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TOTAL PRIME LABELING OF SOME SPECIAL GRAPHS

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ABSTRACT. In this article, we have obtained results for the total prime labeling for some special types of derived graphs.

1. INTRODUCTION

The labeling of graph is assigning the vertices and edges real values with some particular conditions. There are several graph labelings studied by various researchers for past fifty years. Particularly, prime labeling and vertex prime labeling are most attractive. Combining these two labellings a new labeling called a total prime labeling was obtained and established by Kala et al. [1]. They have given some graphs that are total prime labeling graphs. Some constructed graphs such as wheel, gear, carona, triangular book, double comb, and planter graphs are all total prime graphs which are introduced in [2]. For more results related to total prime graphs, see [3,4].

Let $\mathcal{T}$ be a (s,t)-graph.

- A bijection $h : V(\mathcal{T}) \to \{1, 2, 3, \dots, s\}$ is said to be prime labeling if for an edge $e = ab$ the labels assigned to a and b are relatively prime.
- A graph which admits prime labeling is called prime graph.
- A bijection $h : E(\mathcal{T}) \to \{1, 2, 3, \dots, t\}$ is said to be vertex prime labeling if for each vertex of degree at least 2 the gcd of $\{h(a), h(b)\} = 1$.

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- A bijection $h : V \cup E \to \{1, 2, 3, \ldots, (s + t)\}$ is said to be total prime labeling if
  (i) for each edge $e = ab$, the labels assigned to a and b are relatively prime
  (ii) for each vertex of degree at least 2, the gcd of the labels on the incident edge is 1.
- A graph which admits total prime labeling is called total prime graph.

For more details see [3, 4].

2. Main results

In this section, we obtain the total prime labeling for several types derived graphs. An edge union of cycles of same length is called a book. This common edge is called the base of the book and r copies of cycles of length $k \geq 3$ the book is denoted by B_k^r. If $t = 3$, then it is called triangular book and if $t = 4$, then it is called rectangular book.

Theorem 2.1. *For $r \geq 1$, the rectangular book B_4^r is a total prime graph.*

Proof. Let $\mathcal{T}$ be the rectangular book graph. Let $V(B_4^r) = \{u, w_1, v_1, v_2, \ldots, v_{2r}\}$ and $E(B_4^r) = \{uw\} \cup \{wv_{2a-1} | 1 \leq a \leq r\} \cup \{v_{2a-1}v_{2a} | 1 \leq a \leq r\} \cup \{v_{2a}u | 1 \leq a \leq r\}$. The number of vertices and edges are $s = 2r + 2$ and $t = 3r + 1$, respectively. Hence $s + t = 5r + 3$.

Define a mapping $h : V \cup E \to \{1, 2, 3, \ldots, (5r + 3)\}$ by $h(u) = 1, h(w) = 2, h(v_a) = 2 + a; 1 \leq a \leq 2r$ and $h(e_a) = 2(r + 1) + a; 1 \leq a \leq 3r + 1$. According to this pattern,

(i) $gcd\{h(u), h(w)\} = 1$
(ii) $gcd\{h(w), h(v_{2a-1}) | 1 \leq a \leq r\} = 1$
(iii) $gcd\{h(v_{2a}), h(u) | 1 \leq a \leq r\} = 1$
(iv) $gcd\{$for all edges incident to $u\} = gcd\{2r+3, 2r+3+3(1), 2r+3+3(2)+ \ldots..2r + 3 + 3(2)\} = gcd\{2r + 3, 2r + 6, 2r + 9, \ldots, 5r + 3\} = 1$
(v) $gcd\{$for all edges incident to $w\} = gcd\{2r+3, 2r+4, 2r+4+3, \ldots..2r+4 + 3(r - 1)\} = gcd\{2r + 3, 2r + 4, 2r + 7, \ldots, 5r + 1\} = 1$
(vi) $gcd\{$ for all edges incident to $v_{2a-1} | 1 \leq a \leq r\} = gcd\{2r + 1 + 3a, 2r + 3a + 2 | 1 \leq a \leq r\} = 1$

(vii) $gcd\{$for all edges incident to$v_{2a}|1 \leq a \leq r\} = gcd\{2r+3a+2, 2r+3a+3|1 \leq a \leq r\} = 1$.

We conclude that for an edge $e = uv$ the gcd of $\{h(u), h(v)\} = 1$ and for each vertex with at least degree 2 the gcd of all the incident to an edge is one. Thus, the graph B_4^r is a total prime graph. □

The Dragon graph is defined by $\mathcal{T} = C_z@P_r$. Let $V(C_z) = \{v_1, v_2, \dots, v_z\}$ and $V(P_r) = \{u_1, u_2, \dots u_r\}$, where $u_1 = v_1$.

Theorem 2.2. *The Dragon graph $C_z@P_r$ is total prime graph.*

Proof. Let $V(C_z) = \{v_1, v_2, \dots, v_z\}$ and $V(P_r) = \{u_1, u_2, \dots, u_r\}$. Note that $u_1 = v_1$. Take $\mathcal{T} = C_z@P_r$. Clearly, $V(\mathcal{T}) = \{v_1, v_2, \dots, v_z, v_1 = u_1, u_2, \dots, u_r\}$ and $E(\mathcal{T}) = \{v_a v_{a+1}|1 \leq a \leq z-1\} \cup \{v_z v_1\} \cup \{u_a u_{a+1}|1 \leq a \leq r-1\}$. Since $s = z+r-1$ and $t = z+r-1$, $s+t = 2(z+r-1)$. Define a labeling $h : V \cup E \rightarrow \{1, 2, \dots, 2(z+r-1)\}$ by $h : (v_a) = a; 1 \leq a \leq z$,$h : (v_a) = h(v_1) = 1$,$h : (u_a) = z+a-1; 2 \leq a \leq r$ and $h : (e_a) = z+r+a-1; 1 \leq a \leq z+r-1$. According to this pattern,

(i) $gcd\{v_z, v_1\} = gcd\{z, 1\} = 1$
(ii) $gcd\{v_1 = u_1, u_2\} = \{1, z+1\} = 1$
(iii) $gcd\{u_a, u_{a+1}\} = gcd\{z+a-1, z+a\} = 1 for 2 \leq a \leq r-1$
(iv) $gcd\{$for all edges incident to$v_{a+1}\} = gcd\{z+r+a-1, z+r+a\} = 1 for 1 \leq a \leq z-1$
(v) $gcd\{$for all edges incident to$u_a\} = gcd\{2z+r+a-1, 2z+r+a\} = 1 for 1 \leq a \leq r-2$
(vi) $gcd\{$for all edges incident to$v_1\} = gcd\{z+r, 2z+r-1, 2z+r\} = 1$.

Thus for an edge $e = uv$, the $gcd\{h(u), h(v)\} = 1$ and for each vertices with at least degree 2 the gcd of all the incident edges is 1. Hence the dragon graph is a total prime graph. □

Crown graph C_{r*} is the graph obtained from a cycle C_r by attaching edge at each vertices of the r-cycle.

Theorem 2.3. *The Crown graph C_{r*} is a total prime graph.*

Proof. Let $\mathcal{T}$ be the crown graph and let $V(\mathcal{T}) = \{v_1, v_2, \dots, v_r, u_1, u_2, \dots, u_r\}$ and $E(\mathcal{T}) = \{v_a v_{a+1}|1 \leq a \leq r-1\} \cup \{v_r v_1\} \cup \{v_a u_a|1 \leq a \leq r\}$. The total number of vertices $s = 2r$ and the total number of edges of $t = 2r$. Hence $s + t = 4r$.

Define a labeling $h : V \cup E \to \{1, 2, \dots, 2r\}$ by $h : (v_a) = 2i - 1; 1 \leq a \leq r$,$h : (u_a) = 2a; 1 \leq a \leq r$ and $h : (e_a) = 2r + a; 1 \leq a \leq 2r$. According to this pattern,

(i) $gcd\{v_a, v_{a+1}\} = gcd\{2a - 1, 2a + 1\} = 1 for 1 \leq a \leq r - 1$
(ii) $gcd\{v_r, v_1\} = gcd\{2r - 1, 1\} = 1$
(iii) $gcd\{v_a, u_a\} = gcd\{2a - 1, 2a\} = 1 for 1 \leq a \leq r - 1$
(iv) $gcd\{$for all edges incident to$v_1\} = gcd\{2r + 1, 2r + 2, 4r\} = 1$
(v) gcd {for all edges incident to$v_a\} = gcd\{2r + 2i - 2, 2r + 2i - 1, 2r + 2i\} = 1 for 2 \leq a \leq r$.

For an edge $e = uv$, the gcd of $\{h(u), h(v)\} = 1$ and for each vertex with at least degree 2 the gcd of all the incident edges is 1. Hence the crown graph is a total prime graph. □

The joint sum of $\mathcal{T}$ is obtained from two copies of $\mathcal{T}$, connect a vertex of first copy to a vertex of second copy with a new edge. The fan graph F_r is defined by $K_1 + P_r$.

Theorem 2.4. *For $r \geq 3$, the joint sum of fan graph F_r is a total prime graph.*

Proof. Let $\mathcal{T}$ be a joint sum of a fan graph. Let $V(\mathcal{T}) = \{v, v_1, v_2, \dots v_r, u, u_1, u_2, \dots u_r\}$ and $E(\mathcal{T}) = \{vv_a | 1 \leq a \leq r\} \cup \{v_a v_{a+1} | 1 \leq a \leq r - 1\} \cup \{vu\} \cup \{uu_a | 1 \leq a \leq r\} \cup \{u_a u_{a+1} | 1 \leq a \leq r - 1\}$. The number of vertices and edges of F_r are $2r + 2$ and $4r - 1$. Hence, $s + t = 2r + 2 + 4r - 1 = 6r + 1$. Define a labeling $h : V \cup E \to \{1, 2, \dots, (6r+1)\}$ by $h : (v) = 1$,$h : (u) = 2$,$h : (v_a) = 2a+2; 1 \leq a \leq r$,$h : (u_a) = 2a+1; 1 \leq a \leq r$,$h : (e_a) = 4r+2$,$h : (e'_a) = 2r+a+2; 1 \leq a \leq 2r-1$ and $h : (e''_a) = 4r + 2 + a; 1 \leq a \leq 2r - 1$. According to this pattern

(i) $gcd\{v_1, v_a\} = gcd\{1, 2a\} = 1 for 1 \leq a \leq r$
(ii) $gcd\{v_a, v_{a+1}\} = gcd\{2a, 2a + 2\} = 1 for 1 \leq a \leq r - 1$
(iii) $gcd\{v_1, u\} = 1$
(iv) $gcd\{u, u_a\} = gcd\{2, 2a + 1\} = 1 for 1 \leq a \leq r - 1$
(v) $gcd\{$for all edges incident to$v\} = gcd\{2r+2+1, 2r+4+1, \dots 2r+2r, 2r+2r + 1, 4r + 2\} = gcd\{2r + 3, 2r + 5, \dots 4r + 1, 4r + 2\} = 1$
(vi) $gcd\{$for all edges incident to$v_1\} = gcd\{2r + 3, 2r + 4\} = 1$
(vii) $gcd\{$for all edges incident to$v_r\} = gcd\{4r, 4r + 1\} = 1$

(viii) $gcd\{$for all edges incident to$v_{a+1}\} = gcd\{2(r+1)+2a, 2(r+1)+2a+1, 2(r+1)+2a+2\} = 1 for 1 \leq a \leq r-2$

(ix) $gcd\{$for all edges incident to$u\} = gcd\{4r+2, 4r+3, 4r+5, 4r+7, \ldots, 4r+2r+1\} = gcd\{4r+2, 4r+3, \ldots (6r+1)\} = 1$

(x) $gcd\{$for all edges incident to$u_1\} = gcd\{4r+3, 4r+4\} = 1 = gcd\{4r+3, 5r\} = 1$

(xi) $gcd\{$for all edges incident to$u_r\} = gcd\{6r, 6r+1\} = 1$

(xii) $gcd\{$for all edges incident to$u_{a+1}\} = gcd\{(2r+1)+2a, 2(2r+1)+2a+1, 2(2r+1)+2a+2\} = 1 for 1 \leq a \leq r-2$.

If u and v are relatively prime and an edge $e = uv$, the gcd of each vertex of degree at least 2, all the incident edges is 1. Hence the joint sum of fan graph is a total prime graph. □

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