ADV MATH SCI JOURNAL

Advances in Mathematics: Scientific Journal **9** (2020), no.11, 9837–9847
ISSN: 1857-8365 (printed); 1857-8438 (electronic)
https://doi.org/10.37418/amsj.9.11.94 Spec. Iss. on ICRTAMS-2020

SOME FIXED POINT RESULTS IN d-COMPLETE TOPOLOGICAL SPACES

S. RATHEE, P. GUPTA[1], AND VISHNU NARAYAN MISHRA[1]

ABSTRACT. This paper aims to use T-orbitally lower semi-continuous and w-continuous functions in d-complete topological spaces to validate some fixed point theorems and extend various known results. The paper also seeks to establish, in the setting of d-complete topological spaces, Mizoguchi-Takahashi's type coincidence point theorem for single valued map. The results are supported by illustrative examples.

1. INTRODUCTION

In mathematics, the fixed point theory is very important concept. A famous result in the fixed point theory has been created by Banach in 1922 called Banach contraction principle. Later, regarding the fixed point theory, more work have been thoroughly introduced by many authors in various spaces. Problems in sciences and applied mathematics are being increasingly solved by fixed point theorems (FPT) in metric spaces. Many authors like Hicks in [2], Popa in [9], Hicks and Rhoades in [3], Iseki in [4] and Kasahara in [5], [6], have established metric FPT in d-complete topological spaces (TS) under certain conditions. Some examples of d-complete TS are complete metric spaces (CMS) and complete quasi metric spaces.

[1]*corresponding authors*

2010 *Mathematics Subject Classification.* 46T99, 47H10, 54H25.

Key words and phrases. Fixed point, T-orbitally lower semi-continuous function, w-continuous function, MT-function, d-complete topological space.

The present paper uses T-orbitally lower semi-continuous and w-continuous functions in d-complete TS and proves FPT to generalize the results of Hicks, [2], Krayilan and Telci, [7] in d-complete TS and results proved by Shatanawi, [10] in generalized metric spaces. It also proves, in the setting of d-complete TS, Mizoguchi-Takahashi's type coincidence point theorem for single valued map which has been proved by Ali in [1] in CMS.

2. PRELIMINARIES

Before going to the main work, we need some preliminaries which are as follows:

Definition 2.1. *[2], For a TS (U, κ), let $d : U \times U \to [0, \infty)$ be such that $d(p, q) = 0$ if and only if $p = q$. Then, the triplet (U, κ, d) is forenamed as d-complete TS if $\sum_{n=0}^{\infty} d(p_n, p_{n+1}) < \infty \implies$ the sequence $< p_n >$ converges in U.*

Definition 2.2. *[9], Let g and h be self mappings on a TS (U, κ).*

(i) *A point $p \in U$ is forenamed as a fixed point(FP) of g if $p = gp$.*
(ii) *A point $p \in U$ is forenamed as a coincidence point of g and h if $gp = hp$.*

Definition 2.3. *[2], Let $T : U \to U$ be a mapping. T is w-continuous at p if $p_n \to p \implies Tp_n \to Tp$ as $n \to \infty$.*

Definition 2.4. *[2], Let $T : U \to U$ be a mapping. The set $O_T(p, \infty) = \{p, Tp, T^2p, ...\}$ is named as orbit of p.*

Definition 2.5. *[2], Let $T : U \to U$ be a mapping on d-complete TS (U, κ, d), and $p \in U$,*

(i) *a mapping H is forenamed as lower continuous at p if, for any sequence $< p_n >$ in U , $p_n \to p$ as $n \to \infty \implies H(p) \leq liminf_{n\to\infty} H(p_n)$.*
(ii) *A mapping $H : U \to [0, \infty)$ is forenamed as "T-orbitally lower semi-continuous" relative to p if, for any sequence $< p_n >$ in $O_T(p, \infty)$, $p_n \to p$ as $n \to \infty \implies H(p) \leq liminf_{n\to\infty} H(p_n)$.*

Definition 2.6. *[8], A function $\eta : [0, \infty) \to [0, \infty)$ is forenamed as MT-function if it satisfies "Mizoguchi-Takahashi's condition" (i.e. $\limsup_{s\to t^+} \eta(s) < 1$), $\forall\ t \in [0, \infty)$.*

Example 1. *Let* $\eta : [0,\infty) \to [0,1)$ *be defined by*

$$\eta(t) = \begin{cases} \frac{4}{5} & \text{if } 0 \le t \le \frac{1}{2} \\ \frac{2}{3} & t > \frac{1}{2}. \end{cases}.$$

Since $\limsup_{s\to t^+} \eta(s) < 1$*,* η *is an MT-function.*

Example 2. *[8], Let* $\eta : [0,\infty) \to [0,1)$ *be defined by*

$$\eta(t) = \begin{cases} \frac{sint}{t} & \text{if } t \in (0, \frac{\pi}{2}] \\ 0 & \text{otherwise}. \end{cases}.$$

Since $\limsup_{s\to t^+} \eta(s) = 1$*,* η *is not an MT-function.*

3. Main results

This section aims to include our main results. First of all, we prove FPT in d-complete TS with the help of T-orbitally lower semi-continuous function.

Theorem 3.1. *Let* T *be a self mapping on* d*-complete TS* (U,κ,d) *. Suppose* $\exists$ *a* $c_0 \in U$ *such that*

$$d(Tc,Td) \le \varrho(\max\{d(c,d), d(c,Tc), d(d,Td), d(Tc,d)\}), \tag{3.1}$$

$\forall\ c,d \in O_T(c_0,\infty)$ *where* $\varrho : (0,\infty) \to (0,\infty)$ *is a non-decreasing function with* $\sum_{n=1}^{\infty} \varrho^n(t) < \infty$ *and* $\varrho(t) < t\ \forall\ t > 0$*. Then,*

(i) $\lim_{n\to\infty} T^n c_0 = c'$*, exists.*
(ii) $Tc' = c'$ *iff* $F(c) = d(Tc,c)$ *is "*T*-orbitally lower semi-continuous" at* c' *relative to* c_0*.*

Proof.

(i) Let $c_n = T^n c_0\ \forall n \in N$ for some $c_0 \in U$.. Suppose $c_n \neq c_{n-1}$, $\forall\ n \in N$. Thus, for $n \in N$, we get

$$\begin{aligned} d(c_n, c_{n+1}) &= d(T^n c_0, T^{n+1} c_0) = d(T(T^{n-1}c_0), T(T^n c_0)) = d(Tc_{n-1}, Tc_n) \\ &\le \varrho(\max\{d(c_{n-1},c_n), d(c_{n-1},c_n), d(c_n,c_{n+1}), d(c_n,c_n)\}) \\ &= \varrho(\max\{d(c_{n-1},c_n), d(c_n,c_{n+1}), 0\}). \end{aligned}$$

If $\max\{d(c_{n-1}, c_n), d(c_n, c_{n+1})\} = d(c_n, c_{n+1})$, then $d(c_n, c_{n+1}) \leq \varrho(d(c_n, c_{n+1}) < d(c_n, c_{n+1})$, which is impossible. So, necessarily it is the case that

$$\max\{d(c_{n-1}, c_n), d(c_n, c_{n+1})\} = d(c_{n-1}, c_n),$$

and hence $d(c_n, c_{n+1}) \leq \varrho(d(c_{n-1}, c_n))$.

Thus, for $n \in N$, we have

$$\begin{array}{rcl} d(c_n, c_{n+1}) & = & d(T^n c_0, T^{n+1} c_0) \\ & \leq & \varrho(d(T^{n-1} c_0, T^n c_0)) \\ & = & \varrho((d(c_{n-1}, c_n)) \\ & \leq & \varrho^2(d(c_{n-2}, c_{n-1})) \\ & \cdots & \\ & \cdots & \\ & \leq & \varrho^n(d(c_0, c_1)), \quad for \;\; n = 1, 2, 3, \ldots. \end{array}$$

$$\begin{array}{rcl} S_n = \sum_{i=0}^{n} d(c_i, c_{i+1}) & = & d(c_0, c_1) + d(c_1, c_2) + \cdots + d(c_n, c_{n+1}) \\ & \leq & d(c_0, c_1) + \varrho(d(c_0, c_1)) + \cdots + \varrho^n(d(c_0, c_1)) \\ & = & \sum_{i=1}^{n} \varrho^i(d(c_0, c_1)) \leq \sum_{i=1}^{\infty} \varrho^i(d(c_0, c_1)) < \infty. \end{array}$$

In consequence, S_n is bounded above. Also S_n is non-decreasing. So S_n is convergent.

Thus, we get $\sum_{i=0}^{\infty} d(c_i, c_{i+1}) < \infty$. Since (U, κ) is d-complete TS, therefore $\exists\, c' \in U$ such that $\lim_{n\to\infty} T^n c_0 = c'$.

(ii) Suppose $Tc' = c'$ and $< c_n > \in O_T(c_0, \infty)$ with $\lim_{n\to\infty} c_n = c'$. Then, we have

$$F(c') = d(Tc', c') = 0 \leq \lim_{n\to\infty} inf d(Tc_n, c_n) = \lim_{n\to\infty} inf F(c_n)$$

and so F is "T-orbitally lower semi-continuous" at c' relative to c_0.

Conversely, suppose that $F(c) = d(Tc, c)$ is "T-orbitally lower semi-continuous" at c' relative to c_0. Now, sequence $< c_n > \in O_T(c_0, \infty)$, such that $\lim_{n\to\infty} c_n = c'$ then $c' \in O_T(c_0, \infty)$. Since $\sum_{i=0}^{\infty} d(c_i, c_{i+1}) < \infty$, we get $\lim_{n\to\infty} d(c_n, c_{n+1}) = 0$. Therefore

$$0 \le d(Tc', c') = F(c') \le \liminf_{n\to\infty} F(c_n) = \liminf_{n\to\infty} d(Tc_n, c_n)$$
$$= \liminf_{n\to\infty} d(c_{n+1}, c_n) = 0.$$

Thus, $d(Tc', c') = 0$ and so $Tc' = c'$.

This concludes the proof. □

Corollary 3.1. *Let T be a self mapping on a d-complete TS (U, κ, d). Presume $\exists$ a $c_0 \in U$ such that*

$$d(Tc, Td) \le \varrho(d(c, d)),$$

for all $c, d \in O_T(c_0, \infty)$ where $\varrho : [0, \infty) \to [0, \infty)$ $\forall$ $t > 0$. Then

(i) $\lim_{n\to\infty} T^n c_0 = c'$ *exists.*
(ii) $Tc' = c'$ *iff $F(c) = d(Tc, c)$ is "T-orbitally lower semi-continuous" at c' relative to c_0.*

Define $\varrho : [0, \infty) \to [0, \infty)$ as $\varrho(t) = kt$ (here $k < 1$) in above Corollary. Then we get the subsequent corollary:

Corollary 3.2. *Let T be a self mapping on a d-complete TS (U, κ, d). Presume $\exists$ a $c_0 \in U$ such that*

$$d(Tc, Td) \le k(d(c, d)),$$

$\forall$ $c, d \in O_T(c_0, \infty)$ and $t > 0$. Then

(i) $\lim_{n\to\infty} T^n c_0 = c'$ *exists.*
(ii) $Tc' = c'$ *iff $F(c) = d(Tc, c)$ is "T-orbitally lower semi-continuous" at c' relative to c_0.*

Example 3. *Let $U = \{0, 1\}$.*
Let a metric on U be ρ and on U, κ be a metric topology induced by ρ. Outline $d : U \times U \to [0, \infty)$ by

$$d(p, q) = \left|p^2 - q\right|,$$

$\forall$ $p, q \in U$. Then d is niether a quasi-metric nor a metric on U. Also d does not gratify the triangle inequality and symmetry property. So d is not a semi-metric. Though, (U, κ) is d-complete TS with d.

Case 1: *Outline $T : U \to U$ by*

$$Tp = 0.$$

Take $p = 1$. Then, we get $O_T(1,\infty) = \{1,0,0,0,\ldots\}$. Thus, T gratifies the inequality (2.1) $\forall$ $y \in O_T(1,\infty)$, where ϱ is a function as in Theorem 3.1. Also $F(p) = d(Tp,p)$ is "T-orbitally lower semi-continuous" at $p = 0$ relative to $p = 1$. Thus, all the conditions of Theorem 3.1 are gratified. Similarly all the conditions hold for T relative to $p = 0$. Clearly, $p = 0$ is a FP of T.

Case 2: *Outline $T : U \to U$ by*

$$Tp = 1.$$

For $p = 0$, $O_T(0,\infty) = \{0,1,1,1,\ldots\}$. Clearly, we can check that T gratifies the inequality (2.1) $\forall$ $q \in O_T(0,\infty)$ and $F(p) = d(Tp,p)$ is "T-orbitally lower semi-continuous" at $p = 1$ relative to $p = 0$. Thus, all the conditions of Theorem 3.1 are gratified. Similarly, conditions hold for T relative to $p = 1$. Thus, $p = 1$ is FP of T.

Now, we prove the FPT in d-complete TS by using the w-continuity which generalize the result of Krayilan et al., [7].

Theorem 3.2. *Let $T : U \to U$ be a self mapping on a d-complete Hausdorff TS (U,κ). Presume $\exists$ a $c_0 \in U$ such that*

$$d(Tc,Td) \le \varrho(\max\{d(c,d), d(c,Tc), d(d,Td), d(Tc,d)\}), \tag{3.2}$$

$\forall$ $c,d \in O_T(c_0,\infty)$ where $\varrho : (0,\infty) \to (0,\infty)$ is a function which is non-decreasing and $\varrho(0) = 0$.

Assuming T is w-continuous. Then T has FP iff there exists $c \in U$ with

$$\sum_{n=0}^{\infty} \varrho^n(d(Tc,c)) < \infty.$$

Proof. If $Tz = z$, then $d(Tz,z) = 0$. Since $\varrho^n(0) = 0$, the subsequent inequality $\sum_{n=0}^{\infty} \varrho^n(d(Tz,z)) < \infty$ is satisfied.

Conversely, assume that there exists a c in U with $\sum_{n=0}^{\infty} \varrho^n(d(Tc,c)) < \infty$. Let $c = c_0$. Outline the sequence $< c_n >$ inductively by

$$c_n = T^n c_0$$

for $n = 0,1,2,\ldots$. Assume $c_n \neq c_{n-1}$, $\forall$ $n \in N$. So, for $n \in N$, we get

$$\begin{aligned}
d(c_n,c_{n+1}) &= d(T^n c_0, T^{n+1}c_0) = d(T(T^{n-1}c_0), T(T^n c_0)) = d(Tc_{n-1}, Tc_n) \\
&\le \varrho(\max\{d(c_{n-1},c_n), d(c_{n-1},c_n), d(c_n,c_{n+1}), d(c_n,c_n)\}) \\
&= \varrho(\max\{d(c_{n-1},c_n), d(c_n,c_{n+1}), 0\}).
\end{aligned}$$

If $\max\{d(c_{n-1}, c_n), d(c_n, c_{n+1})\} = d(c_n, c_{n+1})$, then $d(c_n, c_{n+1}) \leq \varrho(d(c_n, c_{n+1})) < d(c_n, c_{n+1})$, which is absurd. So it must be the case that

$$\max\{d(c_{n-1}, c_n), d(c_n, c_{n+1})\} = d(c_{n-1}, c_n),$$

and hence $d(c_n, c_{n+1}) \leq \varrho(d(c_{n-1}, c_n))$, and in general,

$$d(c_n, c_{n+1}) \leq \varrho^n(d(c_0, c_1)) = \varrho^n(d(c_0, Tc_0),$$

for $n = 0, 1, 2, \dots$, because ϱ is non-decreasing. It pursues from hypothesis that

$$\sum_{n=0}^{\infty} d(c_n, c_{n+1}) \leq \sum_{n=0}^{\infty} \varrho^n(d(c_0, Tc_0)) < \infty,$$

and so

$$\sum_{n=0}^{\infty} \varrho^n(d(c_0, Tc_0)) < \infty.$$

As (U, κ, d) is d-complete , $\lim_{n\to\infty} c_n = z$ exists. Due to (U, κ) Hausdorff, for w-continuous mapping T, we have

$$z = \lim_{n\to\infty} c_{n+1} = \lim_{n\to\infty} Tc_n = Tz. \tag{3.3}$$

Thus, z is a FP of T.

This finalizes the proof. □

Example 4. *Let $U = \{0, 1\}$. Outline $d : U \times U \to [0, \infty)$ by*

$$d(p, q) = \left|p^2 - q\right|,$$

for all $p, q \in U$. As in Example 3, (U, κ, d) is d-complete TS.

Case 1: *Outline $T : U \to U$ by*

$$Tp = 0.$$

Taking $p = 1$, we get $O_T(1, \infty) = \{1, 0, 0, 0, \dots\}$. To determine that T satisfies the conditions of theorem 3.2, we recognize the function $\varrho : [0, \infty) \to [0, \infty)$ outlined by $\varrho(p) = e^p - 1$, then we get

$$d(Tp, Tq) = d(0, 0) = 0 \leq \varrho(\max\{d(p, q), d(p, Tp), d(q, Tq), d(Tp, q)\}).$$

Also, $\sum_{n=0}^{\infty} \varrho^n(d(Tp, p)) < \infty$ for $p = 0$ and T is w-continuous. Therefore, all conditions of theorem 3.2 are entertained. Thus, 0 is a FP for T.

Case 2: *Outline $T : U \to U$ by $Tp = 1$. For $p = 0$, $O_T(0, \infty) = \{0, 1, 1, 1, \ldots\}$. Clearly, we can check that T gratifies all the conditions of theorem as in case 1. Similarly, conditions hold for T relative to $p = 1$. Thus, $p = 1$ is FP of T.*

Ali in [1], Karayilan in [7] and many other authors gave some FP and coincidence point results using MT-function. Here, we will determine Mizoguchi-Takahashi's type coincidence point theorem for single valued map (proved by Ali in [1] in CMS for multi-valued map) in the setting of d-complete TS.

In 2013, Ali in [1] determined the subsequent Mizoguchi-Takahashi's coincidence and common FPT in metric spaces.

Theorem 3.3. *Let $g : U \to U$ and $S : U \to CL(U)$ be two mappings on a metric space (U, d) such that $SU \subseteq gU$, and*

$$d(gq, Sq) \leq \eta(d(gp, gq))d(gp, gq)$$

$\forall\ p \in U$ and $gq \in Sp$, where $\eta : [0, \infty) \to [0, 1)$ is MT-function. If (gU, d) is a CMS, then

(i) *for any $p_0 \in U$, $\exists$ a g-orbit $\{gp_n\}$ of S and $g\xi \in gU$ such that $lim_{n\to\infty} gp_n = g\xi$.*
(ii) *ξ is a coincidence point of g and S iff the function $h : U \to R$ defined by $h(p) = d(gp, Sp)\ \forall\ p \in U$, is lower semi-continuous at ξ.*
(iii) *If $gg\xi = g\xi$ and g is S-weakly commuting at ξ, then g and S have a common FP.*

Now, we determine Mizoguchi-Takahashi's type "coincidence point theorem" in the setting of d-complete TS.

Theorem 3.4. *Let $g : U \to U$ and $S : U \to U$ be two mappings on TS (U, κ) such that $SU \subseteq gU$, satisfying*

$$d(gq, Sq) \leq \eta(d(gp, gq))d(gp, gq), \tag{3.4}$$

for every $p \in U$ and $gq = Sp$, where η is a function from $(0, \infty)$ into $[0, 1)$ such that

$$\limsup_{r \to t^+} \eta(r) < 1, \tag{3.5}$$

for each $t \in [0, \infty)$.

Let (gU, κ, d) be a d-complete TS. Then,

(i) *for each* $p_0 \in U$, $\exists$ *an* $gp_n \in O_S(p_0, \infty)$ *and* $gp' \in gU$ *such that* $lim_{n\to\infty} gp_n = gp'$.

(ii) *Moreover,* p' *is coincidence point of* g *and* S *iff* $hp = d(gp, Sp)$ *is lower semi-continuous at* p'.

Proof.

(i) Choose $p_0 \in U$. As $SU \subseteq gU$, so $Sp_0 \in gU$. Therefore, $\exists\ p_1 \in U$ such that $Sp_0 = gp_1$. Enduring in this way, we get a sequence $< gp_n >$ in U such that $gp_n = Sp_{n-1}$. If $p_n = p_{n+1}$, then p_n is a coincidence point of g and S. For $p_n \neq p_{n+1}$,

$$\begin{aligned} d(gp_n, gp_{n+1}) &= d(gp_n, Sp_n) \\ &\leq \eta(d(gp_{n-1}, gp_n))d(gp_{n-1}, gp_n) \\ &= \eta(d(gp_{n-1}, gp_n))d(gp_{n-1}, Sp_{n-1}) \end{aligned}$$

Enduring in this way, we get

$$\begin{aligned} d(gp_n, gp_{n+1}) \leq &\eta(d(gp_{n-1}, gp_n))\eta(d(gp_{n-2}, gp_{n-1}))\cdots \\ &\eta(d(gp_0, gp_1))d(gp_0, gp_1). \end{aligned}$$

It pursues from equation (3.4) that we may opt $\epsilon > 0$ and $a \in (0, 1)$ such that $\eta(t) < a$ for $t \in (0, \epsilon)$. Let N be such that

$$d(gp_{n-1}, gp_n) < \epsilon$$

for $n \geq N$. Thus, we have

$$\begin{aligned} d(gp_n, gp_{n+1}) \leq &a^{n-(N-1)}\eta(d(gp_{N-1}, gp_N))\eta(d(gp_{N-2}, gp_{N-1}))\ldots \\ &\eta(d(gp_0, gp_1))d(gp_0, gp_1) < a^{n-(N-1)}d(gp_0, gp_1). \end{aligned}$$

Now, taking summation, we get

$$\sum_{n=N}^{\infty} d(gp_n, gp_{n+1}) < \sum_{n=N}^{\infty} a^{n-(N-1)}d(gp_0, gp_1) < \infty.$$

This signifies that $\sum_{n=0}^{\infty} d(gp_n, gp_{n+1}) < \infty$. This proves that $< gp_n >$ is a "d-cauchy sequence" in gU. Since gU is d-complete, $\exists\ gp' \in gU$ such that $gp_n \to gp'$.

(ii) Since $gp_n = Sp_{n-1}$, it pursues from (i) that

$$\begin{aligned} d(gp_n, gp_{n+1}) = d(gp_n, Sp_n) &\leq \eta(d(gp_{n-1}, gp_n))d(gp_{n-1}, gp_n) \\ &< d(gp_{n-1}, gp_n). \end{aligned}$$

Letting $n \to \infty$, we have $lim_{n\to\infty} d(gp_n, Sp_n) = 0$. Suppose $h(p) = d(gp, Sp)$ is lower semi continuous at p', then

$$d(gp', Sp') = h(p') \leq liminf_{n\to\infty} h(p_n) = liminf_{n\to\infty} d(gp_n, Sp_n) = 0.$$

Thus $d(gp', Sp') = 0$ and so $gp' = Sp'$.

Conversely, if p' is a coincidence point of g and S then

$$h(p') = 0 \leq liminf_{n\to\infty} h(p_n)$$

and hence h is lower semi-continuous at p'.

□

Example 5. *Let $U = \{\frac{1}{n} : n = 1, 2, 3, ...\} \cup \{0\}$. Let a metric on U be ρ and on U, κ be a metric topology induced by ρ. Outline $d : U \times U \to [0, \infty)$ by*

$$d(p,q) = \begin{cases} 1 & \text{if } p = \frac{1}{n}, q = \frac{1}{n+1} \text{ or } p = \frac{1}{n+1}, q = \frac{1}{n} \\ 0 & \text{if } p = q = \frac{1}{n} \\ 2^{-n} & \text{if } p = \frac{1}{n}, q = \frac{1}{m} \text{ where } n \neq m, m \neq n+1, n \neq m+1 \end{cases}$$

$$Sp = \begin{cases} 1 & \text{if } p = 1 \\ \frac{1}{n+1} & \text{if } p = \frac{1}{n}, n \neq 1 \\ 0 & \text{if } p = 0 \end{cases}, \quad gp = \begin{cases} 1 & \text{if } p = 1 \\ \frac{1}{n-1} & \text{if } p = \frac{1}{n}, n \neq 1 \\ 0 & \text{if } p = 0 \end{cases}.$$

Outline $\eta(t) = \frac{1}{2}\ \forall\ t \geq 0$. Here d is niether a quasi metric nor a metric on gU. Also d is not a semi-metric. Though, (gU, κ, d) is d-complete TS. For each $p \in U$ and $gq = Sp\ \forall\ t \geq 0$, we have

$$d(gq, Sq) \leq \eta(d(gp, gq))d(gp, gq).$$

We see that all conditions of theorem 3.4 are entertained. We can easily see that for each $p_0 \in U$, $\exists$ a $gp_n \in O_S(p_0, \infty)$ and $gp' \in gU$ such that $gp_n \to gp'$. Also, g and S have a coincidence point at 0 and 1 iff $h(p) = d(gp, Sp)$ is lower semi-continuous at 0 and 1 respectively.

Corollary 3.3. *Let S be a self mapping on a d-complete TS (U, κ, d) satisfying*

$$d(q, Sq) \leq \eta(d(p, q))d(p, q),$$

for each $p \in U$ and $q = Sp$, where η is a function from $(0, \infty)$ into $[0, 1)$ such that $\limsup_{r\to t^+} \eta(r) < 1$ for every $t \in [0, \infty)$. Then,

(i) *for every $p_0 \in U$, $\exists$ a $p_n \in O_S(p_0, \infty)$ and $p' \in U$ such that $\lim_{n\to\infty} p_n = p'$.*

(ii) $Sp' = p'$ *iff* $gp = d(p, Sp)$ *is S-orbitally lower semi-continuous at* p'.

Proof. This corollary pursues from theorem 3.4 by considering $g = I$. □

REFERENCES

[1] M. U. ALI: *Mizoguchi-Takahashi's type common fixed point theorem*, J. Egypt. Math. Soc., (2014), 272–274.

[2] T. L. HICKS: *Fixed point theorems for d-complete topological spaces*, I. Int. J. Math. Math. Sci. **15**(1992), 435–439.

[3] T. L. HICKS, B. E. RHOADES: *Fixed point theorems for d-complete topological spaces II*, Math. Japonica, **37** (1992), 847–853.

[4] K. ISEKI: *An approach to fixed point theorems*, Math. Semin. Notes Kobe Univ., **3** (1975), 193–202.

[5] S. KASAHARA: *On some generalizations of the Banach contraction theorem*, Math. Semin. Notes Kobe Univ., **3** (1975), 161–169.

[6] S. KASAHARA: *Some fixed point and coincidence theorems in L-spaces*, Math. Semin. Notes Kobe Univ., **3** (1975), 181–187.

[7] H. KARAYILAN, M. TELCI: *Common fixed point theorems for pairs of mappings in d-complete topological spaces*, Vietnam J. Math., **43** (2015), 621–627.

[8] I. J. LIN, H. LAKZIAN, Y. CHOU: *On best proximity point theorems for new cyclic maps*, International Mathematical Forum, **7** (2012), 1839–1849.

[9] V. POPA: *A fixed point theorem for mappings in d-complete topological spaces*, Math. Morav., **6** (2002), 87–92.

[10] W. SHATANAWI: *Fixed point theory for contractive mappings satisfying ϕ-maps in G-metric spaces* Fixed Point Theory and Applications, Article ID 181650 (2010), 9 pages.

DEPARTMENT OF MATHEMATICS
MAHARSHI DAYANAND UNIVERSITY
ROHTAK-124001, HARYANA, INDIA.
Email address: dr.savitarathee@gmail.com

DEPARTMENT OF MATHEMATICS
PT CLS GOVT. COLLEGE
KARNAL-132001, HARYANA, INDIA.
Email address: gupta.priyanka989@gmail.com

DEPARTMENT OF MATHEMATICS,
INDIRA GANDHI NATIONAL TRIBAL UNIVERSITY
LALPUR, AMARKANTAK, ANUPPUR, MADHYA PRADESH-484887, INDIA.
Email address: vishnunarayanmishra@gmail.com