ADV MATH SCI JOURNAL
Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9753–9757
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.84 Spec. Iss. on ICRTAMS-2020

$\mathcal{AC}^*$ AND $\mathcal{AC}_2^*$-PAIRWISE PARACOMPACTNESS IN BITOPOLOGICAL SPACES

S. UMAMAHESWARI[1] AND M. SARASWATHI

ABSTRACT. This paper deals with two new classes of $\mathcal{AC}^*$ pairwise and $\mathcal{AC}_2^*$ pairwise paracompact spaces in bitopological spaces some characterizations and several basic properties of $\mathcal{AC}^*$ and $\mathcal{AC}_2^*$ pairwise paracompact spaces are obtained. Also introduced the $\mathcal{AC}^*$-pairwise normal spaces in bitopological spaces and its properties were discussed.

1. INTRODUCTION

We are focused on the idea of $\mathcal{AC}^*$- pairwise paracompact spaces, $\mathcal{AC}_2^*$-pairwise paracompact spaces and $\mathcal{AC}^*$- pairwise normal in bitopological spaces. Various characterzations of bitopological compactness have showed up in the writing of Fletcher in [5]. The bitopological paracompactness has also been considered by M. C. Datta in [4], C- paracompact spaces and C_2- paracompact spaces were characterized by Arhangelskii. C- paracomapact spaces and C_2- paracompact spaces were concentrated in [7]. S. Alzahrani in [1] research C- normal topological property. In this paper, every bitopological space are presumed to be pairwise Hausdorff. According to D. H. Fermlin in [2], we consider a space angelic. We use the idea of an angelic spaces in C- pairwise paracompact and

[1]*corresponding author*

2020 *Mathematics Subject Classification.* 54E55, 46A50, 54D10, 54D20.
Key words and phrases. Bitopological spaces, Angelic spaces, $\mathcal{AC}^*$-pairwise paracompact, $\mathcal{AC}_2^*$ pairwise paracompact, $\mathcal{AC}^*$- pairwise normal.

C_2- pairwise paracompact spaces and C-pairwise normal in bitopological spaces. Throughout this paper p_w denotes pairwise.

2. PRELIMINARIES

The preliminary definitions of p_w- compact, p_w - Hausdorff, p_w - regular, p_w normal, p_w- paracompact, p_w- continuous and p_w- homeomorphism are studied in the following references in [3], [4], [5] and [6].

3. $\mathcal{AC}^*$ AND $\mathcal{AC}_2^*$ PAIRWISE PARACOMPACT SPACES

Definition 3.1. *Let (L, τ_1, τ_2) be an angelic bitopological space and (Y, τ_1^*, τ_2^*) be an angelic p_w-compact subspace of (L, τ_1, τ_2). If there is a bijection mapping $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$, (M, δ_1, δ_2) is an angelic p_w-paracompact space and the restriction $t|_Y : Y \to t(Y)$ is a p_w- homeomorphism, then (L, τ_1, τ_2) is said to be an $\mathcal{AC}^*$ p_w- paracompact space.*

Definition 3.2. *Let M be an angelic bitopological space and (Y, τ_1^*, τ_2^*) be an angelic p_w-compact subspace of (L, τ_1, τ_2). If there is a bijection function $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$, (M, δ_1, δ_2) is a p_w- Hausdorff angelic p_w- paracompact space and the restriction $t|_Y : Y \to t(Y)$ is a p_w- homeomorphism, then (L, τ_1, τ_2) is said to be an $\mathcal{AC}_2^*$ p_w- paracompact space.*

Theorem 3.1. *Every $\mathcal{AC}^*$ p_w- paracompact space ($\mathcal{AC}_2^*$ p_w- paracompact space) is a topological property.*

Proof. Suppose (M, δ_1, δ_2) is an $\mathcal{AC}^*$-p_w paracompact ($\mathcal{AC}_2^*$ p_w-paracompact) space and $(L, \tau_1, \tau_2) \cong (P, \lambda_1, \lambda_2)$. Let (M, δ_1, δ_2) be an angelic p_w- paracompact (p_w-Hausdorff angelic p_w-paracompact) space and $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a bijective mapping $\ni$ the restriction $t|_Y : Y \to t(Y)$ is p_w-homeomorphism for every angelic p_w-compact subspace $Y \subseteq L$. Let $u : (P, \lambda_1, \lambda_2) \to (L, \tau_1, \tau_2)$ be a p_w-homeomorphism. Hence, (M, δ_1, δ_2) and $t \circ u : (P, \lambda_1, \lambda_2) \to (L, \tau_1, \tau_2)$ has the topological properties. □

Theorem 3.2. *Every $\mathcal{AC}^*$ p_w- paracompact space ($\mathcal{AC}_2^*$ p_w- paracompact space) has an additive property.*

Theorem 3.3. *If (L, τ_1', τ_2') is a submetrizable space of (L, τ_1, τ_2) with $(\tau_1', \tau_2') \subseteq (\tau_1, \tau_2)$, then (L, τ_1', τ_2') is an $\mathcal{AC}_2^*$ p_w- paracompact.*

Theorem 3.4. *If (L, τ_1, τ_2) is an $\mathcal{AC}^*$ p_w- paracompact ($\mathcal{AC}_2^*$ p_w- paracompact) Frechét space and $t_\alpha : (L_\alpha, \tau_1, \tau_2) \to (M_\alpha, \delta_1, \delta_2)$ is a witness of the $\mathcal{AC}^*$-p_w paracompactness ($\mathcal{AC}_2^*$-p_w paracompactness) of (L, τ_1, τ_2), then t is p_w- continuous.*

Corollary 3.1. *If (L, τ_1, τ_2) is an $\mathcal{AC}^*$-p_w paracompact ($\mathcal{AC}_2^*$-p_w paracompact) first countable space and $t_\alpha : (L_\alpha, \tau_1, \tau_2) \to (M_\alpha, \delta_1, \delta_2)$ is a witness of the $\mathcal{AC}^*$-p_w paracompact ($\mathcal{AC}_2^*$-p_w paracompact) of (L, τ_1, τ_2), then t is p_w- continuous.*

Corollary 3.2. *If (L, τ_1, τ_2) is an $\mathcal{AC}_2^*$-p_w paracompact Frechet space, then X is p_w-Hausdorff.*

Theorem 3.5. *If (L, τ_1, τ_2) is a T_1 space $\ni$ the only angelic p_w- compact subsets are the finite subsets, then M is an $\mathcal{AC}_2^*$ p_w- paracompact space.*

Theorem 3.6. *Let (L, τ_1, τ_2) be the p_w- Hausdorff locally angelic p_w- compact space. Then (L, τ_1, τ_2) is an $\mathcal{AC}_2^*$-p_w paracompact space.*

Definition 3.3. *A topological space (L, τ_1, τ_1) is termed as lower angelic p_w- compact if $\exists$ a coarser topology (τ_1', τ_2') on $(L, \tau_1, \tau_1) \ni (X, \tau_1', \tau_2')$ is T_2 angelic p_w-compact.*

Theorem 3.7. *If (L, τ_1, τ_2) lower angelic p_w- compact space, then L is an $\mathcal{AC}_2^*$-p_w paracompact.*

Proof. Suppose (τ_1', τ_1') is a T_2 angelic p_w- compact topology on $(L, \tau_1, \tau_1) \ni (\tau_1', \tau_1') \subseteq (\tau_1, \tau_2')$. Next (L, τ_1', τ_2') is T_2 angelic p_w- paracompact and the identity mapping $id_L : (L, \tau_1, \tau_2) \to (L, \tau_1', \tau_2')$ is a p_w- continuous function. If (Y, τ_1^*, τ_2^*) is some angelic p_w-compact subspace of (L, τ_1, τ_2), next the restriction of the identity mapping on Y onto $id_L(Y)$ is a p_w- homeomorphism as Y is an angelic p_w-compact, $id_L(Y)$ is p_w- Hausdorff being a subspace of the T_2 space (L, τ_1', τ_2') and every p_w-continuous 1-1 function of an angelic p_w-compact space onto a p_w-Hausdorff space is a p_w- homeomorphism. Hence, (L, τ_1, τ_2) is an $\mathcal{AC}_2^*$-p_w paracompact. □

Theorem 3.8. *If (L, τ_1, τ_2) is an $\mathcal{AC}_2^*$-p_w paracompact countably angelic p_w-compact Frechet, then (L, τ_1, τ_2) is lower angelic p_w- compact.*

4. $\mathcal{AC}^*$- PAIRWISE NORMAL AND ITS PROPERTIES

Definition 4.1. *A space (L, τ_1, τ_2) is termed as an $\mathcal{AC}^*$- normal if $\exists$ a p_w normal space (M, δ_1, δ_2) and a bijective $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2) \ni$ the restriction $t|_Y : Y \to t(Y)$ is p_w- homeomorphism for every angelic p_w- compact subspace $(Y, \tau_1^*, \tau_2^*) \subseteq (L, \tau_1, \tau_2)$.*

Definition 4.2. *A space (L, τ_1, τ_2) is termed as an angelic countably p_w- normal if there exists a p_w-normal space (M, δ_1, δ_2) and a bijective $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ $\ni$ the restriction $t|_Y : Y \to t(Y)$ is a p_w-homeomorphism for each angelic countable subspace $(Y, \tau_1^*, \tau_2^*) \subseteq (L, \tau_1, \tau_2)$.*

Example 1. *An $\mathcal{AC}^*$ p_w- normal space is not an $\mathcal{AC}_2^*$-p_w paracompact.*

Lemma 4.1. *If $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ is a bijective function, (L, τ_1, τ_2) is an $\mathcal{AC}^*$-p_w normal space and any finite subset of (L, τ_1, τ_2) is discrete, then (M, δ_1, δ_2) is T_1.*

Theorem 4.1. *Every angelic p_w- compact non-p_w normal space is not an $\mathcal{AC}^*$-p_w normal.*

Proof. Consider (L, τ_1, τ_2) is an angelic p_w- compact non-p_w normal space. Assume (L, τ_1, τ_2) is an $\mathcal{AC}^*$-p_w normal, then $\exists$ a p_w-normal space (M, δ_1, δ_2) and a bijective mapping $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2) \ni$ the restriction $t|_Y : Y \to t(Y)$ is p_w- homeomorphism for every angelic p_w- compact subspace $(Y, \tau_1^*, \tau_2^*) \subseteq (L, \tau_1, \tau_2)$. Since (L, τ_1, τ_2) is an angelic p_w- compact, then $(L, \tau_1, \tau_2) \cong (M, \delta_1, \delta_2)$, and this is a contradiction as (M, δ_1, δ_2) is p_w-normal and (L, τ_1, τ_2) is not an angelic p_w- compact non-p_w normal space. Hence (L, τ_1, τ_2) cannot be an $\mathcal{AC}^*$-p_w normal. □

Theorem 4.2. *Let (L, τ_1, τ_2) be an $\mathcal{AC}^*$-p_w normal space. If every countable subspace of (L, τ_1, τ_2) is included in an angelic p_w- compact subspace, then (L, τ_1, τ_2) is an angelic countably p_w- normal.*

Proof. Take (L, τ_1, τ_2) is any $\mathcal{AC}^*$-p_w normal space $\ni$ if (Y, τ_*, δ_2^*) is any countable subspace of (L, τ_1, τ_2), then $\exists$ an angelic p_w-compact subspace E such that $(Y, \tau_1^*, \tau_2^*) \subseteq E$. Take (M, δ_1, δ_2) is a p_w- normal space and $t : (L, \tau_1, \tau_2) \to (M, \delta_1, \delta_2)$ be a bijective mapping $\ni t|_Y : Y \to t(Y)$ is p_w- homeomorphism for every angelic p_w- compact subspace (Y, τ_1^*, τ_2^*) of (L, τ_1, τ_2). Now, take (Y, τ_1^*, τ_2^*)

is some countable subspace of (L, τ_1, τ_2). Choose an angelic p_w- compact subspace E of $(L, \tau_1, \tau_2) \ni S \subseteq E$, next $t|_E : E \to t(E)$ is p_w- homeomorphism, hence $t|_Y : Y \to t(Y)$ is p_w- homeomorphism as $(t|_E)|_Y = p|_Y$. □

Theorem 4.3. *Let (L, τ_1, τ_2) be an $\mathcal{AC}^*$-p_w normal. If (L, τ_1, τ_2) is a Frechét p_w-Lindelöf space $\ni$ any finite subspace of (L, τ_1, τ_2) is discrete, then (L, τ_1, τ_2) is an $\mathcal{AC}_2^*$-p_w paracompact.*

Theorem 4.4. *If (L, τ_1, τ_2) is p_w-Lindelöf epinormal space then (L, τ_1, τ_2) is an $\mathcal{AC}_2^*$-p_w paracompact.*

Theorem 4.5. *If (L, τ_1, τ_2) is an $\mathcal{AC}_2$- paracompact Frechet space, then (L, τ_1, τ_2) is an epinormal.*

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DEPARTMENT OF MATHEMATICS
KKC-VELUR 638182
TAMIL NADU, INDIA.
Email address: umamaheswari.maths@gmail.com

DEPARTMENT OF MATHEMATICS
KKC-VELUR 638182
TAMIL NADU, INDIA.
Email address: msmathsnkl@gmail.com