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RADIO ANTIPODAL MEAN LABELING OF TRIANGULAR SNAKE FAMILIES

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ABSTRACT. Let $G = (V, E)$ be a graph with vertex set V, edge set E and $diam(G)$ be the diameter of G. Let $u, v \in V(G)$. The *radio antipodal mean labeling* of graph G is a function f that assigns to each vertex u, a non-negative integer $f(u)$ such that $f(u) \neq f(v)$ if $d(u,v) < diam(G)$ and $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq diam(G)$, where $d(u,v)$ represents the shortest distance between any pair of vertices u and v of G. The antipodal mean number of f, denoted by $r_{amn}(f)$ is the maximum number assigned to any vertex of G and is denoted by $amn(f)$. The antipodal mean number of G, denoted by $r_{amn}(G)$ is the minimum value of $r_{amn}(f)$ taken over all antipodal mean labeling f of G. In this paper, we have obtained the upper bounds of radio antipodal mean number of triangular snake families.

1. INTRODUCTION

The graphs considered in this paper are simple, finite and undirected graphs. The process of assigning non negative integer to the vertices, edges or to both in a graph G, subject to certain conditions is known as *graph labeling* [6]. The technique of assigning channels (frequencies) to radio transmitters is popularly known as *channel assignment problem* which was introduced by William Hale in 1980, [8]. This channel assignment problem can be converted into a graph

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theoretical problem by representing the radio transmitters by vertices and connecting the adjacent transmitters by edges. In graph theoretic approach, each of these vertices will be assigned different labels (non negative integer) or colors such that the adjacent vertices receives different colors (or labels), [4]. This motivated Gary Chartrand et al. in [2] to introduce a graph labeling technique called as *radio labeling*. An *radio labeling* of a graph G is a function $f : V(G) \rightarrow N$ such that, $d(u,v)+ \mid f(u) - f(v) \mid \geq 1 + diam(G)$, where $d(u,v)$ represents the shortest distance between the vertices u and v and $diam(G)$ is the diameter of G. M Kchikech et al. in [5] proved that the problem of finding the radio number of an arbitrary network is an NP complete problem.

In 2002, Gary Chartrand et al. in [3] introduced a new graph labeling technique called *radio antipodal labeling* by modifying the existing radio labeling definition.The radio antipodal labeling of a graph G is a function $f : V(G) \rightarrow N$ such that $d(u,v)+ \mid f(u)-f(v) \mid \geq diam(G)$. The benefit of using radio antipodal labeling was based on the concept of reusing the same frequency for transmitters which are at diametric distance. That is a radio labeling is a one to one mapping whereas in an antipodal labeling, two vertices which are at diametric distance can receive the same label.

In 2018, D Antony Xavier and Thivyarathi in [1] modified the condition of radio antipodal labeling and introduced a new graph labeling called *radio antipodal mean labeling*. In their work, they have studied the antipodal mean number of path, wheel, cycle, mesh and its derived architectures. Kins Yenoke et al. in [6] have investigated the radio antipodal mean number of Mongolian tent, triangular ladder and pagoda graph.

In this paper, the upper bounds of radio antipodal mean number of triangular snake families has been discussed.

2. Preliminaries

In this section, we give the basic definitions needed for our main results.

Definition 2.1. *[1] The radio antipodal mean labeling of graph G is a function f that assigns to each vertex u, a non-negative integer $f(u)$ such that $f(u) \neq f(v)$ if $d(u,v) < diam(G)$ and $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq diam(G)$, where $d(u,v)$ represents the shortest distance between any pair of vertices u and v of G.*

Definition 2.2. *[7] The triangular snake T_n is obtained from a path $u_1, u_2, \ldots, u_{n+1}$ by replacing every edge of a path by a triangle C_3. That is T_n is constructed by placing the vertices $v_1, v_2, \ldots, v_n$ above $u_1, u_2, \ldots, u_{n+1}$ so that u_i, v_i and $u_{i+1}, 1 \leq i \leq n$ forms a triangle. T_n has $2n+1$ vertices. Its diameter is n.*

Definition 2.3. *[7] An alternate triangular snake $A(T_n)$ is obtained from the path $u_1, u_2, \ldots, u_{2n}$ by replacing every alternate edge of a path by C_3. That is $A(T_n)$ is constructed by placing the vertices $v_1, v_2, \ldots, v_n$ above $u_1, u_2, \ldots, u_{2n}$ so that u_{2i-1}, v_i and $u_{2i}, 1 \leq i \leq n$ forms a triangle. $A(T_n)$ has $3n$ vertices. Its diameter is $2n-1$.*

Definition 2.4. *[7], A double triangular snake $D(T_n)$ consists of two triangular snakes that have a common path. That is, $D(T_n)$ is constructed by placing the vertices $v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_n$ above and below the path $u_1, u_2, \ldots, u_{n+1}$ so that u_i, v_i and $u_{i+1}, 1 \leq i \leq n$ forms a triangle in the upper part of the path and u_i, w_i and $u_{i+1}, 1 \leq i \leq n$ forms a triangle at the bottom part of the path. $D(T_n)$ has $3n+1$ vertices. Its diameter is n.*

Definition 2.5. *[7], A double alternate triangular snake $DA(T_n)$ consists of two alternate triangular snakes that have a common path. That is, $DA(T_n)$ is constructed by placing the vertices $v_1, v_2, \ldots, v_n$ and $w_1, w_2, \ldots, w_n$ above and below the path $u_1, u_2, \ldots, u_{n+1}$ so that u_{2i-1}, v_i and $u_{2i}, 1 \leq i \leq n$ forms a triangle in the upper part of the path and u_{2i-1}, w_i and $u_{2i}, 1 \leq i \leq n$ forms a triangle at the bottom part of the path. $DA(T_n)$ has $4n$ vertices. Its diameter is $2n-1$.*

3. Main results

The upper bounds of radio antipodal mean number of triangular snake families were studied in this section.

Theorem 3.1. *The radio antipodal mean number of triangular snake, $ramn(T_n) \leq 3n-5, n \geq 4$.*

Proof. Let $V(T_n)$ be the vertex set of T_n and can be written as $V(T_n) = V_1 \cup V_2$, where $V_1 = \{v_i : 1 \leq i \leq n\}$ and $V_2 = \{u_i : 1 \leq i \leq n+1\}$.

In the vertex set V_1, the vertices v_1 and v_n are at diametric distance. Therefore the vertices v_1 and v_n can receive same labeling. That is $f(v_1) = f(v_n)$. Similarly in the vertex set V_2, $f(u_1) = f(u_{n+1})$.

The remaining vertices are labeled by the mapping:

$$(3.1)\quad f(u_i) = \begin{cases} n-2, i=1 \\ n+i-2, 2 \le i \le n \end{cases} \qquad f(v_i) = \begin{cases} n-1, i=1 \\ 2n+i-3, 2 \le i \le n-2. \\ n-3, i=n-1 \end{cases}$$

We have to prove that the radio antipodal mean labeling condition is satisfied for every pair of vertices of T_n.

Claim: The mapping (3.1) is an valid radio antipodal mean labeling.

Let $u, v \in V(T_n)$. Consider the following cases:

Case 1. Let $u, v \in V_1$. In this case, $d(u,v) \ge 2$.

Case 1.1. Let $u = v_1$ and $v = v_j, 2 \le j \le n-2$. Then $f(v_1) = n-1$ and $f(v_j) = 2n+j-3$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 2 + \lceil \frac{n-1+2n+j-3}{2} \rceil \ge n$

Case 1.2. Let $u = v_i$ and $v = v_j, 2 \le i,j \le n-2$. Then $f(v_i) = 2n+i-3$ and $f(v_j) = 2n+j-3$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 2 + \lceil \frac{2n+i-3+2n+j-3}{2} \rceil > n$

Case 1.3. Let $u = v_1$ and $v = v_n$. In this case, $d(u,v) = n$ and $f(u) = n-1$ and $f(v) = n-1$. Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge n + \lceil \frac{n-1+n-1}{2} \rceil > n$

Case 1.4. Let $u = v_{n-1}$ and $v = v_n$. In this case, $d(u,v) = 2$ and $f(u) = n-3$ and $f(v) = n-1$. Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 2 + \lceil \frac{n-3+n-1}{2} \rceil \ge n$.

Case 2. Let vertices $u, v \in V_2$

Case 2.1. Let vertices $u = u_1$ and $v = u_i, 2 \le i \le n$. In this case, $d(u,v) \ge 1$. By mapping (3.1), $f(u) = n-2$ and $f(v) = n+i-2$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 1 + \lceil \frac{n-2+n+i-2}{2} \rceil \ge n$.

Case 2.2. Let $u = u_i$ and $v = u_j, 2 \le i,j \le n$.

Case 2.2.1. Suppose the vertices u and v are adjacent. Then $d(u,v) = 1$. Here $f(u) = n+i-2$ and $f(v) = n+j-2, j = i+1$ or $i-1$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 1 + \lceil \frac{n+i-2+n+j-2}{2} \rceil > n$.

Case 2.2.2. Suppose the vertices u and v are non adjacent. Then $d(u,v) \ge 2$. Here $f(u) = n+i-2$ and $f(v) = n+j-2$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 2 + \lceil \frac{n+i-2+n+j-2}{2} \rceil \ge n$.

Case 2.3. Suppose $u = u_1$ and $v = u_{n+1}$. This case will be similar to Case 1.3.

Case 3. Suppose $u \in V_2$ and $v \in V_1$

Case 3.1. Let $u = u_1$ and $v = v_1$. In this case, $d(u,v) = 1$ and $f(u) = n-2$ and $f(v) = n-1$. Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \ge 1 + \lceil \frac{n-1+n-2}{2} \rceil > n$.

Case 3.2. Let $u = v_i$ and $v = u_j, 2 \le i \le n-2$ and $2 \le j \le n$.

Case 3.2.1. Suppose the vertices u and v are adjacent then $d(u,v) = 1$. Here

$f(u) = 2n + i - 3$ and $f(v) = n + j - 2$. Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq 1 + \lceil \frac{n+j-2+2n+i-3}{2} \rceil > n$.

Case 3.2.2. Suppose the vertices u and v are non adjacent, then $d(u,v) \geq 2$. Here $f(u) = 2n + i - 3$ and $f(v) = n + j - 2$, $d(u,v) + \lceil \frac{n+j-2+2n+i-3}{2} \rceil \geq n$.

Case 3.3. Let $u = u_{n+1}$ and $v = v_{n-1}$. Here, $f(u) = n - 2$ and $f(v) = n - 3$ and $d(u,v) = 2$.

Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq 2 + \lceil \frac{n-2+n-3}{2} \rceil \geq n$. Therefore in all the cases, it can be seen that the radio antipodal mean labeling condition is satisfied by all pairs of vertices of T_n. Therefore, mapping (3.1) is an valid radio antipodal mean labeling. By mapping (3.1), the vertex v_{n-1} receives the maximum label, and is given by $3n - 5$. Therefore, $ramn(T_n) \leq 3n - 5$. □

Theorem 3.2. *The radio antipodal mean number of double triangular snake,* $ramn(D(T_n)) \leq 4n - 6, n \geq 4$.

Proof. Let $V(D(T_n))$ be the vertex set of $D(T_n)$ and can be written as $V(D(T_n)) = V_1 \cup V_2 \cup V_3$ where V_1 and V_2 will be same as defined in Theorem (3.1). The vertex set V_3 is defined as follows: $V_3 = \{w_i : 1 \leq i \leq n\}$. Here, $f(v_1) = f(v_n)$, $f(u_1) = f(u_{n+1})$ and $f(w_1) = f(w_n)$. The remaining vertices are labeled by the mapping:

$$f(u_i) = 2n + i - 5, f(v_i) = n + i - 4, f(w_i) = 3n + i - 5. \tag{3.2}$$

Claim: The mapping (3.2) is an valid radio antipodal mean labeling. Let u, v be any two vertices of $D(T_n)$. Consider the following cases:

Case 1. Suppose the vertices $u, v \in V_1$. In this case, $d(u,v) \geq 2$.

Case 1.1. Let $u = v_i$ and $v = v_j, 1 \leq i, j \leq n - 1$. Then $f(v_i) = n + i - 4$ and $f(v_j) = n + j - 4$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq 2 + \lceil \frac{n+i-4+n+j-4}{2} \rceil \geq n$.

Case 1.2. Let $u = v_1$ and $v = v_n$. In this case, $d(u,v) = n$ and $f(u) = n - 3$ and $f(v) = n - 3$. Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq n + \lceil \frac{n-3+n-3}{2} \rceil > n$.

Case 2. Suppose the vertices $u, v \in V_2$.

Case 2.1. Let $u = u_i$ and $v = u_j, 1 \leq i, j \leq n$.

Case 2.1.1. If the vertices u and v are adjacent then $d(u,v) = 1$. Here $f(u) = 2n + i - 5$ and $f(v) = 2n + j - 5, j = i + 1$ or $i - 1$, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq 1 + \lceil \frac{2n+i-5+2n+j-5}{2} \rceil > n$.

Case 2.1.2. If the vertices u and v are non adjacent then $d(u,v) \geq 2$. Here $f(u) = 2n + i - 5$ and $f(v) = 2n + j - 5$.

$d(u,v) + \lceil \frac{2n+i-5+2n+j-5}{2} \rceil \geq n$.

Case 2.2. Let $u = u_1$ and $v = u_{n+1}$. This case will be similar to Case 1.2.

Case 3. Suppose $u \in V_1$ and $v \in V_2$.

Case 3.1. Let $u = v_i, 1 \leq i \leq n$ and $v = u_j, 1 \leq j \leq n+1$.

Case 3.1.1. Suppose the vertices u and v are adjacent. In this case, $d(u,v) = 1$. Here $f(u) = n+i-4$ and $f(v) = 2n+j-5$. Therefore, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq 1 + \lceil \frac{n+i-4+2n+j-5}{2} \rceil > n$.

Case 3.1.2. Suppose the vertices u and v are non adjacent, then $d(u,v) \geq 2$. Here $f(u) = n+i-4$ and $f(v) = 2n+j-5$.
$d(u,v) + \lceil \frac{n+i-4+2n+j-5}{2} \rceil \geq n$.

Case 4. Let $u, v \in V_3$. The cases will be similar to Case 1.

Case 5. Let $u \in V_2$ and $v \in V_3$. This case will be similar to case 3.

Case 6. Let $u \in V_1$ and $v \in V_3, 1 \leq i \leq n$. In this case, $d(u,v) \geq 2$ and $f(u) = 2n+i-5$ and $f(v) = 3n+i-5$. Hence, $d(u,v) + \lceil \frac{f(u)+f(v)}{2} \rceil \geq n$.

Hence in all the cases, it can be seen that the radio antipodal mean labeling condition is satisfied by all pairs of vertices of $D(T_n)$. Therefore, the mapping (3.2) is an valid radio antipodal mean labeling. By mapping (3.2), the vertex $f(w_{n-1})$ receives the maximum labeling given by $4n-6$.

Hence, $ramn(D(T_n)) \leq 4n-6$ □

Theorem 3.3. *The radio antipodal mean number of alternate triangular snake,* $ramn(A(T_n)) \leq 5n-7, n \geq 3$.

Proof. The vertex set of $A(T_n)$ can be partitioned into 2 disjoint sets V_1 and V_2 where the vertex set V_1 will be same as defined in Theorem (3.1) and V_2 is defined as follows: $V_2 = \{u_i : 1 \leq i \leq 2n\}$. In this graph, we have $f(v_1) = f(v_n)$ and $f(u_1) = f(u_{2n})$. The remaining vertices are labeled by the mapping:

$$f(u_i) = 2n+i-4, 1 \leq i \leq 2n-1, f(v_i) = \begin{cases} 4n+i-5, 1 \leq i \leq n-2 \\ 2n-4, i = n-1. \end{cases} \tag{3.3}$$

Claim: The mapping (3.3) is an valid radio antipodal mean labeling. The cases are similar to Theorem 3.1. Hence, we conclude that the mapping (3.3) is an valid ratio antipodal mean labeling. By the mapping (3.3), the vertex $f(v_{n-1})$ receives the maximum labeling and it's label is given by $5n-7$. Therefore, $ramn(A(T_n)) \leq 5n-7, n \geq 3$. □

Theorem 3.4. *The radio antipodal mean number of double alternate triangular snake,* $ramn(DA(T_n)) \leq 6n-9, n \geq 5$.

Proof. Let $V(DA(T_n))$ be the vertex set of $DA(T_n)$. It can be written as $V(DA(T_n)) = V_1 \cup V_2 \cup V_3$, where the vertex set V_1 and V_3 is already defined in Theorem (3.2). The vertex set V_2 is same as defined in Theorem (3.3). Here, $f(v_1) = f(v_n)$, $f(u_1) = f(u_{2n})$ and $f(w_1) = f(w_n)$. The remaining vertices are labeled by the mapping:

$$f(u_i) = 3n+i-7, f(v_i) = \begin{cases} 2n+i-5, 1 \leq i \leq n-2 \\ 2n-5, i = n-1 \end{cases}, f(w_i) = 5n+i-8. \tag{3.4}$$

Claim: The mapping (3.4) is an valid radio antipodal mean labeling. The cases are similar to Theorem 3.2.

Therefore we conclude that the mapping (3.4) is an valid radio antipodal mean labeling. By mapping (3.4), the vertex $f(w_{n-1})$ receives the maximum label which is $6n-9, n \geq 4$. Therefore, $ramn(DA(T_n)) \leq 6n-9, n \geq 5$. □

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