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# TOTAL PRIME LABELING OF CERTAIN TYPES OF GRAPHS

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ABSTRACT. In this article, we present the several results related to the total prime labeling of certain constructed graphs.

## 1. INTRODUCTION

The labeling of graph where the vertices and edges are assigned real values with some particular conditions. There are several graph labelings studied by various researchers for past fifty years. Particularly, prime labeling and vertex prime labeling are most attractively. Combining these two labellings a new labeling called a total prime labeling were established by Kala et al. [1] and they were given some graphs that are total prime labeling graphs. Some constructed graphs such as wheel, gear, carona, triangular book, double comb, and planter graphs are all total prime graphs which are introdused in [2]. For more results related to total prime graphs, see [3,4].

Let $\mathcal{T}$ be a $(s,t)$-graph.

- A bijection $h : V(\mathcal{T}) \to \{1, 2, 3, \ldots, s\}$ is said to be prime labeling if for an edge $e = ab$ the labels assigned to $a$ and $b$ are relatively prime.
- A graph which admits prime labeling is called prime graph.
- A bijection $h : E(\mathcal{T}) \to \{1, 2, 3, \ldots, t\}$ is said to be vertex prime labeling if for each vertex of degree at least 2 the gcd of $\{h(a), h(b)\} = 1$.

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- A bijection $h : V \cup E \to \{1, 2, 3, \ldots, (s + t)\}$ is said to be total prime labeling if

  (i) for each edge $e = ab$, the labels assigned to $a$ and $b$ are relatively prime

  (ii) for each vertex of degree at least 2, the gcd of the labels on the incident edge is 1.
- A graph which admits total prime labeling is called total prime graph.

For more details see [3, 4].

## 2. MAIN RESULTS

In this section, we obtain the total prime labeling for several constructed graphs.

**Theorem 2.1.** *For all $n$, the graph $\mathcal{T}$ is obtained by attaching $S_2$ at each vertex of a star $S_r$ is a total prime graph.*

*Proof.* Let $\mathcal{T}$ be a graph obtained by attaching $S_2$ at each vertex of a star $S_r$. That is, $\mathcal{T} = S_r \circ S_2$. Let $V(\mathcal{T}) = \{u, u_1, u_2, \ldots, , u_r, v_{11}, v_{21}, ..., v_{r1}, v_{12}, v_{22}, \ldots, v_{r2}\}$ and $E(\mathcal{T}) = \{uu_a | 1 \leq a \leq r\} \cup \{v_{a1}u_a | 1 \leq a \leq r\} \cup \{u_a v_{12} | 1 \leq a \leq r\}$, where $e_a = uu_a, e_{a1} = v_{a1}u_a and e_{a2} = u_a v_{a2} for 1 \leq a \leq r$. Total number of vertices are $s = 3r + 1$ and number of edges are $t = 3r$. This implies that, $s + t = 6r + 1$. Define Labeling $h : V \cup E \to \{1, 2, \ldots, (6r + 1)\}$ by $h(u) = 1$,$h(u_i) = 3a; 1 \leq a \leq r$,$h(v_{a1}) = 3a - 1; 1 \leq a \leq r$,$h(v_{a2}) = 3a + 1; 1 \leq a \leq r$,$h(e_a) = 3r + 1 + a; 1 \leq a \leq r$,$h(e_{a1}) = 4r + 2a; 1 \leq a \leq r$ and $h(e_{a2}) = 4r + 1 + 2a; 1 \leq a \leq r$. According by the pattern

(i) $gcd\{u, u_a\} = gcd\{1, 3a\} = 1 for 1 \leq a \leq r.$
(ii) $gcd\{u_{a1}, u_a\} = gcd\{3a - 1, 3a\} = 1 for 1 \leq a \leq r.$
(iii) $gcd\{u_a, u_{a2}\} = gcd\{3a, 3a + 1\} = 1 for 1 \leq a \leq r.$
(iv) $gcd\{$for all edges incident to$u\} = gcd\{3r + 1 + 1, 3r + 1 + 2, \ldots, .3r + 1 + 4\} = gcd\{3r + 1 + 1, 3n + 1 + 2, \ldots, ..4r + 1\} = 1.$
(v) $gcd\{$for all edges incident to$u_a\} = gcd\{4r + 2a, 3r + 1 + a, 4r + 1 + 2a\} = 1 for 1 \leq a \leq r.$

Therefore, the graph $\mathcal{T}$ is a total prime graph. □

Let $r$ and $z$ be positive integers. The butterfly graph $B_{r,z}$ is obtained from two copies of the same order sharing a common vertex with an arbitrary number of pendent edges attached at the common vertices

**Theorem 2.2.** *For all $m, r > 0$, the butterfly graph $B_{r,z}$ is a total prime graph.*

*Proof.* Let $\mathcal{T}$ be a butterfly $B_{r,z}$ graph. Let $V(\mathcal{T}) = \{u, u_1, u_2, \ldots, u_{r-2}, u_{r-1}, u_{r+1} \ldots, u_{2r-1}, v_1, v_2, \ldots, .v_{2z}\}$ and $E(\mathcal{T}) = \{uu_1\} \cup \{u_a u_{a+1} | 1 \leq a \leq r-2\} \cup \{u_{r-1}u\} \cup \{uu_r\} \cup \{u_a u_{a+1} | r \leq a \leq r-2\} \cup \{u_{r-2}u\} \cup \{uv_a | 1 \leq a \leq 2z\}$. where, $e'_a = uv_a$. Hence $s = 2(r+z) - 1$ and $t = 2(r+z)$. Hence, $s + t = 4(r+z) - 1$. Define a labeling $h : V \cup E \to \{1, 2, \ldots, (4(r+z) - 1\}$ by $h(u) = 1$,$h(u_a) = 1 + a : r \leq a \leq 2(r-1)$,$h(v_a) = 2r - 1 + a; r \leq a \leq 2z$,$h(e_a) = 2(r+z) + a - 1; r \leq a \leq 2r$ and $h(e'_1) = 4r + 2z + a - 1; r \leq a \leq 2z$. According to this pattern

(i) $gcd\{u, u_1\} = gcd\{1, 2\} = 1.$
(ii) $gcd\{u_a, u_{a+1}\} = gcd\{1 + a, 2 + a\} = 1 for 1 \leq a \leq r - 2.$
(iii) $gcd\{u_{r-1}, u\} = gcd\{r, 1\} = 1.$
(iv) $gcd\{u, u_r\} = gcd 1, r + 1 = 1.$
(v) $gcd\{u_i, u_{a+1}\} = \{a, a + 1\} = 1 for r + 1 \leq a \leq 2r - 2.$
(vi) $gcd\{u_{2r-2}, u\} = gcd\{2r - 1, 1\} = 1.$
(vii) $gcd\{u, v_a\} = gcd\{1, 2r + a - 1\} = 1 for 1 \leq a \leq 2z.$
(viii) $gcd\{$for all edges incident to$u\} = gcd\{2(r+z), 2(r+z) + r - 1, 2(r+z) + r, 2(r+z) + 2r - 1, 4r + 2z, 4r + 2z + 1, \ldots, ..4r + 4z - 1\} = 1.$
(ix) $gcd\{$for all edges incident to$u_a\} = gcd\{2(r+z) + a - 1, 2(r+z) + a\} = 1 for 1 \leq a \leq r - 1.$
(x) $gcd\{$for all edges incident to$u_a\} = gcd\{2(r+z) + a, 2(r+z) + a + 1\} = 1 for r \leq a \leq 2r - 2.$

For an edge $e = uv$ the $gcd\{h(u), h(v)\} = 1$ and for each vertex at least degree 2 the gcd of all the incident edges is 1. Hence the butterfly graph is total prime graph. □

The planter graph $R_r, r \geq 3$ can be constructed by joining a fan graph $F_r$ and cycle graph $C_r$ with sharing a common vertex, when $r$ is any positive integer. That is $R_r = F_r + C_r$.

**Theorem 2.3.** *The graph $R_r@P_r$ is a total prime graph (OR). The graph $\mathcal{T}$ obtained by attaching the path $P_r$ at the center vertex of the planter graph $R_r$, $r \geq 3$ is a total prime graph.*

*Proof.* Let $\mathcal{T}$ be a graph obtained by attaching the path $P_r$ at the center vertex of $R_r$. Therefore, $V(\mathcal{T}) = \{v_1, v_2, \ldots, v_r, v_{r+1}, \ldots, v_{2r}, v_{2r+1}, \ldots, ..v_{3r-1}\}$ and $E(\mathcal{T}) = \{v_a v_{a+1} | 1 \leq a \leq r\} \cup \{v_1 v_{r+2}\} \cup \{v_{r+1+a}, v_{r+2+a} | 1 \leq a \leq r-2\} \cup \{v_{2r} v_1\} \cup \{v_1 v_{2r+1}\} \cup \{v_{2r+a} v_{2r+1+a} | 1 \leq a \leq r-2\}$. Since $s = 3r-1$ and $t = 4r-2$. Hence, $s + t = 7r - 3$.

Define a labeling $h : V \cup E \rightarrow \{1, 2, \ldots, .(7r-3)\}$ by $h; (v_a) = a : 1 \leq a \leq 3r-1$,$h : (e_a) = 3r - 1 + a : 1 \leq a \leq 4r - 2$. According to this pattern

(i) $gcd\{h(v_1), h(v_{a+1})\} = gcd\{1, 1+a\} = 1 for 1 \leq a \leq r.$
(ii) $gcd\{h(v_1), h(v_{r+2}\} = gcd\{1, r+2\} = 1.$
(iii) $gcd\{h(v_{r+1+a}), h(v_{r+2+a})\} = gcd\{r+1+a, r+2+a\} = 1 for 1 \leq a \leq r-2.$
(iv) $gcd\{h(v_{2n}), h(v_1)\} = gcd\{2a, 1\} = 1.$
(v) $gcd\{h(v_{a+1}), h(v_{a+2})\} = gcd\{a+1, a+2\} = 1 for 1 \leq a \leq r-1.$
(vi) $gcd\{h(v_1), h(v_{2r+1})\} = gcd\{1, 2r+1\} = 1.$
(vii) $gcd\{h(v_{2r+a}), h(v_{2r+1+a})\} = gcd\{2r+a, 2r+1+a\} = 1 for 1 \leq a \leq r-2.$
(viii) $gcd\{$for all edges incident to$v_1\} = gcd\{3r, 3r+2, \ldots, ..4r, 4r+2, 4r+3, \ldots, 5r-1, 5r+2, 5r+3 \ldots, 6r-1\} = 1.$
(ix) $gcd\{$for all edges incident to$v_2\} = gcd\{3r, 3r+1\} = 1.$
(x) $gcd\{$for all edges incident to$v_a\} = gcd\{2r+2a-1, 2r+2a, 2r+2a+1\} = for 3 \leq a \leq r.$
(xi) $gcd\{$ for all edges incident to$v_{r+1}\} = gcd\{4r+1, 4r+2\} = 1.$
(xii) $gcd\{$for all edges incident to$v_{r+1+a}\} = gcd\{(5r+a-2, 5r+a-1\} = 1 for 1 \leq a \leq r-1.$
(xiii) $gcd\{$for all edges incident to$v_{2r+a}\} = gcd\{5r+2+a, 5r+3+a\} = 1 for 1 \leq a \leq r-2.$

□

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