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MODULAR COLORINGS OF CORONA PRODUCT OF P_m WITH C_n

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ABSTRACT. For $\ell \geq 2$, a modular ℓ-colouring of a graph $\mathcal{G}$ except singleton nodes is a colouring of the nodes of $\mathcal{G}$ with the elements in $\mathbb{Z}_\ell$ have the possessions that for every two adjacent nodes of $\mathcal{G}$, the sums of the colours of that neighbours are different in $\mathbb{Z}_\ell$. The lower ℓ for that $\mathcal{G}$ has a modular ℓ-colouring is the modular chromatic number of $\mathcal{G}$. In this paper, we discussed the modular chromatic number of corona product of paths with cycles.

1. INTRODUCTION

Generally we follow [1] for basic definitions and notations. A node v of a graph $\mathcal{G}$, let $N_\mathcal{G}(v)$, the *neighbor set of* v, shows the collection of elements adjacent to v in $\mathcal{G}$.

In [2], for a graph $\mathcal{G}$ without singleton nodes, let $c : \mathcal{V}(\mathcal{G}) \to \mathbb{Z}_\ell$, $\ell \geq 2$, be a node colouring of $\mathcal{G}$ where adjacent nodes probably the same colour. The *colour sum* $\mathcal{S}(v) = \sum_{u \in N_\mathcal{G}(v)} c(u)$ of a node v of $\mathcal{G}$ is the sum of the colours of the nodes in $N_\mathcal{G}(v)$. The colouring c is called a *modular ℓ-colouring* of $\mathcal{G}$ if $\mathcal{S}(x) \neq \mathcal{S}(y)$ in $\mathbb{Z}_\ell$ for all pairs x, y of adjacent nodes in $\mathcal{G}$. The *modular chromatic number* $mc(\mathcal{G})$ of $\mathcal{G}$ is the minimum ℓ for which $\mathcal{G}$ has a modular ℓ-colouring, [3, 4].

The *corona* of two graphs $\mathcal{G}$ and $\mathcal{H}$ is the graph $\mathcal{G} \circ \mathcal{H}$ formed from one copy of $\mathcal{G}$ and $|\mathcal{V}(\mathcal{G})|$ copies of $\mathcal{H}$, where the ith node of $\mathcal{G}$ is adjacent to every node in the ith copy of $\mathcal{H}$. For convenience, we define $[k] = \{1, 2, 3, \ldots, k\}$.

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2. Corona of P_m with C_n

Define $\mathcal{V}(P_m) = \{u_1, u_2, u_3, \dots, u_m\}$; $\mathcal{V}(C_n) = \{v_1, v_2, v_3, \dots, v_n\}$; $\mathcal{E}(P_m) = \{u_1u_2, u_2u_3, u_3u_4, \dots, u_{m-1}u_m\}$; $\mathcal{E}(C_n) = \{v_1v_2, v_2v_3, v_3v_4, \dots, v_nv_1\}$; $\mathcal{V}(P_m \circ C_n) = \mathcal{V}(P_m) \cup \{v_s^r : r \in [m]$ and $s \in [n]\}$; $\mathcal{E}(P_m \circ C_n) = \mathcal{E}(P_m) \bigcup \{v_s^r v_{s+1}^r : r \in [m]$ and $s \in [n-1]\} \bigcup \{u_r v_s^r : s \in [n]$ and $r \in [m]\} \bigcup \{v_n^r v_1^r : r \in [m]\}$.

In [5,6] the authors obtained modular chromatic number of $C_m \Box P_n$ and modular chromatic number of $C_m \Box C_n$.

Theorem 2.1. *For n even, $m \geq 2$, $n \geq 4$, $m \not\equiv 0 (\mod 2)$ and $n \not\equiv 4 (\mod 6)$, $mc(P_m \circ C_n) = 3$.*

Proof. Let $c : \mathcal{V}(P_m \circ C_n) \to \mathbb{Z}_3$.

Case 1. $n \equiv 0 (\mod 6)$. First, for m odd. Define c as follows: $c(u_r) = 0$ if $r \equiv 0, 1, 2 (\mod 4)$; $c(u_r) = 1$ if $r \equiv 3 (\mod 4)$; $c(v_s^r) = 1$ if $r \equiv 1 (\mod 4)$, $s \equiv 1 (\mod 2)$; $c(v_s^r) = 2$ if $r \equiv 1, 3 (\mod 4)$, $s \equiv 0 (\mod 2)$; $c(v_s^r) = 0$ if $r \equiv 0, 2, 3 (\mod 4)$, $s \equiv 1 (\mod 2)$; $c(v_s^r) = 1$ if $r \equiv 0, 2 (\mod 4)$, $s \equiv 0 (\mod 2)$; then $\mathcal{S}(u_r) = 0$ if r odd; $\mathcal{S}(u_r) = 1$ if r even; $\mathcal{S}(v_s^r) = 0$ if r, s even; $\mathcal{S}(v_s^r) = 1$ if $r \equiv 1 (\mod 4)$, s odd; $\mathcal{S}(v_s^r) = 1$ if $r \equiv 3 (\mod 4)$, s even; $\mathcal{S}(v_s^r) = 2$ if $r \equiv 1 (\mod 4)$, s even; $\mathcal{S}(v_s^r) = 2$ if $r \equiv 3 (\mod 4)$, s odd; $\mathcal{S}(v_s^r) = 2$ if $r \equiv 0 (\mod 2)$, s odd.

Next, for m even. Define c as follows: $c(u_r) = 0$ if $r \equiv 0, 1, 3 (\mod 4)$; $c(u_r) = 1$ if $r \equiv 2 (\mod 4)$; $c(v_s^r) = 1$ if $r \equiv 1 (\mod 2)$, $s \equiv 0 (\mod 2)$; $c(v_s^r) = 2$ if $r \equiv 0 (\mod 2)$, $s \equiv 0 (\mod 2)$; $c(v_s^r) = 0$ if $r \equiv 1, 2, 3 (\mod 4)$, $s \equiv 1 (\mod 2)$; $c(v_s^r) = 1$ if $r \equiv 0 (\mod 4)$, $s \equiv 1 (\mod 2)$; then $\mathcal{S}(u_r) = 1$ if r odd; $\mathcal{S}(u_r) = 0$ if r even; $\mathcal{S}(v_s^r) = 0$ if r odd, s even; $\mathcal{S}(v_s^r) = 2$ if r, s odd; $\mathcal{S}(v_s^r) = 1$ if $r \equiv 2 (\mod 4)$, s even; $\mathcal{S}(v_s^r) = 2$ if $r \equiv 2 (\mod 4)$, s odd; $\mathcal{S}(v_s^r) = 2$ if $r \equiv 0 (\mod 4)$, s even; $\mathcal{S}(v_s^r) = 1$ if $r \equiv 0 (\mod 4)$, s odd.

Case 2. $n \equiv 2 (\mod 6)$. Define c as follows: $c(u_r) = 0$ if $r \in \{1, 2, 3, \dots\}$; $c(v_s^r) = 0$ if $r \in \{1, 2, 3, \dots\}$, s even; $c(v_s^r) = 1$ if r, s odd; $c(v_s^r) = 2$ if r even, s odd; then $\mathcal{S}(u_r) = 1$ if r odd; $\mathcal{S}(u_r) = 2$ if r even; $\mathcal{S}(v_s^r) = 1$ if r, s even; $\mathcal{S}(v_s^r) = 2$ if r odd, s even; $\mathcal{S}(v_s^r) = 0$ if $r \in \{1, 2, 3, \dots\}$, s odd.

Case 3. $n \equiv 4 (\mod 6)$ and m odd. Define c as follows:
$c(u_r) = 0$ if r even; $c(u_r) = 1$ if r odd; $c(v_s^r) = 0$ if $r \in [m]$; s even; $c(v_s^r) = 1$ if $r \in [m]$; s odd; then $\mathcal{S}(u_r) = 1$ if r even; $\mathcal{S}(u_r) = 2$ if r odd; $\mathcal{S}(v_s^r) = 0$ if r odd,

s even; $\mathcal{S}(v_s^r) = 0$ if r even, s odd; $\mathcal{S}(v_s^r) = 1$ if r, s odd; $\mathcal{S}(v_s^r) = 2$ if r, s even. Clearly, $mc(P_m \circ C_n) \geq \chi(P_m \circ C_n) = 3$. Hence, $mc(P_m \circ C_n) = 3$. □

Theorem 2.2. *For n odd, $m \geq 2$, $n \geq 3$, $mc(P_m \circ C_n) = 4$.*

Proof. Let $c : \mathcal{V}(P_m \circ C_n) \to \mathbb{Z}_4$.

Case 1. $n = 3$.

Subcase 1.1. $m \equiv 0, 2, 3 \pmod 4$. Define c as follows: $c(u_r) = 0$ if $r \equiv 0, 1, 3 \pmod 4$; $c(u_r) = 1$ if $r \equiv 2 \pmod 4$; $c(v_1^r) = 0$ if $r \equiv 1, 2, 3 \pmod 4$; $c(v_2^r) = 1$ if $r \equiv 1, 3 \pmod 4$; $c(v_3^r) = 2$ if $r \equiv 1, 3 \pmod 4$; $c(v_1^r) = 1$ if $r \equiv 0 \pmod 4$; $c(v_2^r) = 2$ if $r \equiv 0, 2 \pmod 4$; $c(v_3^r) = 3$ if $r \equiv 0, 2 \pmod 4$; then $\mathcal{S}(u_r) = 0$ if r odd; $\mathcal{S}(u_r) = 1$ if $r \equiv 2 \pmod 4$; $\mathcal{S}(u_r) = 2$ if $r \equiv 0 \pmod 4$; $\mathcal{S}(v_1^r) = 3$ if r odd; $\mathcal{S}(v_2^r) = 2$ if r odd; $\mathcal{S}(v_3^r) = 1$ if r odd; $\mathcal{S}(v_1^r) = 2$ if $r \equiv 2 \pmod 4$; $\mathcal{S}(v_2^r) = 0$ if $r \equiv 2 \pmod 4$; $\mathcal{S}(v_3^r) = 3$ if $r \equiv 2 \pmod 4$; $\mathcal{S}(v_1^r) = 1$ if $r \equiv 0 \pmod 4$; $\mathcal{S}(v_2^r) = 0$ if $r \equiv 0 \pmod 4$; $\mathcal{S}(v_3^r) = 3$ if $r \equiv 0 \pmod 4$.

Subcase 1.2. $m \equiv 1 \pmod 4$. Define c as follows: $c(u_r) = 0$ if $r \equiv 0, 1, 2 \pmod 4$; $c(u_r) = 1$ if $r \equiv 3 \pmod 4$; $c(v_1^r) = 0$ if $r \equiv 0, 2, 3 \pmod 4$; $c(v_2^r) = 1$ if r even; $c(v_3^r) = 2$ if r even; $c(v_1^r) = 1$ if $r \equiv 1 \pmod 4$; $c(v_2^r) = 2$ if r odd; $c(v_3^r) = 3$ if r odd; then $\mathcal{S}(u_r) = 0$ if r even; $\mathcal{S}(u_r) = 1$ if $r \equiv 3 \pmod 4$; $\mathcal{S}(u_r) = 2$ if $r \equiv 1 \pmod 4$; $\mathcal{S}(v_1^r) = 1$ if $r \equiv 1 \pmod 4$; $\mathcal{S}(v_2^r) = 0$ if $r \equiv 1 \pmod 4$; $\mathcal{S}(v_3^r) = 3$ if $r \equiv 1 \pmod 4$; $\mathcal{S}(v_1^r) = 3$ if r even; $\mathcal{S}(v_2^r) = 2$ if r even; $\mathcal{S}(v_3^r) = 1$ if r even; $\mathcal{S}(v_1^r) = 2$ if $r \equiv 3 \pmod 4$; $\mathcal{S}(v_2^r) = 0$ if $r \equiv 3 \pmod 4$; $\mathcal{S}(v_3^r) = 3$ if $r \equiv 3 \pmod 4$.

Case 2. $n \equiv 5 \pmod 8$. Define c as follows: $c(u_r) = 0$ if $r \in [m]$; $c(v_s^r) = 0$ if $s \equiv 0, 2, 3 \pmod 4$, $r \in [m]$; $c(v_s^r) = 2$ if $s \equiv 1 \pmod 4$, $s \neq n$ and $r \in [m]$; $c(v_n^r) = 1$ if r even; $c(v_n^r) = 3$ if r odd; then $\mathcal{S}(u_r) = 1$ if r odd; $\mathcal{S}(u_r) = 3$ if r even; $\mathcal{S}(v_s^r) = 0$ if $r \in [m]$, $s \in \{3, 5, 7, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 2$ if $r \in [m]$, $s \in \{2, 4, 6, \ldots, n-3, n\}$; $\mathcal{S}(v_s^r) = 1$ if r even, $s \in \{1, n-1\}$; $\mathcal{S}(v_s^r) = 3$ if r odd, $s \in \{1, n-1\}$.

Case 3. $n \equiv 7 \pmod 8$. Define c as follows: $c(u_r) = 0$ if $r \in [m]$; $c(v_s^r) = 0$ if $s \equiv 0, 2, 3 \pmod 4$, $r \in [m]$; $c(v_s^r) = 2$ if $s \equiv 1 \pmod 4$, $s \neq n-2$ and $r \in [m]$; $c(v_{n-2}^r) = 1$ if r even; $c(v_{n-2}^r) = 3$ if r odd; then $\mathcal{S}(u_r) = 1$ if r odd; $\mathcal{S}(u_r) = 3$ if r even; $\mathcal{S}(v_s^r) = 0$ if $r \in [m]$, $s \in \{1, 3, 5, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 2$ if $r \in [m]$, $s \in \{2, 4, 6, \ldots, n-5, n\}$; $\mathcal{S}(v_s^r) = 1$ if r even, $s \in \{n-1, n-3\}$; $\mathcal{S}(v_s^r) = 3$ if r odd, $s \in \{n-1, n-3\}$.

Case 4. $n \equiv 1 \pmod 8$. First, for m odd. Define c as follows: $c(u_r) = 0$ if r even; $c(u_r) = 1$ if r odd; $c(v_s^r) = 0$ if $r \in [m]$, $s \equiv 0, 2, 3 \pmod 4$; $c(v_s^r) = 2$ if r odd, $s \in \{1, 5, 9, \ldots, n-8\}$; $c(v_s^r) = 1$ if r odd, $s \in \{n-4, n\}$; $c(v_s^r) = 2$ if r even, $s \in \{1, 5, 9, \ldots, n-4\}$; $c(v_n^r) = 1$ if r even; then $\mathcal{S}(u_r) = 0$ if r odd; $\mathcal{S}(u_r) = 3$ if r even; $\mathcal{S}(v_s^r) = 0$ if r even, $s \in \{3, 5, 7, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 1$ if r odd, $s \in \{3, 5, 7, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 2$ if r even, $s \in \{2, 4, 6, \ldots, n-3\}$; $\mathcal{S}(v_s^r) = 2$ if r odd, $s \in \{1, n-5, n-3, n-1\}$; $\mathcal{S}(v_s^r) = 3$ if r odd, $s \in \{2, 4, 6, \ldots, n-7, n\}$; $\mathcal{S}(v_s^r) = 1$ if r even, $s \in \{1, n-1\}$.

Next, for m even. Define c as follows: $c(u_r) = 0$ if r even; $c(u_r) = 1$ if r odd; $c(v_s^r) = 0$ if $r \in [m]$, $s \equiv 0, 2, 3 \pmod 4$; $c(v_s^r) = 2$ if r odd, $s \in \{1, 5, 9, \ldots, n-8\}$; $c(v_s^r) = 1$ if r odd, $s \in \{n-4, n\}$; $c(v_s^r) = 2$ if $r \in \{2, 4, 6, \ldots m-2\}$, $s \in \{1, 5, 9, \ldots, n-4\}$; $c(v_n^r) = 1$ if $r \in \{2, 4, 6, \ldots m-2\}$; $c(v_s^m) = 1$ if $s \in \{1, 5, 9, \ldots, n-8\}$; $c(v_s^m) = 3$ if $s \in \{n-4, n\}$; then $\mathcal{S}(u_r) = 0$ if $r \in \{1, 3, 5, \ldots, m-1\}$; $\mathcal{S}(u_r) = 3$ if $r \in \{2, 4, 6, \ldots, m-2\}$; $\mathcal{S}(u_m) = 1$; $\mathcal{S}(v_s^r) = 0$ if r even, $s \in \{3, 5, 7, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 1$ if r odd, $s \in \{3, 5, 7, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 2$ if $r \in \{2, 4, 6, \ldots, m-2\}$, $s \in \{2, 4, 6, \ldots, n-3\}$; $\mathcal{S}(v_s^m) = 2$ if $s \in \{2, 4, 6, \ldots, n-7\}$; $\mathcal{S}(v_s^r) = 2$ if r odd, $s \in \{n-5, n-3, n-1, 1\}$; $\mathcal{S}(v_s^r) = 3$ if r odd, $s \in \{2, 4, 6, \ldots, n-7, n\}$; $\mathcal{S}(v_s^m) = 3$ if $s \in \{n-5, n-3, n-1, 1\}$.

Case 5. $n \equiv 3 \pmod 8$.

First, for m odd. Define c as follows: $c(u_r) = 0$ if r even; $c(u_r) = 1$ if r odd; $c(v_s^r) = 0$ if $r \in [m]$, $s \equiv 0, 2, 3 \pmod 4$; $c(v_s^r) = 2$ if r odd, $s \in \{1, 5, 9, \ldots, n-10\}$; $c(v_s^r) = 1$ if r odd, $s \in \{n-6, n-2\}$; $c(v_s^r) = 2$ if r even, $s \in \{1, 5, 9, \ldots, n-6\}$; $c(v_{n-2}^r) = 1$ if r even; then $\mathcal{S}(u_r) = 0$ if r odd; $\mathcal{S}(u_r) = 3$ if r even; $\mathcal{S}(v_s^r) = 0$ if r even, $s \in \{1, 3, 5, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 1$ if r odd, $s \in \{1, 3, 5, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 2$ if r even, $s \in \{2, 4, 6, \ldots, n-5\}$; $\mathcal{S}(v_s^r) = 2$ if r odd, $s \in \{n-7, n-5, n-3, n-1\}$; $\mathcal{S}(v_s^r) = 3$ if r odd, $s \in \{2, 4, 6, \ldots, n-9, n\}$; $\mathcal{S}(v_s^r) = 1$ if r even, $s \in \{n-1, n-3\}$.
Next, for m even. Define c as follows: $c(u_r) = 0$ if r even; $c(u_r) = 1$ if r odd; $c(v_s^r) = 0$ if $r \in [m]$, $s \equiv 0, 2, 3 \pmod 4$; $c(v_s^r) = 2$ if r odd, $s \in \{1, 5, 9, \ldots, n-10\}$; $c(v_s^r) = 1$ if r odd, $s \in \{n-6, n-2\}$; $c(v_s^r) = 2$ if $r \in \{2, 4, 6, \ldots m-2\}$, $s \in \{1, 5, 9, \ldots, n-6\}$; $c(v_{n-2}^r) = 1$ if $r \in \{2, 4, 6, \ldots m-2\}$; $c(v_s^m) = 1$ if $s \in \{1, 5, 9, \ldots, n-10\}$; $c(v_s^m) = 3$ if $s \in \{n-6, n-2\}$; then $\mathcal{S}(u_r) = 0$ if r odd; $\mathcal{S}(u_r) = 3$ if $r \in \{2, 4, 6, \ldots, m-2\}$; $\mathcal{S}(u_m) = 1$; $\mathcal{S}(v_s^r) = 0$ if r even, $s \in \{1, 3, 5, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 1$ if r odd, $s \in \{1, 3, 5, \ldots, n-2\}$; $\mathcal{S}(v_s^r) = 2$ if $r \in \{2, 4, 6, \ldots, m-2\}$, $s \in \{2, 4, 6, \ldots, n-5\}$; $\mathcal{S}(v_s^m) = 2$ if $s \in \{2, 4, 6, \ldots, n-9, n\}$; $\mathcal{S}(v_s^r) = 2$ if r odd,

$s \in \{n-7, n-5, n-3, n-1\}$; $\mathcal{S}(v_s^r) = 3$ if r odd, $s \in \{2, 4, 6, \ldots, n-9, n\}$; $\mathcal{S}(v_s^m) = 3$ if $s \in \{n-7, n-5, n-3, n-1\}$. Clearly, $mc(P_m \circ C_n) \geq \chi(P_m \circ C_n) = 4$. Hence, $mc(P_m \circ C_n) = 4$. □

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