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NEUTROSOPHIC IDEALS OF NEUTROSOPHIC BCH-ALGEBRAS

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ABSTRACT. In this paper, the new concept of a $nBCH$-algebra is introduced and investigated some related properties. Also, $nBCH$-ideals of a $nBCH$-algebra are studied and a few properties are obtained. Furthermore, a few results of ideals under homomorphism are discussed in $nBCH$-algebra.

1. INTRODUCTION AND PRELIMINARIES

In 1966, Imai and Iséki introduced BCK and BCI-algebras, [6, 7]. BCI-algebras are a generalization of BCK-algebras. These algebras have been extensively studied since their introduction. In 1983, Hu and Li in [4,5] introduced the notion of a BCH-algebra, which is a generalization of the notions of BCK and BCI-algebras. They have studied a few properties of these algebras. In this paper, we introduce a neutrosophic BCH-ideal of a neutrosophic BCH-algebra and investigated some related properties. Also, we study a neutrosophic homomorphism of a neutrosophic BCH-algebra and some results are obtained. All other undefined notions are from [1–4] and cited therein.

2. NEUTROSOPHIC BCH-ALGEBRA

Definition 2.1. *Let $(S, *, \Theta)$ be any $BCH\ alg$ & let $S(\mathfrak{I}) = \langle S, \mathfrak{I} \rangle$ be a set produced by S & $\mathfrak{I}$. The triple $(S(\mathfrak{I}), *, (\Theta, \Theta))$ is called a neutrosophic BCH-algebra*

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(briefly, $nBCH$ alg). If $(\varrho_1, \varrho_2\Im)$ & $(\varrho_3, \varrho_4\Im)$ are any two elements of $S(\Im)$ with $\varrho_1, \varrho_2, \varrho_3, \varrho_4 \in S$, we define

$$(\varrho_1, \varrho_2\Im) * (\varrho_3, \varrho_4\Im) = (\varrho_1 * \varrho_3, (\varrho_1 * \varrho_4 \wedge \varrho_2 * \varrho_3 \wedge \varrho_2 * \varrho_4)\Im). \tag{2.1}$$

An element $l \in S$ is represented by $(l, \Theta) \in S(\Im)$ and (Θ, Θ) represents the constant element in $S(\Im)$. $\forall$ $(l, \Theta), (m, \Theta) \in S$, we define

$$(l, \Theta) * (m, \Theta) = (l * m, \Theta) = (l \wedge \neg m, \Theta) \tag{2.2}$$

where $\neg m$ is the negation of m in S.

Example 1. *Let $(S(\Im), *, (\Theta, \Theta\Im))$ is a neutrosophic set. For all $(\varrho_1, \varrho_2\Im), (\varrho_3, \varrho_4\Im) \in S(\Im)$ in which $*$ is defined by $(\varrho_1, \varrho_2\Im) * (\varrho_3, \varrho_4\Im) = (\varrho_1, \varrho_2\Im) - (\varrho_3, \varrho_4\Im) = (\varrho_1 - \varrho_3, (\varrho_2 - \varrho_4)\Im)$. Then $(S(\Im), *, (\Theta, \Theta\Im))$ is a $nBCH$ alg.*

Theorem 2.1. *Every $nBCI$-alg $(S(\Im), *, (\Theta, \Theta))$ is a $nBCH$ alg. But not conversely.*

Example 2. *Let $S(\Im) = \{(\Theta, \Theta\Im), (\varrho_1, \varrho_2\Im), (\varrho_3, \varrho_4\Im), (\varrho_5, \varrho_6\Im)\}$ in which $*$ is defined by:*

$*$	$(\Theta, \Theta\Im)$	$(\varrho_1, \varrho_2\Im)$	$(\varrho_3, \varrho_4\Im)$	$(\varrho_5, \varrho_6\Im)$
$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$
$(\varrho_1, \varrho_2\Im)$	$(\varrho_1, \varrho_2\Im)$	$(\Theta, \Theta\Im)$	$(\varrho_5, \varrho_6\Im)$	$(\varrho_5, \varrho_6\Im)$
$(\varrho_3, \varrho_4\Im)$	$(\varrho_3, \varrho_4\Im)$	$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$	$(\varrho_3, \varrho_4\Im)$
$(\varrho_5, \varrho_6\Im)$	$(\varrho_5, \varrho_6\Im)$	$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$	$(\Theta, \Theta\Im)$

Then $(S(\Im), *, (\Theta, \Theta\Im))$ is a $nBCH$ alg but not $nBCI$-alg.

Theorem 2.2. *Every $nBCH$ alg $(S(\Im), *, (\Theta, \Theta))$ is a BCH alg but not converse.*

Proof. Suppose that $(S(\Im), *, (\Theta, \Theta))$ is a $nBCH$ alg. Let $l = (w_1, w_2\Im)$, $m = (w_3, w_4\Im)$ and $z = (w_5, w_6\Im)$ be arbitrary elements of $S(\Im)$. Then

(BCH_1) We have $l * l = (w_1, w_2\Im) * (w_1, w_2\Im) = (w_1 \wedge \neg w_1, (w_1 \wedge \neg w_2 \wedge w_2 \wedge \neg w_1 \wedge w_2 \wedge \neg w_2)\Im) = (\Theta, \Theta)$. (BCH_2) Suppose that $l * m = \Theta$ and $m * l = \Theta$. Then $(w_1, w_2\Im) * (w_3, w_4\Im) = (\Theta, \Theta)$ and $(w_3, w_4\Im) * (w_1, w_2\Im) = (\Theta, \Theta) \Rightarrow (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3 \wedge w_2 \wedge \neg w_4)\Im) = (\Theta, \Theta)$ and $(w_3 \wedge \neg w_1, (w_3 \wedge \neg w_2 \wedge w_4 \wedge \neg w_1 \wedge w_4 \wedge \neg w_2)\Im) = (\Theta, \Theta) \Rightarrow (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3)\Im) = (\Theta, \Theta)$ and $(w_3 \wedge \neg w_1, (w_3 \wedge \neg w_2 \wedge w_4 \wedge \neg w_1)\Im) = (\Theta, \Theta)$ and therefore, $w_1 \wedge \neg w_3 = \Theta$, $w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3 = \Theta$, $w_3 \wedge \neg w_1 = \Theta$ and $w_3 \wedge \neg w_2 \wedge w_4 \wedge \neg w_1 = \Theta$ from which we obtain $w_1 = w_3$ and $w_2 = w_4$. Hence $(w_1, w_2\Im) = (w_3, w_4\Im)$; that is,

$l = m$. (BCH_3) Put $LHS = (l * m) * n = ((w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I})) * (w_5, w_6\mathfrak{I})$, $RHS = (l * n) * m = ((w_1, w_2\mathfrak{I}) * (w_5, w_6\mathfrak{I})) * (w_3, w_4\mathfrak{I})$

Now, $l * m = (w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I}) = (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3 \wedge w_2 \wedge \neg w_4)\mathfrak{I}) = (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3)\mathfrak{I}) \equiv (\rho_3, \rho_4\mathfrak{I})$. LHS $\Rightarrow (l * m) * n = ((w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I})) * (w_5, w_6\mathfrak{I}) = (\rho_3, \rho_4\mathfrak{I}) * (w_5, w_6\mathfrak{I}) = (\rho_3 * w_5, (\rho_3 * w_6 \wedge \rho_4 * w_5 \wedge \rho_4 * w_6)\mathfrak{I}) = (\rho_3 \wedge \neg w_5, (\rho_3 \wedge \neg w_6 \wedge \rho_4 \wedge \neg w_5 \wedge \rho_4 \wedge \neg w_6)\mathfrak{I}) = (\rho_3 \wedge \neg w_5, (\rho_3 \wedge \neg w_6 \wedge \rho_4 \wedge \neg w_5)\mathfrak{I}) = (w_1 \wedge \neg w_3 \wedge \neg w_5, (w_1 \wedge \neg w_3 \wedge \neg w_6 \wedge w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3 \wedge \neg w_5)\mathfrak{I})$

$$= (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_3 \wedge \neg w_4 \wedge \neg w_5 \wedge \neg w_6 \wedge w_2)\mathfrak{I}). \tag{2.3}$$

Now, $l * n = (w_1, w_2\mathfrak{I}) * (w_5, w_6\mathfrak{I}) = (w_1 \wedge \neg w_5, (w_1 \wedge \neg w_6 \wedge w_2 \wedge \neg w_5 \wedge w_2 \wedge \neg w_6)\mathfrak{I}) = (w_1 \wedge \neg w_5, (w_1 \wedge \neg w_6 \wedge w_2 \wedge \neg w_5)\mathfrak{I}) \equiv (\rho_3, \rho_4\mathfrak{I})$. RHS $\Rightarrow (l * n) * m = ((w_1, w_2\mathfrak{I}) * (w_5, w_6\mathfrak{I})) * (w_3, w_4\mathfrak{I}) = (\rho_3, \rho_4\mathfrak{I}) * (w_3, w_4\mathfrak{I}) = (\rho_3 \wedge \neg w_3, (\rho_3 \wedge \neg w_4 \wedge \rho_4 \wedge \neg w_3 \wedge \rho_4 \wedge \neg w_4)\mathfrak{I}) = (\rho_3 \wedge \neg w_3, (\rho_3 \wedge \neg w_4 \wedge \rho_4 \wedge \neg w_3)\mathfrak{I}) = (w_1 \wedge \neg w_5 \wedge \neg w_3, (w_1 \wedge \neg w_5 \wedge \neg w_4 \wedge w_1 \wedge \neg w_6 \wedge w_2 \wedge \neg w_5)\mathfrak{I})$.

$$= (w_1 \wedge \neg w_3 \wedge \neg w_5, (w_1 \wedge \neg w_3 \wedge \neg w_4 \wedge \neg w_5 \wedge \neg w_6 \wedge w_2)\mathfrak{I}) \tag{2.4}$$

From (2.3) and (2.4), $LHS = RHS$.

From (BCH_1) - (BCH_4), we have $(S(\mathfrak{I}), *, (\Theta, \Theta))$ is a $BCH\ alg$. □

Example 3. *Let $(S, *, \Theta)$ is a non-empty set. For all $\varrho_1, \varrho_2 \in S$ in which $*$ is defined by $\varrho_1 * \varrho_2 = \varrho_1 - \varrho_2$. Then $(S, *, \Theta)$ is a $BCH\ alg$ but not $nBCH\ alg$.*

Lemma 2.1. *$(S(\mathfrak{I}), *, (\Theta, \Theta))$ be a $BCH\ alg$. Then $(b_1, b_2\mathfrak{I}) * (\Theta, \Theta) = (b_1, b_2\mathfrak{I})$, iff $b_1 = b_2$.*

Lemma 2.2. *Let $(S(\mathfrak{I}), *, (\Theta, \Theta))$ be a $nBCH\ alg$. Then for all $(w_1, w_2\mathfrak{I}), (w_3, w_4\mathfrak{I}) \in S(\mathfrak{I})$,*

(i) $(\Theta, \Theta) * ((w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I})) = ((\Theta, \Theta) * (w_1, w_2\mathfrak{I})) * ((\Theta, \Theta) * (w_3, w_4\mathfrak{I}))$,
(ii) $(\Theta, \Theta) * ((\Theta, \Theta) * ((w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I}))) = (\Theta, \Theta) * ((w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I}))$.

Proof.

(i) We have $(w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I}) = (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3)\mathfrak{I}) \equiv (\rho_3, \rho_4\mathfrak{I})$. LHS $\Rightarrow (\Theta, \Theta) * ((w_1, w_2\mathfrak{I}) * (w_3, w_4\mathfrak{I})) = (\Theta, \Theta) * (\rho_3, \rho_4\mathfrak{I}) = (\Theta * \rho_3, (\Theta * \rho_4 \wedge \Theta * \rho_3 \wedge \Theta * \rho_4)\mathfrak{I}) = (\rho_3, (\rho_4 \wedge \rho_3)\mathfrak{I})$

$$\equiv (\rho_3, \rho_4\mathfrak{I}). \tag{2.5}$$

Now $(\Theta, \Theta) * (w_1, w_2\mathfrak{I}) = (\Theta * w_1, (\Theta * w_2 \wedge \Theta * w_1 \wedge \Theta * w_2)\mathfrak{I}) = (w_1, (w_2 \wedge w_1)\mathfrak{I}) = (w_1, w_2\mathfrak{I})$ and $(\Theta, \Theta) * (w_3, w_4\mathfrak{I}) = (\Theta * w_3, (\Theta * w_4 \wedge \Theta * w_3 \wedge \Theta * w_4)\mathfrak{I}) =$

$(w_3, (w_4 \wedge w_3)\mathfrak{J}) \equiv (w_3, w_4\mathfrak{J})$ therefore RHS $= ((\Theta, \Theta) * (w_1, w_2\mathfrak{J})) * ((\Theta, \Theta) * (w_3, w_4\mathfrak{J})) = (w_1, w_2\mathfrak{J}) * (w_3, w_4\mathfrak{J}) = (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3 \wedge w_2 \wedge \neg w_4)\mathfrak{J}) = (w_1 \wedge \neg w_3, (w_1 \wedge \neg w_4 \wedge w_2 \wedge \neg w_3)\mathfrak{J})$

$$\equiv (\rho_3, \rho_4\mathfrak{J}). \tag{2.6}$$

Therefore from (2.5) & (2.6) $LHS = RHS$ (ii) It is similar. □

Theorem 2.3. *Let* $(S(\mathfrak{J}), *, (\Theta, \Theta))$ *be a* $nBCH\ alg$. *Then* $\forall\ (w_1, w_2\mathfrak{J}), (w_3, w_4\mathfrak{J}), (w_5, w_6\mathfrak{J}) \in S(\mathfrak{J})$. *(i)* $(w_1, w_2\mathfrak{J}) * (w_3, w_4\mathfrak{J}) = (\Theta, \Theta)$ *implies that* $((w_1, w_2\mathfrak{J}) * (w_5, w_6\mathfrak{J})) * ((w_3, w_4\mathfrak{J}) * (w_5, w_6\mathfrak{J})) = (\Theta, \Theta)$ *and* $((w_5, w_6\mathfrak{J}) * (w_3, w_4\mathfrak{J})) * ((w_5, w_6\mathfrak{J}) * (w_1, w_2\mathfrak{J})) = (\Theta, \Theta)$, *(ii)* $((w_1, w_2\mathfrak{J}) * (w_5, w_6\mathfrak{J})) * ((w_3, w_4\mathfrak{J}) * (w_5, w_6\mathfrak{J})) * ((w_1, w_2\mathfrak{J}) * (w_3, w_4\mathfrak{J})) = (\Theta, \Theta)$.

Theorem 2.4. *Let* $(S(\mathfrak{J}), *, (\Theta, \Theta))$ *be a* $nBCH\ alg$. *Then* $S(\mathfrak{J})$ *is not commutative (resp. implicative) even if* S *is commutative (resp. implicative).*

Definition 2.2. *Let* $(S(\mathfrak{J}), *, (\Theta, \Theta))$ *be a* $nBCH\ alg$. *A nonempty subset* $A(\mathfrak{J})$ *is called a neutrosophic subalgebra (briefly, nsubalg) of* $S(\mathfrak{J})$ *if*

(i) $(\Theta, \Theta) \in A(\mathfrak{J})$,
(ii) $(\varrho_1, \varrho_2\mathfrak{J}) * (\varrho_3, \varrho_4\mathfrak{J}) \in A(\mathfrak{J})$ *for all* $(\varrho_1, \varrho_2\mathfrak{J}), (\varrho_3, \varrho_4\mathfrak{J}) \in A(\mathfrak{J})$,
(iii) $A(\mathfrak{J})$ *contains a proper subset which is a* $BCH\ alg$.

If $A(\mathfrak{J})$ *does not contain a proper subset which is a* $BCH\ alg$, *then* $A(\mathfrak{J})$ *is called a pseudo nsubalg of* $S(\mathfrak{J})$.

Example 4. *In Example 1, then* $A(\mathfrak{J}) = \{(\Theta, \Theta\mathfrak{J}), (\varrho_1, \varrho_2\mathfrak{J}), (\varrho_3, \varrho_4\mathfrak{J})\}$ *is a* $nBCH$ *subalg of* $S(\mathfrak{J})$.

Theorem 2.5. *Let* $(S(\mathfrak{J}), *, (\Theta, \Theta))$ *be a* $nBCH\ alg$ *and for* $d_1 \neq \Theta$ *let* $D_{(d_1, d_1\mathfrak{J})}(\mathfrak{J})$ *be a subset of* $S(\mathfrak{J})$ *defined by* $D_{(d_1, d_1\mathfrak{J})}(\mathfrak{J}) = \{(l, m\mathfrak{J}) \in S(\mathfrak{J}) : (l, m\mathfrak{J}) * (d_1, d_1\mathfrak{J}) = (\Theta, \Theta)\}$. *Then,*

(i) $D_{(d_1, d_1\mathfrak{J})}(\mathfrak{J})$ *is a nsubalg of* $S(\mathfrak{J})$.
(ii) $D_{(d_1, d_1\mathfrak{J})}(\mathfrak{J}) \subseteq D_{(\Theta, \Theta)}(\mathfrak{J})$.

Theorem 2.6. *Let* $(S(\mathfrak{J}), *, (\Theta, \Theta))$ *be a* $nBCH\ alg$ & $S_T(\mathfrak{J})$ *be a subset of* $S(\mathfrak{J})$ *defined by* $S_T(\mathfrak{J}) = \{(l, l\mathfrak{J}) : l \in S\}$. *Then* $S_T(\mathfrak{J})$ *is a nsubalg of* $S(\mathfrak{J})$.

Remark 2.1. *Since* $(S_T(\mathfrak{J}), *, (\Theta, \Theta))$ *is a nsubalg, then* $S_T(\mathfrak{J})$ *is a* n *commutative* BCH *alg in its own right.*

Definition 2.3. *Let $(S(\mathfrak{I}), *, (\Theta, \Theta))$ be a $nBCH\ alg$. A non-empty subset $B(\mathfrak{I})$ is called a neutrosophic ideal (briefly, nI) of $S(\mathfrak{I})$ if*

(i) $(\Theta, \Theta) \in B(\mathfrak{I})$,

(ii) *If* $(b_1, b_2\mathfrak{I}) * (b_3, b_4\mathfrak{I}) \in B(\mathfrak{I})$ & $(b_1, b_2\mathfrak{I}) \in B(\mathfrak{I}) \implies (b_3, b_4\mathfrak{I}) \in B(\mathfrak{I})$, $\forall\ (b_1, b_2\mathfrak{I}), (b_3, b_4\mathfrak{I}) \in B(\mathfrak{I})$.

Definition 2.4. *A non-empty subset $B_T(\mathfrak{I})$ is called a neutrosophic BCH-ideal (briefly, $nBCHI$) of $S_T(\mathfrak{I})$ if ($\mathfrak{I}1$) $(\Theta, \Theta) \in B_T(\mathfrak{I})$, ($\mathfrak{I}2$) If $(l, l\mathfrak{I}) * ((m, m\mathfrak{I}) * (n, n\mathfrak{I})) \in b_T(\mathfrak{I})$ and $(m, m\mathfrak{I}) \in B_T(\mathfrak{I})$ implies $(l, l\mathfrak{I}) * (n, n\mathfrak{I}) \in B_T(\mathfrak{I})$, for all $(l, l\mathfrak{I}),\ (m, m\mathfrak{I}),\ (l, l\mathfrak{I}) \in B_T(\mathfrak{I})$.*

Theorem 2.7. *Every $nBCHI$ of $S_T(\mathfrak{I})$ is a nI of $S_T(\mathfrak{I})$.*

Definition 2.5. *Let $(S(\mathfrak{I}), *, (\Theta, \Theta))$ & $(S'(\mathfrak{I}), \circ, (\Theta', \Theta'))$ be two $nBCH\ alg$'s. A mapping $\varrho : S(\mathfrak{I}) \to S'(\mathfrak{I})$ is called a neutrosophic homomorphism (briefly, $nhom$) if:*

(i) $\varrho((b_1, b_2\mathfrak{I}) * (b_3, b_4\mathfrak{I})) = \varrho((b_1, b_2\mathfrak{I})) \circ \varrho((b_3, b_4\mathfrak{I})), \forall (b_1, b_2\mathfrak{I})(b_3, b_4\mathfrak{I}) \in S(\mathfrak{I})$,

(ii) $\varrho((\Theta, \mathfrak{I})) = (\Theta, \mathfrak{I})$.

(iii) *if ϱ is injective (resp. surjective & bijection), then ϱ is called a neutrosophic monomorphism (resp. epimorphism & isomorphism) (briefly, $nmonomor$ (resp. $nepimor$ & $nisomor$)).*

A bijective $nhom$ from $S(\mathfrak{I})$ onto $S(\mathfrak{I})$ is called a neutrosophic automorphism (briefly, $nautomor$).

Definition 2.6. *Let $\varrho : S(\mathfrak{I}) \to S'(\mathfrak{I})$ be a $nhom$ of $nBCH\ alg$'s. Then*

(i) $Ker\ \varrho = \{(b_1, b_2\mathfrak{I}) \in S(\mathfrak{I}) : \varrho((b_1, b_2\mathfrak{I})) = (\Theta, \Theta)\}$.

(ii) $\mathfrak{I}m\ \varrho = \{\varrho((b_1, b_2\mathfrak{I})) \in S'(\mathfrak{I}) : (b_1, b_2\mathfrak{I}) \in S(\mathfrak{I})\}$.

Example 5. *In Example 1, let $(S(\mathfrak{I}), *, (\Theta, \Theta))$ be a $nBCH\ alg$ and $\varrho : S(\mathfrak{I}) \to S(\mathfrak{I})$ be a mapping defined by $\varrho(b_1, b_2\mathfrak{I}) = (b_1, b_2\mathfrak{I})\ \forall (b_1, b_2\mathfrak{I}) \in S(\mathfrak{I})$. Then ϱ is a $nBCH$ isomor.*

Theorem 2.8. *Let $\varrho : S(\mathfrak{I}) \to S'(\mathfrak{I})$ be a $nhom$ of $nBCH\ alg$'s. Then,*

(i) *If $(\Theta, \Theta\mathfrak{I})$ is the identity in $S(\mathfrak{I})$, then $\varrho(\Theta, \Theta\mathfrak{I})$ is the identity in $S'(\mathfrak{I})$,*

(ii) *If S is a nsubalg of $S(\mathfrak{I})$, then $\varrho(S)$ is a nsubalg of $S'(\mathfrak{I})$,*

(iii) *If S is a nsubalg of $S'(\mathfrak{I})$, then $\varrho^{-1}(S)$ is a nsubalg of $S(\mathfrak{I})$.*

Theorem 2.9. *Let $\varrho : S(\Im) \to S'(\Im)$ be a $nhom$ from a $nBCH\ alg$ $S(\Im)$ into a $nBCH\ alg$ $S'(\Im)$. Then the kernel ϱ is a $nBCHI$ of $S(\Im)$.*

Proof. Since $\varrho(\Theta, \Theta\Im) = (\Theta', \Theta'\Im)$, then $(\Theta, \Theta\Im) \in Ker\ \varrho$. Let $(b_1, b_2\Im) * ((b_3, b_4\Im) * (\rho_1, \rho_2\Im)) \in Ker\ \varrho$ and $(b_3, b_4\Im) \in Ker\ \varrho$, then $\varrho((b_1, b_2\Im) * ((b_3, b_4\Im) * (\rho_1, \rho_2\Im))) = (\Theta', \Theta'\Im)$ and $\varrho(b_3, b_4\Im) = (\Theta', \Theta'\Im)$, since $(\Theta', \Theta'\Im) = \varrho((b_1, b_2\Im) * ((b_3, b_4\Im) * (\rho_1, \rho_2\Im))) = \varrho(b_1, b_2\Im) * \varrho((b_3, b_4\Im) * (\rho_1, \rho_2\Im)) = \varrho(b_1, b_2\Im) * (\varrho(b_3, b_4\Im) * \varrho(\rho_1, \rho_2\Im)) = \varrho(b_3, b_4\Im) * (\varrho(b_1, b_2\Im) * \varrho(\rho_1, \rho_2\Im)) = (\Theta', \Theta'\Im) * (\varrho(b_1, b_2\Im) * \varrho(\rho_1, \rho_2\Im)) = (\varrho(b_1, b_2\Im) * \varrho(\rho_1, \rho_2\Im)) = \varrho((b_1, b_2\Im) * (\rho_1, \rho_2\Im))$ We get $((b_1, b_2\Im) * (\rho_1, \rho_2\Im)) \in Ker\ \varrho$, so $Ker\ \varrho$ is $nBCHI$ of $S(\Im)$. □

Lemma 2.3. *Let $\varrho : S(\Im) \to S'(\Im)$ be a $nhom$ from a $nBCH\ alg$ $S(\Im)$ into a $nBCH\ alg$ $S'(\Im)$. Then $\varrho((\Theta, \Theta)) = (\Theta', \Theta')$.*

Theorem 2.10. *Let $\varrho : S(\Im) \to S'(\Im)$ be a $nhom$ of $nBCH\ alg$'s. Then ϱ is a $nmonomor$ iff $Ker\ \varrho = \{(\Theta, \Theta)\}$.*

Theorem 2.11. *Let $S(\Im), S'(\Im)$ & $S''(\Im)$ be $nBCH\ alg$. Let $\varrho : S(\Im) \to S'(\Im)$ be a $nepimor$ and let $\varpi : S(\Im) \to S''(\Im)$ be a (a) $nhom$. If $Ker\ \varrho \subseteq Ker\ \varpi$, then $\exists$ a unique $nhom$ $\nu : S'(\Im) \to S''(\Im) \ni \nu\varrho = \varpi$. Then (i) $Ker\ \nu = \varrho(Ker\ \varpi)$, (ii) $\Im m\ \nu = Im\ \varpi$, (iii) ν is a $nmonomor$ iff $Ker\ \varrho = Ker\ \varpi$, (iv) ν is a $nepimor$ iff ϖ is a $nepimor$. (b) $nhom$ & let $\varpi : S'(\Im) \to S''(\Im)$ be a $nmonomor \ni \Im m\ \varrho \subseteq Im\ \varpi$. Then $\exists$ a unique $nhom$ $\mu : S(\Im) \to S'(\Im) \ni \varrho = \varpi\mu$. Also, (i)$Ker\ \mu = Ker\ \varrho$, (ii) $\Im m\ \mu = \varpi^{-1}(\Im m\ \varrho)$, (iii) μ is a $nmonomor$ iff ϱ is a $nmonomor$, (iv) μ is a $nepimor$ iff $\Im m\ \varpi = Im\ \varrho$.*

REFERENCES

[1] M. A. CHAUDHRY: *On BCH-algebras*, Math. Japon., **36**(4) (1991), 665–676.

[2] W. A. DUDEK, J. THOMYS: *On decompositions of BCH-algebras*, Math. Japon., **35**(6) (1990), 1131–1138.

[3] B. DAVVAZ, S. M. MOSTAFA, F. F. KAREEM: *Neutrosophic ideals of neutrosophic KU-algebras*, GU J Sci., **30**(4) (2017), 463–472.

[4] Q. P. HU, X. LI: *On BCH-algebras*, Math. Sem. Notes., **11** (1983), 313–320.

[5] Q. P. HU, X. LI: *On proper BCH-algebras*, Math. Japonica, **30**(4) (1985), 659–661.

[6] Y. IMAI, K. ISEKI: *On axioms systems of propositional calculi XIV*, Proc. Japan Academy, **42** (1966), 19–22.

[7] K. ISEKI: *An algebra related with a propositional calculus*, Proc. Japan Academy, **42** (1966), 26–29.

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