ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9669–9677
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.73 Spec. Iss. on ICRTAMS-2020

SOME RESULTS ON DOUBLE TWIN DOMINATION NUMBER OF A GRAPH

S. ANUTHIYA[1] AND G. MAHADEVAN

ABSTRACT. The total number of vertices that dominates every pair of vertices $SDTwin(G) = \sum DTwin(G)$ for $u, v \in V(G)$, where DTwin (u, v) is sum of number of a $u - v$ paths of length less than or equal to four. The double twin domination number of G is defined as $DTD(G) = \frac{SDTwin(G)}{\binom{n}{2}}$. In this paper, we discuss this parameter for some standard graphs, corona product graph, Fan graph and umbrella graph.

1. INTRODUCTION

In a communication network, if some vertices and lines are interrupted, its effectiveness has been lost. A network can generally be modelled through a graph. In network design a more stable model is favoured. Vulnerability attributed of a communication of network is the resistance of network to disturbance of certain vertices before communication of network breakdown. Duygu Vargor and Pinar in [1], introduced the concept of the medium domination number. dom(u,v) of G is sum of number of $u - v$ paths of length one and two of a graph G. The total number of vertices of a graph G that dominate every pair of vertices $DTV(G) = \sum dom(u, v)$ for $u, v \in V(G)$.

The medium domination number of a graph G is defined as $MD(G) = \frac{DTD(G)}{\binom{n}{2}}$. Motivated by the above definition, G. Mahadevan and Vijayalakshmi in [2–5],

[1]*Corresponding author*

2020 *Mathematics Subject Classification.* 05C69.

Key words and phrases. Medium domination number, extended medium domination number, double twin domination number.

introduced the concept of the extended medium domination number of a graph. $edom(u, v)$ of G is sum of number of $u - v$ paths of length less than or equal to three of a graph. The total number of verices of a graph G that dominate every pair of vertices $ETDV(G) = \sum edom(u, v)$ for $u, v \in V(G)$.

The extended medium domination number of a graph G is defined as $EMD(G) = \frac{ETDV(G)}{\binom{n}{2}}$. G. Mahadevan et al. already obtained the parameter double twin domination number in [6, 7].

In this paper, we initiate this parameter of double twin domination number of a graph and discuss this number for some standard and special types of graphs. We investigate the double twin domination number for some special type of graph.

The Corona product $G_1 \odot G_2$ is defined as the graph G obtained by taking one copy of G_1 of order n and n copies of G_2 and then joining the i^{th} vertex of G_1 to every vertex in the i^{th} copy of G_2. The graph $F_{(1,m)} = P_n + K_1$ is called a Fan graph. For any integer $m > 2$ and $n > 1$, an umbrella graph $U(m, n)$ is the graph obtained by appending a path P_n to the central vertex of a Fan $F_{(1,m)} = P_n + K_1$. Here vertex set $V(U(m, n)) = \{u_1, u_2, \ldots, u_m, (u = v_1), v_2, \ldots, v_n\}$ and edge set $E(U(m, n)) = \{(u_i, u_{i+1})/1 \leq i \leq m - 1\} \cup \{(u_i, v_1 = u_1)/1 \leq i \leq m\} \cup \{(v_i, v_{i+1})/1 \leq i \leq n - 1\}$. Hence, $|V(U(m, n))| = m + n$ and $|E(U(m, n))| = 2m + n - 1$.

Notation 1.

- *DTwin(G) denotes Double Twin Domination number of a graph.*
- *SDTwin(G) denotes Sum of Double Twin Domination number of a graph.*
- *DTD(G) denotes Double Twin Total Domination number of a graph.*

2. Double Twin Domination Number of Standard graph

In this section, we discuss about the concept of double twin domination number for many standard types of graphs.

Definition 2.1. *Let G=(V, E) be a graph. Let V, E be the vertex set and edge set, respectively. DTwin (u, v) is sum of number of $u - v$ path of length one, two, three and four. Let G be a graph. The total number of vertices that dominate every pair of vertices. $SDTwin(G) = \sum DTwin(u, v)$ for $u, v \in V(G)$. In any simple graph*

G of n number of vertices, the double twin domination number of G is defined as $DTD(G) = \frac{SDTwin(G)}{\binom{n}{2}}$.

Illustration 1. *For the graph 3.1,* $DTwin(1,2) = 6; DTwin(1,3) = 6;$

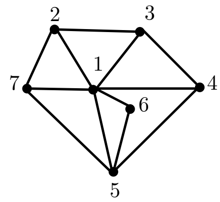

Figure 3.1

$Dom(1,4) = 6; DTwin(1,5) = 6; DTwin(1,6) = 5; DTwin(1,7) = 6;$
$DTwin(2,3) = 5; DTwin(2,4) = 6; DTwin(2,5) = 8; DTwin(2,6) = 6;$
$DTwin(2,7) = 6; DTwin(3,4) = 5; DTwin(3,5) = 6; DTwin(3,6) = 6;$
$DTwin(3,7) = 5; DTwin(4,5) = 6; DTwin(4,6) = 6; DTwin(4,7) = 6;$
$DTwin(5,6) = 4; DTwin(5,7) = 6; DTwin(6,7) = 6. SDTwin(G) = 122;$
DTD(G)$=\frac{SDTwin(G)}{\binom{n}{2}} = \frac{122}{21}$

Observation 1. *[2] For any graph (G), $EDTV(G) \leq SDTwin(G)$. For example, equality occurs in the graph $K_{1,n}$, i.e., $ETDV(K_{1,n}) = SDTwin(K_{1,n})$*

Theorem 2.1. *If $G = P_n$ for $n > 4$, then DTD(G)=* $\frac{4n-10}{\binom{n}{2}}$.

Proof. Consider the graph P_n with n vertices. Let the vertices are $\{x_1, x_2, x_3, \ldots, x_n\}$.
DTwin(x_i, x_{i+1})= 1; for $i = 1$ to n-1;Therefore $\sum_{i=1}^{n-1}$DTwin(x_i, x_{i+1}) = n-1.
DTwin(x_i, x_{i+2})= 1; for $i = 1$ to n-2;Therefore $\sum_{i=1}^{n-2}$DTwin(x_i, x_{i+2})= n-2.
DTwin(x_i, x_{i+3})= 1; for $i = 1$ to n-3;Therefore $\sum_{i=1}^{n-3}$DTwin(x_i, x_{i+3})= n-3.
DTwin(x_i, x_{i+4})= 1; for $i = 1$ to n-4;Therefore $\sum_{i=1}^{n-4}$DTwin(x_i, x_{i+4}) = n-4.
SDTwin(G)=$n - 1 + n - 2 + n - 3 + n - 4 = 4n - 10 = 2(2n - 5)$.
DTD(G)=$\frac{SDTwin(G)}{\binom{n}{2}} = \frac{2(2n-5)}{\binom{n}{2}}$. □

Illustration 2. *For the graph P_n, For the graph 3.2,* $DTwin(v_1, v_2) = 1;$

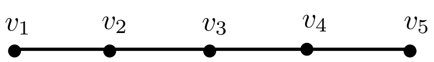

Figure 3.2 - P_5

$DTwin(v_1, v_3) = 1; \ldots$ *By considering as an illustration all possible instances, it can be checked that* $SDTwin(G) = 10; DTD(G) = \frac{4n-10}{\binom{n}{2}} = \frac{10}{10} = 1.$

Theorem 2.2. *If* $G = C_n$ *for* $n \geq 5$ *then DTD(G)* $= \frac{4n}{\binom{n}{2}}$.

Proof. Consider the graph C_n with n vertices. let the vertices are $\{x_1, x_2, x_3, \ldots, x_n\}$.

DTwin (x_i, x_{i+1})=1; for i=1 to n-1;Therefore $\sum_{i=1}^{n-1} DTwin(x_i, x_{i+1})$= n-1.

DTwin$(x_n, x_1) = 1$.

DTwin (x_i, x_{i+2})=1; for i=1 to n-2;Therefore$\sum_{i=1}^{n-2} DTwin(x_i, x_{i+2})$=n-2.

DTwin$(x_n, x_2) = 1$, DTwin$(x_{n-1}, x_1) = 1$.

DTwin (x_i, x_{i+3})=1; for i=1 to n-3;Therefore $\sum_{i=1}^{n-3} DTwin(x_i, x_{i+3})$=n-3.

DTwin$(x_n, x_3) = 1$, DTwin$(x_{n-1}, x_2) = 1$, DTwin$(x_{n-2}, x_1) = 1$.

DTwin (x_i, x_{i+4})=1; for i=1 to n-4;Therefore$\sum_{i=1}^{n-4} DTwin(x_i, x_{i+4})$=n-4.

DTwin$(x_n, x_4) = 1$, DTwin$(x_{n-1}, x_3) = 1$, DTwin$(x_{n-2}, x_2) = 1$,

DTwin$(x_{n-3}, x_1) = 1$

$SDTwin(G) = n-1+1+n-2+2+n-3+3+n-4+4 = 4n.$ $DTD(G) = \frac{4n}{\binom{n}{2}}.$ □

Illustration 3. *For the graph* C_n, *For the graph 3.3,* $DTwin(v_1, v_2) = 2;$

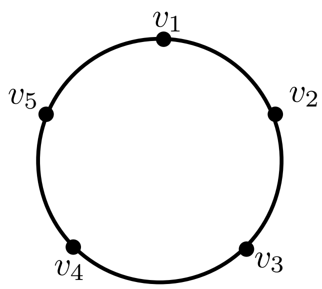

Figure 3.3 - C_5

$DTwin(v_1, v_3) = 2; \ldots$ *By considering as an illustration all possible instance, it can be checked that* $SDTwin(G) = 4n = 20; DTD(G) = \frac{20}{10} = 2.$

Theorem 2.3. *If* $G = K_n$ *for* $n > 4$*, then DTD(G)*$=2n^2 - 9n + 11$.

Proof. Consider the complete graph K_n with n vertices. Let the vertices $\{x_1, x_2, x_3, \ldots, x_n\}$. For the vertices x_1, x_2;

$k_1(x_1, x_2) = 1, k_2(x_1, x_2) = n - 2,$

$k_3(x_1, x_2) = (n - 2)(n - 3), k_4(x_1, x_2) = (n - 2)(n - 3)$

$DTwin(x_1, x_2) = 1 + (n - 2) + (n - 2)(n - 3) + (n - 2)(n - 3)$

$DTwin(x_1, x_2) = 2n^2 - 9n + 11$

Similarly, we can collect $\binom{n}{2}$ pairs of vertices. Therefore SDTwin(G)$=\binom{n}{2}(2n^2 - 9n + 11)$

DTD(G)$=\frac{SDTwin(G)}{\binom{n}{2}}=\frac{\binom{n}{2}(2n^2-9n)+11)}{\binom{n}{2}}=2n^2 - 9n + 11.$ □

Illustration 4. *For the graph K_n, For the graph 3.4,* $DTwin(v_1, v_2) = 16;$

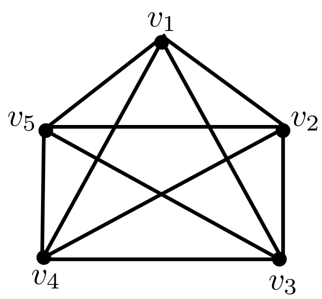

Figure 3.4 - Complete graph K_5

$DTwin(v_1, v_3) = 16; \ldots$ *By considering as an illustration all possible instance, it can be check that SDTwin(G)*$=\binom{n}{2}(2n^2 - 9n + 11) = 160;$
DTD(G)$=(2n^2 - 9n + 11) = 16.$

Theorem 2.4. *If $G = W_{1,n}$ for $n > 8$, then* $DTD(G) = \frac{13n^2-11n}{2\binom{n+1}{2}}.$

Proof. Consider the wheel graph $W_{1,n}$ with $n + 1$ vertices. Let the root vertices be a and the outer cycle vertices are $\{x_1, x_2, x_3, \ldots, x_n\}$.

DTwin$(a, x_i) = 7$; for i=1 to n; therefore $\sum_{i=1}^{n} DTwin(a, x_i) = 7n.$

DTwin$(x_i, x_{i+1}) = 7$; for i=1 to $n - 1$; therefore $\sum_{i=1}^{n-1}$ DTwin$(x_i, x_{i+1}) = 7(n - 1)$. DTwin$(x_n, x_1) = 7.$

DTwin(x_i, x_{i+2})=11; for i=1 to $n - 2$; therefore $\sum_{i=1}^{n-2} DTwin(x_i, x_{i+2}) = 11(n - 2)$,

DTwin$(x_n, x_2) = 11$; DTwin$(x_{n-1}, x_1) = 11.$

DTwin$(x_i, x_{i+3}) = 14$; for i=1 to $n - 3$; therefore $\sum_{i=1}^{n-3} DTwin(x_i, x_{i+3}) = 14(n - 3).$

DTwin$(x_n, x_3) = 14; DTwin(x_{n-1}, x_2) = 14; DTwin(x_{n-2}, x_1) = 14$

DTwin$(x_i, x_{i+4}) = 14$;for i=1 to $n - 4$; therefore $\sum_{i=1}^{n-4} DTwin(x_i, x_{i+4}) = 14(n - 4)$. DTwin$(x_n, x_4) = 14;$

DTwin$(x_{n-1}, x_3) = 14$; DTwin$(x_{n-2}, x_2) = 14$; DTwin$(x_{n-3}, x_1) = 14.$

DTwin$(x_1, x_i) = 13$; for i=6 to $n-4$; therefore $\sum_{i=6}^{n-4} DTwin(x_1, x_i) = 13(n-9)$
DTwin$(x_2, x_i) = 13$; for i=7 to $n-3$; therefore $\sum_{i=7}^{n-3} DTwin(x_2, x_i) = 13(n-9)$
DTwin$(x_3, x_i) = 13$; for i=8 to $n-2$; therefore $\sum_{i=8}^{n-2} DTwin(x_3, x_i) = 13(n-9)$
DTwin$(x_4, x_i) = 13$; for i=9 to $n-1$; therefore $\sum_{i=9}^{n-1} DTwin(x_4, x_i) = 13(n-9)$
DTwin$(x_5.x_i) = 13$; for i=10 to n ; therefore $\sum_{i=10}^{n} DTwin(x_5, x_i) = 13(n-9)$.
DTwin $(x_6, x_i) = 13$; for i = 11 to n; therefore $\sum_{i=11}^{n} DTwin(x_6, x_i) = 13(n-10) \ldots$ DTwin$(x_{n-5}, x_n) = 13$.
SDTwin(G)=$\frac{13n^2-11n}{2}$; DTD(G)=$\frac{13n^2-11n}{2\binom{n+1}{2}}$ □

3. Double Twin Domination Number for a Some Special type of graph

In this section, we discuss about the concept of double twin domination number for the some special types of graphs.

Theorem 3.1. *Let G be a graph $F_{1,n}$ where $n \geq 9$ then $DTD(G) = \frac{108n-579}{\binom{n+1}{2}}$.*

Proof. Consider the Fan graph $F_{1,n}$ with $n+1$ vertices. Let the root vertices be a and the outer vertices are $\{b_1, b_2, \ldots, b_n\}$.
DTwin$(a, b_i) = 4$; i=1, n; therefore $\sum_{i=1,n} DTwin(a, b_i) = 8$.
DTwin$(a, b_i) = 5$; i=2, $n-2$; therefore $\sum_{i=2,n-1} DTwin(a, b_i) = 10$.
DTwin$(a, b_i) = 6$; i=3, $n-2$; therefore $\sum_{i=3,n-2} DTwin(a, b_i) = 12$.
DTwin$(a, b_i) = 7$; i=4 to $n-3$; therefore $\sum_{i=4}^{n-3} DTwin(a, b_i) = 7(n-6)$.
DTwin$(b, b_{i+1}) = 4$; i=1 to $n-1$; therefore $\sum_{i=1,n-1} DTwin(b, b_{i+1}) = 8$.
DTwin$(b, b_{i+1}) = 6$; i=2, $n-2$; therefore $\sum_{i=2,n-2} DTwin(b, b_{i+1}) = 12$.
DTwin$(b, b_{i+1}) = 7$; i=3 to $n-3$; therefore $\sum_{i=3}^{n-3} DTwin(b, b_{i+1}) = 7(n-5)$.
DTwin$(b, b_{i+2}) = 7$; i=1, $n-2$; therefore $\sum_{i=1,n-2} DTwin(b, b_{i+2}) = 14$.
DTwin$(b, b_{i+2}) = 10$; i=2, n-3; therefore $\sum_{i=2,n-3} DTwin(b, b_{i+2}) = 20$.
DTwin$(b, b_{i+2}) = 11$; i=3 to $n-4$; therefore $\sum_{i=3}^{n-4} DTwin(b, b_{i+2}) = 11(n-6)$.
DTwin$(b, b_{i+3}) = 10$; i=1, n-3; therefore $\sum_{i=1,n-3} DTwin(b, b_{i+3}) = 20$.
DTwin$(b, b_{i+3}) = 13$; i=2, n-4; therefore $\sum_{i=2,n-4} DTwin(b, b_{i+3}) = 26$.
DTwin$(b, b_{i+3}) = 14$; i=3 to $n-5$; therefore $\sum_{i=3}^{n-5} DTwin(b, b_{i+3}) = 14(n-7)$.
DTwin$(b, b_{i+4}) = 10$; i=1, n-4; therefore $\sum_{i=1,n-4} DTwin(b, b_{i+4}) = 20$.
DTwin$(b, b_{i+4}) = 13$; i=2, n-5; therefore $\sum_{i=2,n-5} DTwin(b, b_{i+4}) = 26$.
DTwin$(b, b_{i+4}) = 14$; i=3 to $n-5$; therefore $\sum_{i=3}^{n-5} DTwin(b, b_{i+4}) = 14(n-8)$.
DTwin$(b_i, b_n) = 9$; i=3 to $n-5$; therefore $\sum_{i=3}^{n-5} DTwin(b_i, b_n) = 9(n-7)$.

DTwin$(b_i, b_{n-1}) = 12$; i=3 to $n-6$; therefore $\sum_{i=3}^{n-6} DTwin(b_i, b_{n-1}) = 12(n-8)$.
DTwin$(b_1, b_i) = 9$; i=6 to $n-2$; therefore $\sum_{i=6}^{n-2} DTwin(b_1, b_i) = 9(n-7)$.
DTwin$(b_1, b_{n-1}) = 8$; DTwin$(b_1, b_n) = 6$.
DTwin$(b_2, b_i) = 12$; i=7 to $n-2$; therefore $\sum_{i=7}^{n-2} DTwin(b_2, b_i) = 12(n-8)$.
DTwin$(b_2, b_{n-1}) = 11$; DTwin$(b_2, b_n) = 8$.
DTwin$(b_i, b_j) = 13$; i = 3 to $n-7$; j = 7 to $n-2$; $j-i > 4$; therefore $\sum_{i=3}^{n-6} DTwin(b_i, b_j) = 13(n-9)$.
SDTwin(G)=$108n - 579$; $DTD(G) = \frac{108n-579}{\binom{n+1}{2}}$. □

Theorem 3.2. *Let G be a graph $U(n,m)$ where $n > 2$ and $m > 6$ then DTD(G)=* $\frac{4m+117n-597}{\binom{n+m}{2}}$.

Proof. We consider the umberlla graph $U(n,m)$ with $n+m$ vertices. Let $\{a_1, a_2, \ldots, a_m\}$ be the vertices of the path P_m and $\{b_1, b_2, \ldots, b_n\}$ be the pendent vertices of the star $K_{1,n}$. Now attach the root vertex of $K_{1,n}$ to end vertex (say) a_1 of the path P_m. Now join b_i to $b{+}i+1$ for $i = 1$ to $n{-}1$. For any path P_m, SDTwin(G)= $4m - 10$.

For any Fan graph $F_{1,m}, SDTwin(F_{1,m}) = 108m - 579$.
DTwin$(a_2, b_i) = 3$; for i=1, n; therefore $\sum_{i=1,n} DTwin(a_2, b_i) = 6$
DTwin$(a_2, b_i) = 4$; for i=2, $n-1$; therefore $\sum_{i=2,n-1} DTwin(a_2, b_i) = 8$
DTwin$(a_2, b_i) = 5$; for i= 3 to $n-2$; therefore $\sum_{i=3}^{n-2} DTwin(a_2, b_i) = 5(n-4)$
DTwin$(a_3, b_i) = 2$; for i=1, n; therefore $\sum_{i=1,n} DTwin(a_3, b_i) = 4$
DTwin$(a_3, b_i) = 3$; for i= 2 to $n-1$; therefore $\sum_{i=2}^{n-1} DTwin(a_3, b_i) = 3(n-2)$
DTwin$(a_4, b_i) = 1$; for i= 1 to n; therefore $\sum_{i=1}^{n} DTwin(a_4, b_i) = n$.
$\sum_{i=1,n-2} DTwin(b, b_{i+2}) = 14$.
SDTwin(G)=$4m + 117n - 597$; $DTD(G) = \frac{4m+117-579}{\binom{n+m}{2}}$. □

4. Double twin domination number for corona product

In this section, we discuss about the concept of double twin domination number for corona product of two distict path of a graph.

Theorem 4.1. *If G is $P_n \odot P_m$, then $DTD(G) = \frac{6m^2n-7m^2+122nm-591n-24m+10}{\binom{nm}{2}}$ where* $n > 5, m > 9$.

Proof. Consider the graph $P_n \odot P_m$. Let $\{a_1, a_2, \ldots, a_n\}$ be the vertices of the path P_n. $\{a_{i1}, a_{i2}, \ldots, a_{im}\}$ be the vertices of the fan graph $F_{1,m}$, where $i = 1, 2, \ldots, n$.

Now attach the root vertex of the fan graph $F_{1,m}$ to each vertex of the path P_n say a_i. For any path graph P_n, $SDTwin(P_n) = 4n - 10$.

For any Fan graph

SDTwin(n copies of $F_{1,m}$) $= 108nm - 579n$

DTwin $(a_i, a_{(i\pm1)j}) = 3$; for i = 1 to $n-1$; $j = 1, m$; therefore $\sum_{i=1}^{n-1}(a_i, a_{(i\pm1)j}) = 12(n-1)$.

DTwin $(a_i, a_{(i\pm1)j}) = 4$; for i = 1 to $n-1$; $j = 2, m-1$; therefore $\sum_{i=1}^{n-1}(a_i, a_{(i\pm1)j}) = 16(n-1)$.

DTwin $(a_i, a_{(i\pm1)j}) = 5$; for i = 1 to $n-1$; j= 3 to $m-2$; therefore $\sum_{i=1}^{n-1}(a_i, a_{(i\pm1)j}) = 10(n-1)(m-4)$.

DTwin $(a_i, a_{(i\pm2)j}) = 2$; for i = 1 to $n-2$; $j = 1, m$; therefore $\sum_{i=1}^{n-2}(a_i, a_{(i\pm2)j}) = 8(n-2)$.

DTwin $(a_i, a_{(i\pm2)j}) = 3$; for i = 1 to $n-2$; j=2 to $m-1$; therefore $\sum_{i=1}^{n-2}(a_i, a_{(i\pm2)j}) = 6(n-2)(m-2)$.

DTwin $(a_i, a_{(i\pm3)j}) = 1$; for i = 1 to $n-3$; j=2 to m; therefore $\sum_{i=1}^{n-3}(a_i, a_{(i\pm3)j}) = 2(n-3)m$.

DTwin $(a_{ij}, a_{(i+1)k}) = 3$; for i = 1 to $n-1$; j = 1, m; k = 1,m; therefore $\sum_{i=1}^{n-1}(a_{ij}, a_{(i+1)k}) = 12(n-1)$.

DTwin $(a_{ij}, a_{(i+1)k}) = 4$; for i = 1 to $n-1$; j = 1, m; k = 2 to $m-1$; therefore $\sum_{i=1}^{n-1}(a_{ij}, a_{(i+1)k}) = 8(n-1)(m-2)$.

DTwin $(a_{ij}, a_{(i+1)k}) = 4$; for i = 1 to $n-1$; j = 2 to $m-1$; k = 1, m; therefore $\sum_{i=1}^{n-1}(a_{ij}, a_{(i+1)k}) = 8(n-1)(m-2)$.

DTwin $(a_{ij}, a_{(i+1)k}) = 5$; for i = 1 to $n-1$; j = 2 to $m-1$; k = 2 to $m-1$; therefore $\sum_{i=1}^{n-1}(a_{ij}, a_{(i+1)k}) = 5(n-1)(m-2)^2$.

DTwin $(a_{ij}, a_{(i+2)k}) = 1$; for i = 1 to $n-1$; j = 1 to m; k = 1, m; therefore $\sum_{i=1}^{n-2}(a_{ij}, a_{(i+2)k}) = (n-2)m^2$.

SDTwin(G)=$6m^2n - 7m^2 + 122nm - 591n - 24m + 10$

$DTD(G) = \frac{6m^2n-7m^2+122nm-591n-24m+10}{\binom{nm}{2}}$. □

5. CONCULUSION

In this paper, we discussed the parameter called Double twin domination number of a graph. We obtained this number for some standard graphs, fan graph, umberlla graph and corona product of two distinct path. The authors investigated this number for many product related graph and some more special types of graphs which will be reported in the sub sequent papers.

ACKNOWLEDGMENT

The research work was supported by the Research seed money for the Project proposal of the Department of Mathematics, Gandhigram Rural Institute-Deemed to be University, Gandhigram.

REFERENCES

[1] D. VARGOR, P. DUNDAR: *The medium domination number of a graph*, International Journal of Pure and Applied Mathematics, **70**(3) (2011), 297–306.

[2] G. MAHADEVAN, V. VIJAYALAKSHMI, C. SIVAGNANAM: *Extended medium domination number of a graph*, International Journal of Applied Engineering Research, **10**(92) (2015), 355–360.

[3] G. MAHADEVAN, V. VIJAYALAKSHMI, C. SIVAGNANAM: *Investigation of the medium domination number of some special types of graphs*, Aust. J. Basic and Appl. Sci., **9**(35) (2015), 126–129.

[4] G. MAHADEVAN, V. VIJAYALAKSHMI, C. SIVAGNANAM: *Medium domination number of* $J(m,n), Fn$ *and* $S`(Bk,k)$, Journal of Graph Labeling, **2**(2) (2016), 263–272.

[5] G. MAHADEVAN, V. VIJAYALAKSHMI, C. SIVAGNANAM: *Extended medium domination number of a Jahangir graph*, International Journal of Engineering and Applied Sciences, **13**(11) (2018), 8843–8846.

[6] G. MAHADEVAN, S. ANUTHIYA: *Double twin domination number of peculiar graph*, Journal of physics, Conference series, **1543** (2020), Third International Conference on Current Scenario in Pure and Applied Mathematics, 30-31 January 2020, Tamil Nadu, India.

[7] G. MAHADEVAN, S. ANUTHIYA: *Double twin domination number of special types of graph*, International Journal of Recent Technology and Engineering (IJRTE), **8**(4S5) (2019).

DEPARTMENT OF MATHEMATICS, THE GANDHIGRAM RURAL INSTITUTE - DEEMED TO BE UNIVERSITY, GANTHIGRAM-624 302, DINDUGUL DISTRICT, TAMILNADU.
Email address: anuthiya96@gmail.com

DEPARTMENT OF MATHEMATICS, THE GANDHIGRAM RURAL INSTITUTE - DEEMED TO BE UNIVERSITY, GANTHIGRAM-624 302, DINDUGUL DISTRICT, TAMILNADU.
Email address: drgmaha2014@gmail.com