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INTERVAL-VALUED FUZZY INTERIOR IDEALS IN ORDERED SEMIRINGS

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ABSTRACT. In this article the notion of interval-valued (i-v) fuzzy interior ideals in ordered semirings (OSs) are conferred and regularity criterion is examined.

1. INTRODUCTION

Semiring is an algebraic structure which is a typical hypothesis of rings and distributive lattice. Starting late, much interest is seemed to summarize numerical structures of groups, rings, semi-rings and near-rings.

Fuzzy set hypothesis has unprecedented luxuriousness in applications than the traditional set hypothesis. Following the exposure of fuzzy sets, much thought has been paid to summarize the key thoughts of old style classical mathematics in a fuzzy framework and thus the ascent of fuzzy variable based mathematics is unavoidable. Fuzzy thought of ordered and interior idelas of an OS have been amassed in [2].

IVFSs were introduced by Zadeh [13] as an extension of fuzzy sets. For works on fuzzy semirings see [1]. The author has studied i-v fuzzy interior ideals in [8]. For more works on i-v fuzzy ideals and i-v intuitionistic fuzzy near-rings see [4–7, 9, 12].

In Section 2, we introduce IVFII in OS and study some of its related properties. We define and characterize regularity criterion in OS in terms of IVFSs in Section 3.

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2. Preliminaries

For definitions and further concepts on semi-rings and fuzzy semirings see [1, 3]. For basic definition and for further results on IVF ideals and i-v intuitionistic fuzzy near-rings see [4–7, 9–12]. From here onwards $\mathscr{O}_s$ means OS S, $\mathscr{I}_{VFII}$ means IVFII and $\mathscr{I}_{VF}$ represents IVF.

3. Interval-valued fuzzy interior ideal and regularity criterion

Definition 3.1. An IVFS $\overline{\zeta}$ of an $\mathscr{O}_s$ is called an $\mathscr{I}_{VFII}$ if for all $u, v, w \in S$, we have

(i) $\overline{\zeta}(u+v) \geq min\{\overline{\zeta}(u), \overline{\zeta}(v)\}$,
(ii) $\overline{\zeta}(uv) \geq min\{\overline{\zeta}(u), \overline{\zeta}(v)\}$,
(iii) $\overline{\zeta}(uvw) \geq \overline{\zeta}(v)$,
(iv) $u \leq v \Rightarrow \overline{\zeta}(u) \geq \overline{\zeta}(v)$.

Example 1. Let $S = \{0, \alpha, \beta, \gamma\}$ with the ordered relation $0 < \alpha < \beta < \gamma$. Define operations on S by following:

$\oplus$	0	α	β	γ
0	0	α	β	γ
α	α	α	β	γ
β	β	β	β	γ
γ	γ	γ	γ	γ

and

$\odot$	0	α	β	γ
0	0	0	0	0
α	0	α	α	α
β	0	α	α	α
γ	0	α	α	α

Then $(S, \oplus, \odot)$ forms an $\mathscr{O}_s$.
Now if we define an IVFS $\overline{\zeta}$ of $\mathscr{O}_s$ by $\overline{\zeta}(0) = \overline{1}, \overline{\zeta}(\alpha) = [0.7, 0.75], \overline{\zeta}(\beta) = [0.4, 0.45]$ and $\overline{\zeta}(\gamma) = \overline{0.2}$, then $\overline{\zeta}$ will be an $\mathscr{I}_{VFII}$ S.

Example 2. Let $S = \{0, \alpha_1, \beta_1, \gamma_1\}$ with the ordered relation $0 < \alpha_1 < \beta_1 < \gamma_1$. Define operations on S by following:

$\oplus$	0	α_1	β_1	γ_1
0	0	α_1	β_1	γ_1
α_1	α_1	α_1	β_1	γ_1
β_1	β_1	β_1	β_1	γ_1
γ_1	γ_1	γ_1	γ_1	γ_1

and

$\odot$	0	α_1	β_1	γ_1
0	0	0	0	0
α_1	0	α_1	α_1	α_1
β_1	0	α_1	β_1	β_1
γ_1	0	α_1	β_1	γ_1

Then $(S, \oplus, \odot)$ forms an $\mathscr{O}_s$ with $\alpha_1 \leq \beta_1$ if and only if $\alpha_1 \oplus \beta_1 = \beta_1$ and $\alpha_1 \odot \beta_1 = \alpha_1$. Now if we define an IVFS $\overline{\zeta}$ of S by $\overline{\zeta}(0) = \overline{1}, \overline{\zeta}(\alpha_1) = \overline{0.8}, \overline{\zeta}(\beta_1) = \overline{0.6}$ and $\overline{\zeta}(\gamma_1) = \overline{0.3}$, then $\overline{\zeta}$ will be an $\mathscr{I}_{VFII}$ of S.

Theorem 3.1. *An IVFS $\overline{\zeta}$ of $\mathscr{O}_s$ is an $\mathscr{I}_{VFII}$ if and only if its level subset $\overline{\zeta}_{\overline{t}}, \overline{t} \in D[0,1]$ is an II of $\mathscr{O}_s$*

Theorem 3.2. *Let $I(\neq \emptyset) \subseteq \mathscr{O}_s$. Then I is an II of S if and only if χ_I is an $\mathscr{I}_{VFII}$ of $\mathscr{O}_s$*

Definition 3.2. An $\mathscr{O}_s$ is called regular (resp. IR) if for all $u \in S$, we have $v, w \in S \ni u \leq uvu$ (resp. $u \leq vu^2w$).

Proposition 3.1. *Every $\mathscr{I}_{VFII}$ of a regular $\mathscr{O}_s$ is an $\mathscr{I}_{VF}$ ideal of $\mathscr{O}_s$.*

Proof. Assume that $\overline{\zeta}$ is an $\mathscr{I}_{VFII}$ of a regular $\mathscr{O}_s$ and $a, b \in S$. It is sufficient to prove that $\overline{\zeta}(ab) \geq \overline{\zeta}(a)$ and $\overline{\zeta}(ab) \geq \overline{\zeta}(b)$.
Then

$$\begin{aligned}\overline{\zeta}(ab) &\geq \overline{\zeta}((axa)b)[since\ ab \leq axab\ and\ \overline{\zeta}\ is\ an\ ordered\ ideal]\\ &= \overline{\zeta}((ax)ab) \geq \overline{\zeta}(a)[since\ \overline{\zeta}\ is\ an\ interior\ ideal].\end{aligned}$$

Likewise, for $b \in S$, there exists $y \in S \ni b \leq byb$ and hence $ab \leq abyb$. Now $\overline{\zeta}(ab) \geq \overline{\zeta}(ab(yb)) \geq \overline{\zeta}(b)$. Therefore $\overline{\zeta}$ is an $\mathscr{I}_{VFII}$ of S. □

Proposition 3.2. *Each $\mathscr{I}_{VFII}$ of an IR $\mathscr{O}_s$ is an $\mathscr{I}_{VF}$ ideal of $\mathscr{O}_s$.*

Proof. Let $\overline{\zeta}$ be an $\mathscr{I}_{VFII}$ of an IR $\mathscr{O}_s$ and $a, b \in S$. It is sufficient to prove that $\overline{\zeta}(ab) \geq \overline{\zeta}(a)$ and $\overline{\zeta}(ab) \geq \overline{\zeta}(b)$.

As S is IR, there exists $x, y \in S \ni a \leq xa^2y$ and hence we have

$$\begin{aligned}\overline{\zeta}(ab) &\geq \overline{\zeta}((xa^2yb)[since\ ab \leq xa^2yb\ and\ \overline{\zeta}\ is\ an\ ordered\ ideal]\\ &= \overline{\zeta}((xa)a(yb)) \geq \overline{\zeta}(a)[since\ \overline{\zeta}\ is\ an\ interior\ ideal].\end{aligned}$$

Similarly, one can show that $\overline{\zeta}(ab) \geq \overline{\zeta}(b)$. Hence $\overline{\zeta}$ is an $\mathscr{I}_{VFII}$ of S. □

Definition 3.3. An $\mathscr{O}_s$ is simple if it does not contain any proper ideal.

Definition 3.4. An $\mathscr{O}_s$ is called semisimple(S_s) if for $a \in S$, there exists $x, y, z \in S \ni a \leq xayaz$.

Proposition 3.3. *Let $\mathscr{O}_s$ be S_s . Then every $\mathscr{I}_{VFII}$ is an $\mathscr{I}_{VF}$ ideal of S.*

Proof. $\overline{\zeta}$ be an $\mathscr{I}_{VFII}$ of a S_s $\mathscr{O}_s$ and $a, b \in S$. It is sufficient to prove that $\overline{\zeta}(ab) \geq \overline{\zeta}(a)$ and $\overline{\zeta}(ab) \geq \overline{\zeta}(b)$.

Since $\mathscr{O}_s$ is S_s, there have $x, y, z \in S \ni a \leq xayaz$ and hence we have

$$\begin{aligned}\overline{\zeta}(ab) &\geq \overline{\zeta}((xayazb)[since\ ab \leq xayazb\ and\ \overline{\zeta}\ is\ an\ ordered\ ideal]\\ &= \overline{\zeta}((xay)a(zb)) \geq \overline{\zeta}(a)[since\ \overline{\zeta}\ is\ an\ interior\ ideal].\end{aligned}$$

Similarly, we can show that $\overline{\zeta}(ab) \geq \overline{\zeta}(b)$. Hence the result follows. □

Definition 3.5. An $\mathscr{O}_s$ is called $\mathscr{I}_{VF}$ simple if for any $\mathscr{I}_{VFII}$ $\overline{\zeta}$ of S, we have $\overline{\zeta}(a) \geq \overline{\zeta}(b)$ for all $a, b \in S$.

Theorem 3.3. *An $\mathscr{O}_s$ is simple if and only if it is $\mathscr{I}_{VF}$ simple.*

Proof. $\overline{\zeta}$ be an $\mathscr{I}_{VFII}$ of a simple $\mathscr{O}_s$ and $a, b \in S$. By Definition 3.3 and Proposition 3.2 of [8], I_a is an ideal of S. As S is simple, it follows $I_a = S$ thereby $b \in I_a$ from which it follows that $\overline{\zeta}(b) \geq \overline{\zeta}(a)$.

Conversely, suppose S contains proper ideals and let I be such an ideal of S i.e. $I \subseteq S$. We know that χ_I, is an $\mathscr{I}_{VFII}$ i of S. Indeed, let $x \in S$. Since S is $\mathscr{I}_{VFII}$ simple $\chi_I(x) \geq \chi_I(b)$ for all $b \in S$. Now let $a \in I$. Then we have $\chi_I(x) \geq \chi_I(a) = 1$ which implies $x \in I$. Consequently $S \subseteq I$ and so $S = I$.

Hence the result follows. □

Theorem 3.4. *An $\mathscr{O}_s$ is simple if and only if for every $\mathscr{I}_{VFII}$ $\overline{\zeta}$ of $S, \overline{\zeta}(a) \geq \overline{\zeta}(b)$ for all $a, b \in S$.*

Proof. Let $\overline{\zeta}$ be an $\mathscr{I}_{VFII}$ of a simple $\mathscr{O}_s$ and $a, b \in S$. Since S is simple, we have $S = (SbS]$ and as $a \in S, a \in (SbS]$. Then there exist $x, y \in S$ such that $a \leq xby$ and so $\overline{\zeta}(a) \geq \overline{\zeta}(xby) \geq \overline{\zeta}(b)$.

Conversely, suppose for every $\mathscr{I}_{VFII}$ of $\overline{\eta}$ of $S, \overline{\eta}(a) \geq \overline{\eta}(b)$ for all $a, b \in S$.

Now let $\overline{\zeta}$ be an $\mathscr{I}_{VF}$ ideal of S. Then it is $\mathscr{I}_{VFII}$ of S. So by definition S is $\mathscr{I}_{VFII}$ simple. □

We end this paper with the following characterization.

Theorem 3.5. *An $\mathscr{O}_s$ is regular if and only if for any $\mathscr{I}_{VF}$ right ideal $\overline{\zeta}$ and $\mathscr{I}_{VF}$ left ideal $\overline{\eta}$, we have $\overline{\zeta} \circ_1 \overline{\eta} = \overline{\zeta} \cap \overline{\eta}$.*

Proof. Let $x \in S$. Then there stand $a \in S \ni x \leq xax$.
Then

$$(\overline{\zeta} \circ_1 \overline{\eta})(x) = sup_{x\leq yz}\{min\{\overline{\zeta}(y), \overline{\eta}(z)\}\} \geq min_{x\leq xax}\{\overline{\zeta}(xa), \overline{\eta}(x)\} \\ \geq min\{\overline{\zeta}(x), \overline{\eta}(x)\} = (\overline{\zeta} \cap \overline{\eta})(x). \tag{3.1}$$

Now since $x \leq yz, \overline{\zeta}(x) \geq \overline{\zeta}(yz) \geq \overline{\zeta}(y)$ and $\overline{\eta}(x) \geq \overline{\eta}(yz) \geq \overline{\eta}(z)$ which implies

$$(\overline{\zeta} \cap \overline{\eta})(x) = min\{\overline{\zeta}(x), \overline{\eta}(x)\} \geq min\{\overline{\zeta}(y), \overline{\eta}(z)\}$$

and this relation is true for all representations of x. Therefore

$$(\overline{\zeta} \cap \overline{\eta}) \geq sup_{x\leq yz}\{min\{\overline{\zeta}(y), \overline{\eta}(z)\}\} \\ = (\overline{\zeta} \circ_1 \overline{\eta})(x). \tag{3.2}$$

Therefore (3.1) and (3.2) imply that $\overline{\zeta} \cap \overline{\eta} = \overline{\zeta} \circ_1 \overline{\eta}$.

The other way around, let L and R be respectively right and left ideals of S. Then χ_L and χ_R are respectively $\mathscr{I}_{VF}$ right ideal and $\mathscr{I}_{VF}$ left ideal. Moreover, $LR \subseteq L \cap R$. Let $a \in L \cap R$. Then $\chi_L(a) = 1 = \chi_R(a)$.
Thus $(\chi_L \circ_1 \chi_R)(a) = (\chi_L \cap \chi_R)(a) = min\{\chi_L(a), \chi_R(a)\} = 1$.

So, $min\{\chi_L(a_1), \chi_R(a_2)\} = 1$ for some $a_1, a_2 \in S$ satisfying $a \leq a_1a_2$ i.e. $a \in LR$.
Hence $L \cap R = LR$ and so S is regular. □

4. Conclusion

In this paper, we study some results on $\mathscr{I}_{VFII}$ over $\mathscr{O}_s$. Further we examined regularity criterion and obtain some of its characterizations.

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