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GENERALIZED THEORY OF MAGNETO-THERMO-VISCOELASTIC SPHERICAL CAVITY PROBLEM UNDER FRACTIONAL ORDER DERIVATIVE: STATE SPACE APPROACH

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ABSTRACT. This paper is dealing the modified Ohm's law with the temperature gradient of generalized theory of magneto-thermo-viscoelastic for a thermally, isotropic and electrically infinite material with a spherical region using fractional order derivative. The general solution obtained from Laplace transform, numerical Laplace inversion and state space approach. The temperature, displacement and stresses are obtained and represented graphically with the help of mathcad software.

1. INTRODUCTION

Sherief et al. in [1] presented the new theory of coupled and generalized thermoelasticity using time using the method of fractional calculus. Povstenko in [2,3] solved some thermoelastic problem based on the 1D and 2D thermoelastic problem with a time fractional derivative. Gaikwad in [4] analysed the thermoelasticity of thin disk under partially heat supply. Many researchers in [5–9] studied the various problems on thermoelasticity and fractional order thermoelasticity.

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2. Basic equations

We consider a isotropic, homogeneous, thermo-viscoelastic solid: $R \ll r < \infty$, where R is the radius of the shell. First we shall consider the medium is perfect electrical conductor and secondly permeated by an initial constant magnetic field H_0. This produces an induced magnetic field h and electric field E.

(i) Linear equations of electromagnetism valid for slowly moving medium are:

$$\text{curl } h = J + \epsilon_0 \frac{\partial E}{\partial t} \tag{1}$$

$$\text{curl } E = -\mu_0 \frac{\partial E}{\partial t} \tag{2}$$

$$B = \mu_0 H$$

$$\text{div } B = 0$$

where J- current density vector, H- total magnetic field.

(ii) The modified Ohm's law as:

$$E = -\mu_0 \frac{\partial u}{\partial t} \times H_0 + k_0 \text{grad}\theta \tag{3}$$

here μ_0 - magnetic permeability, k_0 - coefficient of temperature gradient.

The equation of Lorentz force, whose expression is:

$$F = J \times B \tag{4}$$

(iii) The displacement equation:

$$\rho \frac{\partial^2 u_i}{\partial t^2} = 2\mu(\beta_1 + \lambda\beta_2)u_{j,ij} - (3\lambda + 2\mu)\alpha_t \beta \theta_{,i} + \mu_0(J \times H_0). \tag{5}$$

where:

$$\beta_1 = 1 + \alpha_1 \frac{\partial}{\partial t},\ \beta_2 = 1 + \alpha_2 \frac{\partial}{\partial t},\ \beta = 1 + \frac{3\lambda\alpha_1 + 2\mu\alpha_2}{3\lambda + 2\mu}\frac{\partial}{\partial t}$$

where ρ, λ, μ, α_1, α_2 are density , Lame's constant, and thermoviscoelastic relaxation times.

(iv) Heat conduction equation:

$$K\theta_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0^\alpha \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)(\rho C_E \theta + (3\lambda + 2\mu)\alpha_t \beta T_0 e + \pi_0 divJ) \tag{6}$$

where π_0 - coefficient of the current density, T_0 - reference uniform temperature, α_t - coefficients of linear thermal expansion, K - thermal conductivity, θ - temperature increment, τ_0 - thermal relaxation time, C_E - specific heat constant.

The Caputo fractional derivative defined in [10] as:

$$D^\alpha f(t) = \begin{cases} \dfrac{1}{\Gamma(n-\alpha)} \int_0^t \dfrac{f^n(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau, & n-1<\alpha<n; \\ \dfrac{df(t)}{dt}, & n=1. \end{cases}$$

For finding the Laplace transform, the Caputo derivative requires information of the initial values of the function $f(t)$ and its integer derivative of the order $k = 1, 2, ..., n-1$

$$L\{D^\alpha f(t); s\} = s^\alpha F(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \qquad n-1<\alpha<n\text{”}$$

(v) Constitutive equations:

$$\sigma_{ij} = 2\mu\beta_1 e_{ij} + \lambda\beta_2 e\delta_{ij} + (3\lambda+2\mu)\alpha_t\beta\theta\delta_{ij} \tag{7}$$

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})$$

where σ_{ij} and δ_{ij} are the stress tensor components and Kronecker's constants. The equations (1) to (7) constitute the problem formulation under consideration.

3. Problem formulation

The spherical coordinates (r, ϕ, θ) are taken for any representative point of the body at time t and the origin is the center of the spherical cavity. Due the symmetry, all the quantities appearing in equation (1)–(7) are depends of variables r and t only. The displacement and strain components as:

$$u_r = u(r,t),\ u_\phi = u_\theta = 0$$

$$e_{rr} = \frac{\partial u}{\partial r},\ e_{\theta\theta} = \frac{u}{r} = e_{\phi\phi},\ e_{r\phi} = e_{r\theta} = e_{\theta\phi} = 0$$

$$e = \frac{\partial u}{\partial r} + \frac{2u}{r} = \frac{1}{r^2}\frac{\partial(r^2 u)}{\partial r}. \tag{8}$$

From equation (7), we obtained the stress tensor components as:

$$\sigma_{rr} = 2\mu\beta_1\frac{\partial u}{\partial r} + \lambda\beta_2 e - (3\lambda+2\mu)\beta\alpha_t\theta \tag{9}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = 2\mu\beta_1\frac{u}{r} + \lambda\beta_2 e - (3\lambda+2\mu)\beta\alpha_t\theta \tag{10}$$

$$\sigma_{r\theta} = \sigma_{r\phi} = \sigma_{\theta\phi} = 0. \tag{11}$$

Due to the application of H_0 their induced magnetic field $h = (0, 0, h)$ which will be negligible. Applying $H_0 = (0, 0, H_0)$ to equations (1-3), is obtained:

$$J = H_0 \frac{\partial e}{\partial r} + \frac{k_0}{\mu_0} \frac{\partial \theta}{\partial r} \tag{12}$$

$$h = -H_0 \left(\frac{\partial u}{\partial r} + \frac{u}{r} \right) - \frac{k_0}{\mu_0} \frac{\partial \theta}{\partial r} \tag{13}$$

$$E = \mu_0 H_0 \frac{\partial u}{\partial t} + k_0 \frac{\partial \theta}{\partial r}. \tag{14}$$

Using equations (4) and (12) the Lorentz force is:

$$F_r = \mu_0 H_0^2 \frac{\partial e}{\partial r} + k_0 H_0 \frac{\partial \theta}{\partial r}$$

Using equation (8) in equation (5), we obtain:

$$\rho \frac{\partial^2 u}{\partial t^2} = (2\mu\beta_1 + \lambda\beta_2 + \mu_0 H_0^2) \frac{\partial e}{\partial r} - (3\lambda + 2\mu)\alpha_t \beta \frac{\partial e}{\partial r} + k_0 H_0 \frac{\partial \theta}{\partial r}. \tag{15}$$

The equation (6) becomes

$$K \nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_0^\alpha \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}} \right) (\rho C_E \theta + (3\lambda + 2\mu)\alpha_t \beta T_0 e + \pi_0 div J) \tag{16}$$

where

$$\nabla^2 = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right).$$

Now, we will introduced the non-dimensional variables:

$$r' = c\eta r, \qquad t' = c^2 \eta t, \qquad \beta_1' = c^2 \eta \beta_1, \qquad \beta_2' = c^2 \eta \beta_2, \qquad u' = c\eta u,$$

$$\theta' = \frac{\theta_0}{T_0}, \qquad \sigma_{ij}' = \frac{\sigma_{ij}}{\mu}, \qquad h' = \frac{h}{H_0}, \qquad E' = \frac{E}{\mu_0 H_0 c}, \qquad J' = \frac{J}{\eta H_o c}.$$

where $\eta = \frac{\rho C_E}{K}$, $c = \sqrt{\frac{\lambda + 2\mu}{\rho}}$.

Equations (9)-(16) takes the forms (dropping the primes for convenience):

$$\sigma_{rr} = \frac{2\mu}{\lambda + 2\mu} \beta_1 \frac{\partial u}{\partial r} + \frac{\lambda}{\lambda + 2\mu} \beta_2 e - \frac{(3\lambda + 2\mu)\alpha_t \theta_0}{\lambda + 2\mu} \beta \theta \tag{17}$$

$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \frac{2\mu}{\lambda + 2\mu} \beta_1 \frac{u}{r} + \frac{\lambda}{\lambda + 2\mu} \beta_2 e - \frac{(3\lambda + 2\mu)\alpha_t \theta_0}{\lambda + 2\mu} \beta \theta \tag{18}$$

$$\sigma_{r\theta} = \sigma_{r\phi} = \sigma_{\theta\phi} = 0 \tag{19}$$

(20) $$J = \frac{\partial e}{\partial r} + \frac{k_0}{H_0\mu_0}\frac{\partial \theta}{\partial r}$$

(21) $$h = -\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) - \frac{k_0}{H_0\mu_0}\frac{\partial \theta}{\partial r}$$

(22) $$E = \frac{\partial u}{\partial t} + \frac{k_0}{H_0\mu_0}\frac{\partial \theta}{\partial r}$$

(23)
$$\frac{\partial^2 u}{\partial t^2} = \left(1 + \frac{k_0}{H_0\mu_0} + \mu_0 H_0^2 + \frac{2\mu\beta_1 + \lambda\beta_2}{\rho c^2}\right)\frac{\partial e}{\partial r} - \frac{(3\lambda + 2\mu)\alpha_t\theta_0}{\rho c^2}\left(\beta + \frac{k_0}{H_0\mu_0}\right)\frac{\partial \theta}{\partial r}$$

(24) $$\nabla^2\theta = \frac{\eta c^2}{K}\left(\frac{\partial}{\partial t} + \tau_0^\alpha \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)\theta + \frac{(3\lambda + 2\mu)\alpha_t c^2}{K}\beta\left(\frac{\partial}{\partial t} + \tau_0^\alpha \frac{\partial^{\alpha+1}}{\partial t^{\alpha+1}}\right)e.$$

The equation (23) becomes:

(25)
$$\ddot{e} = \left(1 + \frac{k_0}{H_0\mu_0} + \mu_0 H_0^2 + \frac{2\mu\beta_1 + \lambda\beta_2}{\rho c^2}\right)\nabla^2 e - \frac{(3\lambda + 2\mu)\alpha_t\theta_0}{\rho c^2}\left(\beta + \frac{k_0}{H_0\mu_0}\right)\nabla^2\theta$$

The boundary conditions:

(26) $$\theta(R,t) = \theta_0, \text{ at } r = R$$

(27) $$e(R,t) = 0, \text{ at } r = R.$$

Applying the Laplace transform to the equations (17-22) and (24-25) by using the homogeneous initial conditions, defined as:

$$\bar{f}(r,s) = \mathcal{L}[f(r,t)] = \int_0^\infty f(r,t)e^{-st}dt,$$

we obtain

(28) $$\bar{\sigma}_{rr} = l_1\frac{d\bar{u}}{dr} + l_2\bar{e} - l_3\bar{\theta}$$

(29) $$\bar{\sigma}_{\theta\theta} = \bar{\sigma}_{\phi\phi} = l_1\frac{\bar{u}}{r} + l_2\bar{e} - l_3\bar{\theta}$$

$$\bar{J} = \frac{d\bar{e}}{dr} + \frac{k_0}{H_0\mu_0}\frac{d\bar{\theta}}{dr}$$

$$\bar{h} = -\left(\frac{d\bar{u}}{dr} + \frac{\bar{u}}{r}\right) - \frac{k_0}{H_0\mu_0}\frac{d\bar{\theta}}{dr}$$

$$\bar{E} = s\bar{u} + \frac{k_0}{H_0\mu_0}\frac{d\bar{\theta}}{\partial r}$$

(30)
$$\nabla^2\bar{\theta} = L_1\bar{\theta} + L_2\bar{e}$$

(31)
$$\nabla^2\bar{e} = M_1\bar{\theta} + M_2\bar{e}$$

(32)
$$\bar{\theta}(r,s) = \frac{\theta_0}{s}, \text{ at } r = R$$

(33)
$$\bar{e}(R,t) = \bar{e}_0 = 0, \text{ at } r = R$$

where

$l_1 = \frac{2\mu}{\lambda+2\mu}\left(\frac{k_0}{H_0\mu_0} + \beta_1^*\right)$, $l_2 = \frac{\lambda}{\lambda+2\mu}\left(\frac{k_0}{H_0\mu_0} + \beta_2^*\right)$, $l_3 = \frac{(3\lambda+2\mu)\alpha_t\theta_0}{\lambda+2\mu}\left(\frac{k_0}{H_0\mu_0} + \beta^*\right)$,

$\beta_1^* = 1 + \alpha_1 s$, $\beta_2^* = 1 + \alpha_2 s$, $\beta^* = 1 + \frac{3\lambda\alpha_1+2\mu\alpha_2}{3\lambda+2\mu}s$,

$L_1 = \frac{\eta c^2(s+\tau_0^\alpha s^{\alpha+1})}{K}$, $L_2 = \frac{(3\lambda+2\mu)\alpha_t c^2\beta^*(s+\tau_0^\alpha s^{\alpha+1})}{K}$,

$M_1 = \frac{(3\lambda+2\mu)\alpha_t\theta_0(k_0+H_0\mu_0\beta^*)L_1}{H_0\mu_0(\mu_0 H_0^2\rho c^2+\rho c^2+2\mu\beta_1^*+\lambda\beta_2^*)}$, $M_1 = \frac{\rho c^2 s^2+(3\lambda+2\mu)\alpha_t\theta_0(k_0+H_0\mu_0\beta^*)L_2}{H_0\mu_0(\mu_0 H_0^2\rho c^2+\rho c^2+2\mu\beta_1^*+\lambda\beta_2^*)}$.

4. Solution of problem in state pace domain

We chosen as state variables the temperature increment $\bar{\theta}$ and strain component $\bar{e}$, then the equations (30) and (31) in matrix form:

(34)
$$\nabla^2 V = X\bar{V}$$

where

$$\bar{V} = \begin{bmatrix} \bar{\theta} \\ \bar{e} \end{bmatrix}, \; X = \begin{bmatrix} L_1 & L_2 \\ M_1 & M_2 \end{bmatrix}.$$

The solution of equation (34) is

(35)
$$\bar{V}(r,s) = B_1\frac{e^{-\sqrt{X(s)}r}}{r} + B_2\frac{e^{-\sqrt{X(s)}r}}{r}.$$

Consider r large for bounded solution, cancelled the positive power of exponential part. Also at $r = R$ the value of B_1 is given by $B_1 = R\bar{V}(R,s)e^{\sqrt{X(s)}R}$, then equation (35) reduces $r \gg R$ to,

$$\bar{V}(R,s) = \frac{R}{r}\bar{V}(R,s)e^{-\sqrt{X(s)}R}.$$

The Characteristic equation corresponding to $X(s)$ as:

$$m^2 - (M_2 + L_1)m + (M_2L_1 - M_1L_2) = 0.$$

The eigen values m_1 and m_2 must satisfy:

$$m_1 + m_2 = M_2 + L_1 \quad m_1 m_2 = M_2 L_1 - M_1 L_2.$$

The Tailor's series expansion for $e^{-\sqrt{X(s)}(r-R)}$ has the form:

$$e^{-\sqrt{X(s)}(r-R)} = \sum_{n=0}^{\infty} \frac{\left[-\sqrt{X(s)}(r-R)\right]^n}{n!}. \tag{36}$$

Making use of Cayley-Hamiltonian theorem, the equation (36) becomes:

$$e^{-\sqrt{X(s)}(r-R)} = b_1 I + b_2 X, \tag{37}$$

where b_1 and b_2 are constants.

By Cayley-Hamiltonian theorem, the eigen values m_1 and m_2 of the matrix X satisfy the equation (37).

$$e^{-\sqrt{m_1}(r-R)} = b_1 + b_2 m_1 \quad \text{and} \quad e^{-\sqrt{m_2}(r-R)} = b_1 + b_2 m_2,$$

Solving the above system of equations, is obtained:

$$b_1 = \frac{m_1 e^{-\sqrt{m_2}(r-R)} - m_2 e^{-\sqrt{m_1}(r-R)}}{m_1 - m_2} \quad \text{and} \quad b_2 = \frac{e^{-\sqrt{m_1}(r-R)} - e^{-\sqrt{m_2}(r-R)}}{m_1 - m_2}.$$

Hence, we have:

$$e^{-\sqrt{X(s)}(r-R)} = L_{ij}, \quad i, j = 1, 2,$$

where

$$\begin{aligned} L_{11} &= \frac{e^{-\sqrt{m_1}(r-R)}(L_1 - m_2) - e^{-\sqrt{m_2}(r-R)}(L_1 - m_1)}{m_1 - m_2}, \\ L_{12} &= \frac{L_2 e^{-\sqrt{m_1}(r-R)} - L_1 e^{-\sqrt{m_2}(r-R)}}{m_1 - m_2}, \\ L_{21} &= \frac{M_1 e^{-\sqrt{m_2}(r-R)} - M_2 e^{-\sqrt{m_1}(r-R)}}{m_1 - m_2}, \\ L_{22} &= \frac{e^{-\sqrt{m_1}(r-R)}(M_2 - m_2) + e^{-\sqrt{m_2}(r-R)}(m_1 - M_2)}{m_1 - m_2}. \end{aligned} \tag{38}$$

Using the equation (32) and (33) into equation (35) and using equation(38), we obtain:

$$\bar{\theta}(r, s) = \frac{R\theta_0}{s(m_1 - m_2)r} \left[(m_1 - L_2)e^{-\sqrt{m_2}(r-R)} - (L_1 - m_2)e^{-\sqrt{m_1}(r-R)}\right] \tag{39}$$

$$\bar{e}(r, s) = \frac{RM_1\theta_0}{s(m_1 - m_2)r} \left[(m_2 - L_1)e^{-\sqrt{m_1}(r-R)} - (L_2 - m_1)e^{-\sqrt{m_2}(r-R)}\right]. \tag{40}$$

Taking Laplace transform for equation (23) using equations (39) and (40), we get displacement function as follows:

(41)

$$\bar{u}(r,s) = \frac{R\theta_0}{sr^2(m_1-m_2)r}\left[\left((D_2(L_1-m_2)(1+r\sqrt{m_1}))-D_1M_1\right)e^{-\sqrt{m_1}(r-R)}\right.$$
$$\left.+\left((1+r\sqrt{m_2})(D_2(m_1-L_2))+D_1M_1\right)e^{-\sqrt{m_2}(r-R)}\right],$$

where

$$D_1 = \frac{1}{s^2}+\frac{2\mu\beta_1^*+\lambda\beta_2^*}{\rho c^2 s^2}, \qquad D_2 = \frac{(3\lambda+2\mu)\alpha_t\theta_0\beta^*}{\rho c^2 s^2}.$$

Using equations (28-29) and (39-41), we obtain the stresses as:

(42)

$$\bar{\sigma}_{rr} = \frac{R\theta_0}{sr^2(m_1-m_2)r}\left\{e^{-\sqrt{m_1}(r-R)}\left((l_3+l_1)M_1r^2+2l_1M_1D_1(1+r\sqrt{m_1})\right.\right.$$
$$\left.-(L_1-m_2)(l_3r^2+2l_1D_2(1+r\sqrt{m_1}))\right)+e^{-\sqrt{m_2}(r-R)}\left(-M_1(l_1+l_2)r^2\right.$$
$$\left.\left.+2l_1D_1M_1(1+r\sqrt{m_1})-(L_1-m_2)(l_3r^2+2l_1D_2(1+r\sqrt{m_2}))\right)\right\}$$

(43)

$$\bar{\sigma}_{\theta\theta} = \frac{R\theta_0}{sr^2(m_1-m_2)r^2}\left\{e^{-\sqrt{m_1}(r-R)}\left[l_1D_1M_1-(L_1-m_2)(l_1D_1(1+r\sqrt{m_1})\right.\right.$$
$$\left.+(l_2M_1+l_3)r^3)\right]+\ e^{-\sqrt{m_2}(r-R)}\left[l_1D_1M_1+(m_1-L_2)(l_1D_1(1+r\sqrt{m_2})\right.$$
$$\left.\left.+(l_2M_1+l_3)r^3)\right]\right\}.$$

5. Inversion of the Laplace transforms

The solution of temperature, displacement and stresses is obtained numerically by the inversion of Laplace transform method in time domain in [11].

$$f(t) = \frac{e^{\gamma t}}{t}\left(\frac{1}{2}\bar{f}(\gamma)+Re\left(\sum_{k=1}^{N}(-1)^k\bar{f}(\gamma+ik\pi/t)\right)\right).$$

6. Results and Discussion

The copper material was chosen for the purpose of numerical evaluation. We have taken the parameters as following Table 1.

The computational mathematical software PTC Mathcad Prime-3.1 was used to obtained the numerical calculation and graphs. Figures 1-4, depicts the variation of temperature, displacement, stresses in radial distance r at instants

TABLE 1. Material constants

T_0 = 293 K	ρ = 8954 kg.m^{-3}	K = 386 W. m^{-1} K^{-1}
λ = 7.76 $\cdot 10^{10}$ kg.m^{-1} s^{-2}	α_t = 1.78 $\cdot$ 10^{-5}K^{-1}	θ_0 = 1 , R=1
C_E = 383.1 J. kg^{-1} K^{-1}	μ = 3.86 $\cdot 10^{10}$ kg.m^{-1} s^{-2}	μ_0 = 4 π $\cdot 10^{-7}$ H m^{-1}
H_0=4 π $\cdot 10^{-7}$ H m^{-1}	α_1 = 0.325 s, α_2 = 0.325 s	τ_0 =0.02 s

$\alpha = 0.25$ for time parameter $t = 0.25, 0.50, 0.75, 1$. From Figure 1 the temperature profile decreases with time increases within the region $0 \leq r \leq 0.5$ and then increases with increasing the radius. From Figure 2. the displacement increases with time increases within the region $0 \leq r \leq 0.5$ and then decreases with increasing the radius. Figure 3. and Figure 4. shows the radial and angular stress distribution for different time parameters. it is clear that the time increases the stresses increases with increasing the radial distance.

Figure 5-8, depicts the temperature, displacement, stresses in radial direction at time $t = 0.50$ for different values of $\alpha = 0.25, 0.50, 0.75, 1$. From Figure 5. we can see that temperature distribution initially decreases with the value of α increases and then increases with increasing the radial distance. Figure 6. depicts the displacement in radial direction with different fractional order parameter at time $t = 0.5$. It is clear that displacement increases with the value of α increases and then decreases with increasing the radial distance.

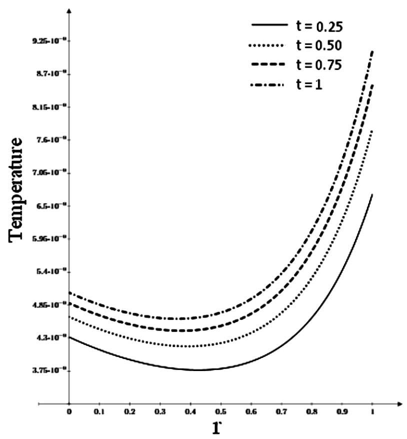

Figure 1. Temperature.

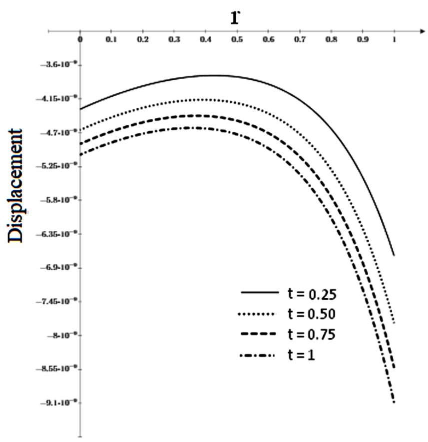

Figure 2. Displacement.

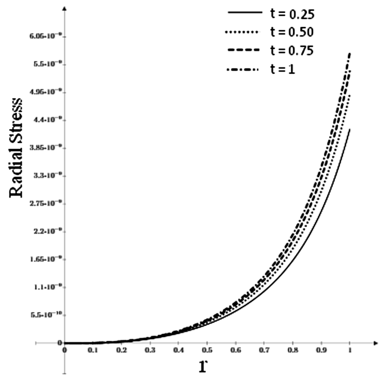

Figure 3. Radial stress.

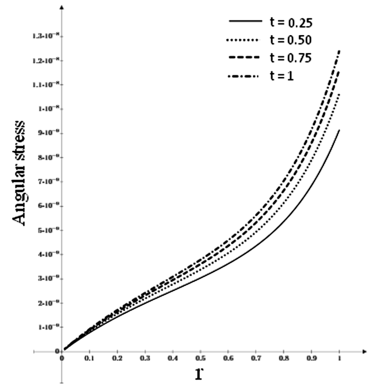

Figure 4. Angular stress.

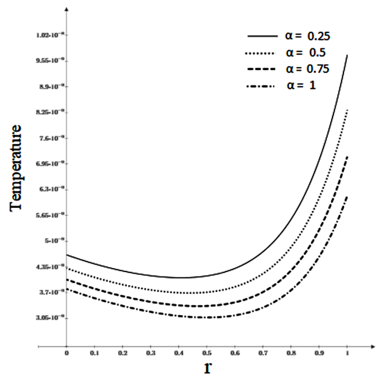

Figure 5. Temperature.

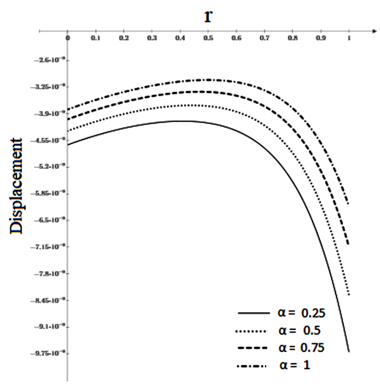

Figure 6. Displacement.

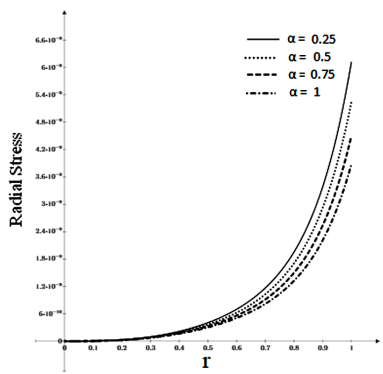

Figure 7. Radial stress.

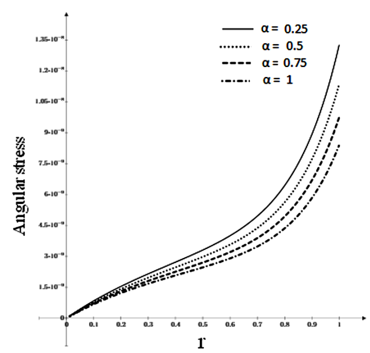

Figure 8. Angular stress.

Figure 7 and Figure 8 shows the radial and angular stress distribution for different fractional order parameter at time $t = 0.5$. We observed that the fractional order parameter increases the stresses increases with increasing the radius.

7. Conclusion

This article analyzed the modified Ohm's law with the temperature gradient of generalized theory of magneto-thermo-viscoelastic for a thermally, isotropic and electrically infinite material with a spherical region using fractional order derivative. The general solution obtained from Laplace transform, numerical Laplace inversion and state space approach. Figures 1-4, shows the temperature, displacement and thermal stresses at $\alpha = 0.25$ for times $t = 0.25, 0.50, 0.75, 1$. From Figures 5-8, we see that the time fractional derivatives play the significant role on the all quantities in the given field and changes in the values of the parameter α. We conclude that the speed of propagation of strain is finite, coincide the behavior of viscoelastic material. This work may prove helpful in material science, designers, real life engineering problems, physicists and in understanding the concept of a theory of magneto-thermo-viscoelasticity.

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