy set on (Y, η_1, η_2). Suppose $f^{-1}(\beta)$ is not pairwise δ-dense fuzzy set on (X, γ_1, γ_2). Then there exists a γ_1-fuzzy δ-closed or γ_2-fuzzy δ-closed set α on (X, γ_1, γ_2) such that $f^{-1}(\beta) < \alpha < 1$. Since f is somewhat pairwise fuzzy irresolute δ-open and α^c is a γ_1-fuzzy δ-open or

γ_2-fuzzy δ-open set on (X, γ_1, γ_2), there exists an η_1-fuzzy δ-open or η_2-fuzzy δ-open set $\delta \neq 0_Y$ on (Y, η_1, η_2) such that $\delta \leq f(Int\alpha^c) \leq f(\alpha^c)$. Since f is surjective, $\delta \leq f(\alpha^c) < f(f^{-1}(\beta^c)) = \beta^c$. Thus there exists an η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set δ^c on (Y, η_1, η_2) such that $\beta < \delta^c < 1$. This is a contradiction. Hence $f^{-1}(\beta)$ is pairwise δ-dense fuzzy set on (X, γ_1, γ_2).

(2) $\Rightarrow$ (1): Let α be a γ_1-fuzzy open or γ_2-fuzzy open set on (X, γ_1, γ_2) and $f(\alpha) \neq 0_Y$. Suppose there exists no η_1-fuzzy δ-open or η_2-fuzzy δ-open set $\beta \neq 0_Y$ on (Y, η_1, η_2) such that $\beta \leq f(\alpha)$. Then $(f(\alpha))^c$ is an η_1-fuzzy set or η_2-fuzzy set δ on (Y, η_1, η_2) such that there exists no η_1-fuzzy δ-closed or η_2-fuzzy δ-closed set δ on (Y, η_1, η_2) with $(f(\alpha))^c < \delta < 1$. This means that $(f(\alpha))^c$ is pairwise δ-dense fuzzy set on (Y, η_1, η_2) . Thus $f^{-1}((f(\alpha))^c)$ is pairwise δ-dense fuzzy set on (X, γ_1, γ_2). But $f^{-1}((f(\alpha))^c) = (f^{-1}(f(\alpha)))^c \leq \alpha^c < 1$. This is a contradiction to the fact that $f^{-1}(f(\beta))^c$ is pairwise δ-dense fuzzy set on (X, γ_1, γ_2). Hence there exists an η_1-fuzzy δ-open or η_1-fuzzy δ-open set $\beta \neq 0_Y$ on (Y, η_1, η_2) such that $\beta \leq f(\alpha)$. Therefore, f is somewhat pairwise fuzzy irresolute δ-open. □

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