ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9747–9751
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.83 Spec. Iss. on ICRTAMS-2020

A NOTE ON FUZZY *BRK* TOPOLOGICAL ACTION OF SUBGROUP

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ABSTRACT. In this paper, fuzzy BRK topological action can be extended to a subgroup S. We study some theorems and properties of a fuzzy BRK topological action on subgroup. Also, we discuss about normal fuzzy BRK topological action in a subgroup of G.

1. INTRODUCTION AND PRELIMINARIES

The concept of a fuzzy set was introduced in [12], provides a general topology called fuzzy topological spaces. The structure of a fuzzy topological spaces by Foster in [2] combined with a fuzzy group. Rosenfeld in [6] has formulate the elements of a theory of fuzzy topological groups. Ma and Yu in [4] and Yalvac in [11] changed the definition to ordinary topological group is a special case of a ftg. In 2012, Bandaru in [5] introduced BRK-algebra, which is a generalization of $BCK/BCI/BCH/Q/QS/BM$-algebras. Sivakumar et al. introduced a topology on BRK-algebra in [7] and also studied there properties. Haddadi in [3] and Rosenfeld in [6] study fuzzy actions of fuzzy submonoids and fuzzy subgroups from an algebraic point of view. Boixader and Recasens in [1] has made some interesting in group with fuzzy actions. The fuzzy actions in a BRK-topological spaces has explained in a paper, [10]. In this paper, we

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2010 *Mathematics Subject Classification.* 20F19.

Key words and phrases. fuzzy BRK topological action, fuzzy BRK topological action of S, normal fuzzy $BRKtAS_g$.

extend fuzzy BRK topological action to a subgroup S. Also, we study a normal $fBRKtA$ of subgroup and their properties. All other undefined notions are from [2, 5, 8–10, 12] and cited therein.

2. Fuzzy BRK topological action on subgroup

Definition 2.1. Let G be a $fBRKtg$ $(G, \star, 0, \tau)$ on X and S be a subgroup of G, then the map $\mu : (G \times S, \tau \times \tau) \to (S, \star, 0, \tau)$ is a $fBRKtA$ of S (briefly, $fBRKtAS_g$) on X if (a*) $\mu(e\star(f\star s)) = \mu((e\star f)\star s)$, (a**) $\mu(0\star s) = \mu(s)\ \forall\, e, f \in G\ \&\ s \in S$, (b) $\mu(e \star (s \star t)) \geq \min(\mu(e \star s), \mu(e \star t))$, (c) $\mu((e \star f) \star s) \geq \min(\mu(e \star s), \mu(f \star t))$, (d) $\mu(e \star s^{-1}) \geq \mu(e \star s)\ \forall\, e, f \in G\ \&\ s, t \in S$.

Example 1. Let $(X = \{0_c, 1_c, 2_c, 3_c\}, \star, 0)$ be a $fBRKtg$ in which $\star$ is defined by

$\star$	0_c	1_c	2_c	3_c
0_c	0_c	1_c	2_c	3_c
1_c	1_c	0_c	3_c	2_c
2_c	2_c	3_c	0_c	1_c
3_c	3_c	2_c	1_c	0_c

Now define a fuzzy set $\mu : X \to [0, 1]$ by $\mu(0_c) = 0.8$, $\mu(1_c) = 0.7$, $\mu(2_c) = \mu(3_c) = 0.6$, $\{0, 1, 2\} \in G$ and $\{0, 1\} \in S$. Then, it is a $fBRKtAS_g$.

Theorem 2.1. Let G be a $fBRKtg$ on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$, then every subgroup H of G acts on S.

Proof. A $fBRKtg$ G on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$. Then (a*), G acting on S. Also there exists a map $\mu : (G \times S, \tau \times \tau) \to (S, \star, 0, \tau)$ with the conditions $\mu(g \star (h \star s)) = \mu((g \star h) \star s)$ and $\mu(0 \star s) = \mu(s)\ \forall\, g, h \in G,\ s \in S$. In particular, the restriction map of μ on $(H \times S, \tau \times \tau) \to (S, \star, 0, \tau)$ satisfies the conditions $\mu(g \star (h \star s)) = \mu((g \star h) \star s)$ and $\mu(0 \star s) = \mu(s)\ \forall\ g, h \in H,\ s \in S$. (b) $\mu(g \star (s \star t)) \geq \min(\mu(g \star s), \mu(g \star t))$, (c) $\mu((g \star h) \star s) \geq \min(\mu(g \star s), \mu(h \star t))$, (d) $\mu(g \star s^{-1}) \geq \mu(g \star s)\ \forall\, g, h \in H,\ s, t \in S$. Hence every subgroup of G acts on the given $fBRKtAS_g$ μ. Similarly G acts on each fuzzy subgroup H_μ of μ under S. □

Theorem 2.2. Let G be a $fBRKtg$ on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$. Then:

(i) $\mu(g \star s) \leq \mu(0 \star s)$ for all $g \in G$ and $s \in S$.

(ii) The subset $G_\mu = \{g \in G / \mu(g \in s) = \mu(0 \star s)\}$ is a subgroup of G acting on $fBRKtAS_g\ \mu$.

Proof. Let $g \in G$. Then $\mu(g * s) = \min\{\mu(g * s), \mu(g * s)\} = \min\{\mu(g * s), \mu((g^{-1}) * s)\} \leq \mu((g * g^{-1}) * s) = \mu(0 * s)$ implies (i).To verify (ii), it follows that $0 \in G_\mu$, and $G_\mu \neq 0$. Let $g, h \in G_\mu$ and $s \in S$. $\mu((g \star h^{-1}) \star s) \geq \min\{\mu(g \star s), \mu((h^{-1}) \star s)\} = \min\{\mu(g \star s), \mu(h \star s)\} = \min\{\mu(0 \star s), \mu(0 \star s)\} = \mu(0 \star s)$ but from (i) $\mu((g \star h^{-1}) \star s) \leq \mu(0 \star s)$ for $g, h \in G$ and $s \in S$, so $\mu((g \star h^{-1}) \star s) = \mu(0 \star s)$ which means $(g \star h^{-1}) \in G_\mu$. Thus G_μ is a subgroup of G. So G_μ acts on $fBRKtAS_g\ \mu$ on S by Theorem 2.1. □

Corollary 2.1. Let G be a $fBRKtg$ on a $fBRKtAS_g\ \mu$ on $(S, \star, 0, \tau)$. Consider the subset H of G given by $H = \{g \in G / \mu(g \star s) = \mu(0 \star s)\}$. Then H is a crisp subgroup of G acting on S.

Theorem 2.3. Let G be a $fBRKtg$ on a $fBRKtAS_g$'s μ_1 and μ_2 on $(S, \star, 0, \tau)$. Then G acts on a $fBRKtAS_g$'s $\mu_1 \cap \mu_2$ on S.

Proof. Let G be a $fBRKtg$ on a $fBRKtAS_g$'s μ_1 and μ_2 on $(S, \star, 0, \tau)$. It is given that μ_1 and μ_2 are $fBRKtAS_g$'s on S. It follows that $\mu_1 \cap \mu_2$ on S.

Since G acts on S, then there exists a map $\mu : (G \times S, \tau \times \tau) \to (S, \star, 0, \tau)$ such that $e \star (f \star s) = (e \star f) \star s$ and $0 \star s = s$ or all $s \in S$, and $\forall\, e, f \in G$ which gives (a*) & (a**). Let $e, f \in G$ and $s \in S$. (b) $(\mu_1 \cap \mu_2)(e \star (s \star t)) = \min\{\mu_1(e \star (s \star t)), \mu_2(e \star (s \star t))\} \geq \min\{\min\{\mu_1(e \star s), \mu_1(e \star t)\}, \min\{\mu_2(e \star s), \mu_2(e \star t)\}\} = \min\{\min\{\mu_1(e \star s), \mu_2(e \star s)\}, \min\{\mu(e \star t), \mu_2(e \star t)\}\} = \min\{(\mu_1 \cap \mu_2)(e \star s), (\mu_1 \cap \mu_2)(e \star t)\}$. (c) $(\mu_1 \cap \mu_2)((e \star f) \star s) = \min\{\mu_1((e \star f) \star s), \mu_2((e \star f) \star s)\} \geq \min\{\min\{\mu_1(e \star s), \mu_1(f \star s)\}, \min\{\mu_2(e \star s), \mu_2(f \star s)\}\} = \min\{\min\{\mu_1(e \star s), \mu_2(e \star s)\}, \min\{\mu_1(f \star s), \mu_2(f \star s)\}\} = \min\{(\mu_1 \cap \mu_2)(e \star s), (\mu_1 \cap \mu_2)(f \star s)\}$. (d) $(\mu_1 \cap \mu_2)(e \star s^{-1}) = \min\{\mu_1(e \star s^{-1}), \mu_2(e \star s^{-1})\} \geq \min\{\mu_1(e \star s), \mu_2(e \star s)\} = (\mu_1 \cap \mu_2)(e \star s)$. Thus G acts on a $fBRKtAS_g$'s $\mu_1 \cap \mu_2$ on S. □

Theorem 2.4. If G acts each member in the family $\{\mu_i\}_i \in G$ of $fBRKtg$ under S, then G acts on $fBRKtg \cap \mu_i$ under S.

Theorem 2.5. Let G be a $fBRKtg$ on a $fBRKtAS_g\ \mu$ on $(S, \star, 0, \tau)$, then G acts on anti-$fBRKtAS_g\ \mu^c$ on S.

Theorem 2.6. Let G be a $fBRKtg$ on a $fBRKtAS_g\ \mu$ on $(S, \star, 0, \tau)$. Then G acts on level cut set $U(\mu; t)$ on S where t is in $[0, 1]$.

3. NORMAL $fBRKtAS_g$

Definition 3.1. Let G be a $fBRKtg$ on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$ and θ is a homomorphism from S into S. Define $\mu^\theta = \{< s \in S, \inf_{x \in G} \mu(\theta(x)) \star s >\}$ under S.

Definition 3.2. A $fBRKtAS_g$ μ on S is normal $fBRKtAS_g$ (briefly, $nfBRKtAS_g$) if $\inf_{c \in G} \mu(c \star s) = \inf_{c \in G} \mu(c \star (t^{-1} \star s \star t))$ $\forall$ $s, t \in S$.

Definition 3.3. Let G be a $fBRKtg$ on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$ is a fuzzy BRK characteristic of G acting on S if $\mu^\theta = \mu$ for some homomorphism θ on S. In a group S, define $[s, t] = (t^{-1} \star s^{-1} \star s \star t)$.

Theorem 3.1. Let G be a $fBRKtg$ on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$ and θ is a homomorphism from S into S. The group G acts on fuzzy BRK subgroup $\mu^\theta = \{< s \in S, \inf_{x \in G} \mu(\theta(x)) \star s >\}$ under S.

Theorem 3.2. Let G be a $fBRKtg$ on a $fBRKtAS_g$ μ on $(S, \star, 0, \tau)$. Let μ^+ be a fuzzy set $\{< s \in S, \mu^+(s) >: \mu^+(s) = \inf_{c \in G} \mu(c \star s) + 1 - \mu(0 \star s)$. Then G acts on the fuzzy BRK group μ^+ under S.

Proof. It follows that $\mu^+(s) = \inf_{c \in G} \mu(c \star s) + 1 - \mu(0 \star s)$ for all $s \in S$. So $\mu^+(s \star t^{-1}) = \inf_{c \in G} \mu(c \star (s \star t^{-1})) + 1 - \mu(0 \star (s \star t^{-1})) = [\inf_{c \in G} \mu(c \star s) + 1 - \mu(0 \star s)], \inf_{c \in G} \mu(c \star t) + 1 - \mu(0 \star t)$ $\forall$ $s, t \in S$. Thus $s, t \in \mu^+$ implies that $s \star t^{-1} \in \mu^+$. Then μ^+ is a fuzzy BRK subgroup of S. Hence G acts on the fuzzy BRK group μ^+ under S. So $\mu^+(c \star (t \star s \star t^{-1})) = \inf_{c \in G} \mu(c \star (t \star s \star t^{-1})) + 1 - \mu(0 \star (t \star s \star t^{-1})) = \mu(c \star s) + 1 - \mu(0 \star s) = \mu^+(c \star s)$ $\forall$ $s, t \in S$. Hence μ^+ is normal fuzzy BRK group of G acting on S. □

Theorem 3.3. Let G be a $fBRKtg$ act on a $nfBRKtAS_g$ μ on $(S, \star, 0, \tau)$. Then $\inf_{x \in G} \mu(x \star [s, t]) = \mu(0 \star s)$ $\forall$ $s, t \in S$.

Proof. Since G acts on a normal fuzzy BRK group μ under S, it gives that $\inf_{x \in G} \mu(x \star s) = \inf_{x \in G} \mu(x \star (t \star s \star t^{-1})$ $\forall$ $s, t \in S$. Replacing s by (s^{-1}) and t by (t^{-1}), $\inf_{x \in G} \mu(x \star s^{-1}) = \inf_{x \in G} \mu(x \star (t^{-1} \star s \star t))$ $\forall$ $s, t \in S$. Then it becomes that $\inf_{x \in G} \mu(x \star [s, t]) = \mu(0 \star s)$ $\forall$ $s, t \in S$. □

Theorem 3.4. Let G act on a fuzzy BRK characteristic group μ with respect to θ under S. Then μ is a $nfBRKtAS_g$ of S.

Proof. Let $g \in G$. Consider the map $\theta_t : S \to S$ by $\theta(s) = t \star s \star t^{-1}$, where t fixed in S. Clearly θ_t is an automorphism of S. Now $\mu(g \star s) = \mu^{\theta_t}(x \star s) = \mu(\theta_t(g \star s)) \; \forall \; s \in S = \mu((g \star (t^{-1} \star s \star t)) \; \forall \; s, t \in S$. It follows that $\inf_{g \in G} \mu(g \star s) = \inf_{g \in G} \mu(g \star (t \star s \star t^{-1})) \; \forall \; s, t \in S$. So μ is a $nfBRKtAS_g$ of S and hence G acts on a $nfBRKtAS_g$ under S. □

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