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# INVERTIBLE NEUTROSOPHIC TOPOLOGICAL SPACES

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ABSTRACT. Aim of this present paper is, the new Type of Neutrosophic topological spaces called invertible and completely invertible Neutrosophic topological spaces are introduced. Also several properties of this newly introduced Neutrosophic topological space besides giving some characterizations of these spaces are established.

## 1. INTRODUCTION

C. L. Chang [2] introduced fuzzy topological space by using Zadeh's [8] (uncertain) fuzzy sets. Further Coker [3] developed the notion of intuitionistic fuzzy topological spaces by using Atanassov's [1] intuitionistic fuzzy set. Smarandache [4] defined the neutrosophic set of three component neutrosophic set $(t, f, i)$ =(Truth, Falsehood, Indeterminacy). The neutrosophic crisp set concept converted to neutrosophic topological spaces by A. A. Salama [6, 7]. Neutrosophic concepts have wide range of applications in the area of decision making artificial intelligence. Information systems, computer science, medicine, applied mathematics, mechanics, electrical and electronic and, management science, etc,. The mapping is the one of the important concept in topology. M.Parimala et.al., [5] introduced neutrosophic $\alpha\psi$ homeomorphism in neutrosophic topological spaces. Aim of this present paper

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is, the new type of neutrosophic topological spaces called invertible and completely invertible neutrosophic topological spaces are introduced. also several properties of this newly introduced neutrosophic topological space besides giving some characterizations of these spaces are established.

## 2. PRELIMINARIES

In this section, we introduce the basic definition for Neutrosophic sets and its operations.

**Definition 2.1.** *[4] Let $\mathcal{S}^1_N$ be a non-empty fixed set. A Neutrosophic set $A_{S^1{}_N}$ is the form $A_{\mathcal{S}^1_N} = \{\xi^*, \mu_{A_{\mathcal{S}^1_N}}(\xi^*), \sigma_{A_{\mathcal{S}^1_N}}(\xi^*), \gamma_{A_{\mathcal{S}^1_N}}(\xi^*) >: \xi^* \in \mathcal{S}^1_N\}$. Where $\mu_{\mathcal{S}^1_N}(\xi^*) : \mathcal{S}^1_N \to [0,1], \sigma_{\mathcal{S}^1_N}(\xi^*) : \mathcal{S}^1_N \to [0,1], \gamma_{A_{S^1{}_N}}(\xi^*) : \mathcal{S}^1_N \to [0,1]$ are represent Neutrosophic of the degree of membership function, the degree indeterminacy and the degree of non membership function respectively of each element $\xi^* \in \mathcal{S}^1_N$ to the set $A_{\mathcal{S}^1_N}$ with $0 \leq \mu_{A_{\mathcal{S}^1_N}}(\xi^*) + \sigma_{A_{\mathcal{S}^1_N}}(\xi^*) + \gamma_{A_{\mathcal{S}^1_N}}(\xi^*) \leq 1$. This is called standard form generalized fuzzy sets. But also Neutrsophic set may be $0 \leq \mu_{A_{\mathcal{S}^1_N}}(\xi^*) + \sigma_{A_{\mathcal{S}^1_N}}(\xi^*) + \sigma_{A_{\mathcal{S}^1_N}}(\xi^*) \leq 3$.*

**Definition 2.2.** *[4] We must introduce the Neutrosophic set $0_N$ and $1_N$ in $\mathcal{S}^1_N$ as follows:*

$$0_N = \{< \xi,\ 0, 0, 1 >: \xi \in\ \mathcal{S}^1_N\} \quad \textit{and} \quad 1_N = \{< \xi,\ 1, 1, 0 >: \xi \in\ \mathcal{S}^1_N\}.$$

**Definition 2.3.** *[4] Let $\mathcal{S}^1_N$ be a non-empty set and Neutrosophic sets $A_{\mathcal{S}^1_N}$ and $B_{\mathcal{S}^1_N}$ in the form NS*

$$A_{\mathcal{S}^1_N} = \{< \xi^*,\ \mu_{A_{\mathcal{S}^1_N}}(\xi^*), \sigma_{A_{\mathcal{S}^1_N}}(\xi^*), \gamma_{A_{\mathcal{S}^1_N}}(\xi^*)) >: \xi^* \in \mathcal{S}^1_N\} \qquad \textit{and}$$
$$B_{\mathcal{S}^1_N} = \{< \xi^*,\ \mu_{B_{\mathcal{S}^1_N}}(\xi^*), \sigma_{B_{\mathcal{S}^1_N}}(\xi^*), \gamma_{B_{\mathcal{S}^1_N}}(\xi^*) >: \xi^* \in \mathcal{S}^1_N\}$$

*defined as:*

*(1)* $A_{\mathcal{S}^1_N} \subseteq B_{\mathcal{S}^1_N} \Leftrightarrow \mu_{A_{\mathcal{S}^1_N}}(\xi^*) \leq \mu_{B_{\mathcal{S}^1_N}}(\xi^*), \sigma_{A_{\mathcal{S}^1_N}}(\xi^*), \leq \sigma_{B_{\mathcal{S}^1_N}}(\xi^*)$ *, and* $\sigma_{A_{\mathcal{S}^1_N}}(\xi^*) \geq \gamma_{B_{\mathcal{S}^1_N}}(\xi^*)$;
*(2)* $A^C_{\mathcal{S}^1_N} = \{< \xi^*,\ \gamma_{A_{\mathcal{S}^1_N}}(\xi^*),\ \sigma_{A_{\mathcal{S}^1_N}}(\xi^*),\ \mu_{A_{\mathcal{S}^1_N}}(\xi^*) >: \xi^* \in \mathcal{S}^1_N\}$;
*(3)* $A_{\mathcal{S}^1_N} \cap B_{\mathcal{S}^1_N} = \{< \xi^*,\ \mu_{A_{\mathcal{S}^1_N}}(\xi^*)) \wedge \mu_{B_{\mathcal{S}^1_N}}(\xi^*),\ \sigma_{A_{\mathcal{S}^1_N}}(\xi^*)) \wedge \sigma_{B_{\mathcal{S}^1_N}}(\xi^*),\ \gamma_{A_{\mathcal{S}^1_N}}(\xi^*) \vee \gamma_{B_{\mathcal{S}^1_N}}(\xi^*) >: \xi^* \in R^1_N\}$;

*(4)* $A_{\mathcal{S}_N^1} \cup B_{\mathcal{S}_N^1} = \{< \xi^*,\ \mu_{A_{\mathcal{S}_N^1}}(\xi^*) \vee \mu_{B_{\mathcal{S}_N^1}}(\xi^*),\ \sigma_{A_{\mathcal{S}_N^1}}(\xi^*) \vee \sigma_{B_{\mathcal{S}_N^1}}(\xi^*), \gamma_{A_{\mathcal{S}_N^1}}(\xi^*) \wedge \gamma_{B_{\mathcal{S}_N^1}}(\xi^*) >: \xi^* \in \mathcal{S}_N^1\}$.

**Definition 2.4.** *[6] A Neutrosophic topology is a non -empty set $\mathcal{S}_N^1$ is a family $\tau_{N\mathcal{S}_N^1}$ of Neutrosophic subsets in $\mathcal{S}_N^1$ satisfying the following axioms:*

*(i)* $0_N$, $1_N \in \tau_{N\mathcal{S}_N^1}$;
*(ii)* $G_{\mathcal{S}_N^1} \cap H_{\mathcal{S}_N^1} \in \tau_{N\mathcal{S}_N^1}$ *for any* $G_{S^N{}_1}, H_{S^N{}_1} \in \tau_{N\mathcal{S}_N^1}$;
*(iii)* $\bigcup_i Gi_{\mathcal{S}_N^1} \in \tau_{N\mathcal{S}_N^1}$ *for every* $Gi_{\mathcal{S}_N^1} \in \tau_{NS^1{}_N}, I \in J$.

*The pair* $(\mathcal{S}_N^1 \tau_{N\mathcal{S}_N^1})$ *is called a Neutrosophic topological space.*

*The element Neutrosophic topological spaces of* $\tau_{N\mathcal{S}_N^1}$ *are called Neutrosophic open sets.*

*A Neutrosophic set* $A_{\mathcal{S}_N^1}$ *is closed if and only if* ${A_{\mathcal{S}_N^1}}^C$ *is Neutrosophic open.*

**Definition 2.5.** *[4] Let* $\mathcal{S}_N^1$ *and* $\mathcal{S}_N^2$ *be two finite sets. Define* $\psi_1 : \mathcal{S}_N^1 \to \mathcal{S}_N^2$.

*If* $A_{\mathcal{S}_N^2} = \{< \theta, \mu_{A_{\mathcal{S}_N^2}}(\theta), \sigma_{A_{\mathcal{S}_N^2}}(\theta), \gamma_{A_{\mathcal{S}_N^2}}(\theta)) >: \theta \in \mathcal{S}_N^2\}$ *is an NS in* $\mathcal{S}_N^2$*, then the inverse image( pre image)* $A_{\mathcal{S}_N^2}$ *under* $\psi_1$ *is an NS defined by* ${\psi_1}^{-1}(A_{\mathcal{S}_N^2}) =< \xi$, $\psi_1^{-1}\mu_{A_{\mathcal{S}_N^2}}(\xi), {\psi_1}^{-1}\sigma_{A_{\mathcal{S}_N^2}}(\xi), {\psi_1}^{-1}\sigma_{A_{\mathcal{S}_N^2}}(\xi) : \xi \in \mathcal{S}_N^2 >$. *Also define image NS* $U =< \xi, \mu_U(\xi), \sigma_U(\xi), \gamma_U(\xi) : \xi \in S_N^1 :>$ *under* $\psi_1$ *is an NS defined by* $\psi_1(U) =< \psi_1(\mu_{A_{\mathcal{S}_N^2}} \psi_1(\sigma_{A_{\mathcal{S}_N^2}} \psi_1(\gamma_{A_{\mathcal{S}_N^2}} \theta \in \mathcal{S}_N^2 >$ *where*

$$\psi_1(\mu_{A_{\mathcal{S}_N^2}}(\theta)) = \begin{cases} \sup \mu_{A_{\mathcal{S}_N^2}}(\xi), & \textit{if } \psi_1^{-1}(\theta) \neq \phi, \xi \in \psi_1^{-1}(\theta) \\ 0 & \textit{elsewhere} \end{cases},$$

$$\psi_1(\sigma_{A_{\mathcal{S}_N^2}}(\theta)) = \begin{cases} \sup \sigma_{A_{\mathcal{S}_N^2}}(\xi) & \textit{if } \psi_1^{-1}(\theta) \neq \phi, \xi \in \psi_1^{-1}(\theta) \\ 0, & \textit{elsewhere} \end{cases},$$

$$\psi_1(\gamma_{A_{\mathcal{S}_N^2}}(\theta)) = \begin{cases} \inf(\gamma_{A_{\mathcal{S}_N^2}}(\xi), & \textit{if } \psi_1^{-1}(\theta) \neq \phi, \xi \in \psi_1^{-1}(\theta) \\ 0, & \textit{elsewhere} \end{cases}.$$

**Definition 2.6.** *[6] A mapping* $\psi_1 : (\mathcal{S}_N^1 \tau_{N_{\mathcal{S}_N^1}}) \to (\mathcal{S}_N^2, \tau_{N_{\mathcal{S}_N^1}})$ *is called a Neutrosophic continuous(Neu-continuous) if* $\psi_1^{-1}(A_{\mathcal{S}_N^2}) \in C(NUTS\mathcal{S}_N^1)$ *whenever* $A_{\mathcal{S}_N^2} \in C(NUTS\mathcal{S}_N^2)$.

TYPE I AND TYPE II INVERTIABLE

*In this section the concepts of invertible and completely invertible Neutrosophic topological spaces are introduced. Also Type 1 and Type 2 invertible Neutrosophic topological spaces are defined with examples.*

**Definition 2.7.** *An Neutrosophic topological spaces* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *is said to be invertible with respect to a proper Neutrosophic open set* $A_{\mathcal{S}_N^1}$ *if there is a Neutrosophic topological homeomorphism* $\psi_1 : (\mathcal{S}_N^1 \tau_{N_{\mathcal{S}_N^1}}) \rightarrow (\mathcal{S}_N^2,\ \tau_{N_{\mathcal{S}_N^1}})$ *such that* $\psi_1(\overline{A_{\mathcal{S}_N^1}}) \subseteq A_{\mathcal{S}_N^1}$. $\psi_1$ *is called an Inverting map for* $A_{\mathcal{S}_N^1}$ *and* $A_{\mathcal{S}_N^1}$ *is said to be an Inverting Neutrosophic set of* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$.

*If an Neutrosophic topological spaces* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *is invertible, then there exists an Inverting Neutrosophic set* $A_{R_N^1}$ *and an Inverting map* $\psi_1$ *of* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$. *This* $A_{\mathcal{S}_N^1}$ *and* $\psi_1$ *together called an Inverting pair of* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *and is denoted by* $(A_{\mathcal{S}_N^1}, \psi_1)$.

**Definition 2.8.** *An Neutrosophic topological spaces* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *is said to be completely invertible if for every proper* $NOS$, $A_{\mathcal{S}_N^1} \in \tau_{N_{\mathcal{S}_N^1}}$,*there is an Neutrosophic topological spaces homeomorphism* $\psi_1$ *of* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *such that* $\psi_1(\overline{A_{\mathcal{S}_N^1}}) \subseteq A_{\mathcal{S}_N^1}$.

**Definition 2.9.** *An Neutrosophic topological spaces* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *is said to be Type 1 if identity is an Inverting map for any one of the Inverting Neutrosophic set.*

**Definition 2.10.** *An Neutrosophic topological spaces* $(\mathcal{S}_N^1,\ \tau_{N_{\mathcal{S}_N^1}})$ *is said to be Type 2 if identity is an Inverting map for all the Inverting Neutrosophic topological sets.*

**Example 1.** *Let* $\mathfrak{R}_N^1 = \{a_n, b_n\}$ *and* $\Im_N^1 = \{0_N, A_{\mathcal{S}_N^1},\ 1_N\}$ *be a Neutrosophic Topological spces on* $\mathfrak{R}_N^1$ *where* $A_{\mathfrak{R}_N^1} = \{< x,\ (a_n,\ 0.6, 0.5, 0.4),\ (b_n,\ 0.8, 0.5, 0.2) > ; x \in \mathfrak{R}_N^1\}$ *is in* $\mathfrak{R}_N^1$.

*Define an Neutrosophic mapping* $\psi_1 : (\mathfrak{R}_N^1,\ \Im_N^1) \rightarrow (\mathfrak{R}_N^1,\ \Im_N^1)$, $\psi_1(a_n) = a_n \psi_1(b_n) = b_n$.

*Clearly* $\psi_1 is$ *Neutrosophic topological spaces homeomorphism.* $A_{R_N^1}$*is an* $NOS$ *in* $\Im_N^1$.

$$\psi_1(A_{R_N^1}) = \{< y,\ (a_n,\ 0.6, 0.5, 0.4),\ (b_n,\ 0.8, 0.5, 0.2) >; y \in \mathfrak{R}_N^1\} \qquad \textit{and}$$

$$\psi_1(\overline{A_{R_N^1}}) = \{< y,\ (a_n,\ 0.4, 0.5, 0.6),\ (b_n,\ 0.2, 0.5, 0.8) >; y \in \mathfrak{R}_N^1\}.$$

*So $\psi_1(\overline{A_{R^1_N}}) \subseteq A_{R^1_N}$. Then $\psi_1$ is is completely invertible Neutrosophic topological spaces and also is of Type 2.*

**Example 2.** *Let $\mathfrak{R}^1_N = \{a_n, b_n\}$ and $\Im^1_N = \{0_N, A_{R^1_N},\ B_{R^1_N},\ 1_N\}$ be a Neutrosophic Topological spaces on $\mathfrak{R}^1_N$ where*

$$A_{R^1_N} = \{< x, (a_n, 0.6, 0.5, 0.4),\ (b_n, 0.8, 0.5, 0.2) >; x \in \mathfrak{R}^1_N\},$$
$$B_{R^1_N} = \{< x, (a_n, 0.7, 0.5, 0.3), (b_n, 0.5, 0.5, 0.5) >; x \in \mathfrak{R}^1_N\}.$$

*Define an Neutrosophic mapping $\psi_1 : (\mathfrak{R}^1_N,\ \Im^1_N) \to (\mathfrak{R}^1_N,\ \Im^1_N)\psi_1(a_n) = a_n, \psi_1(b_n) = b_n$. Clearly $\psi_1$ is Neutrosophic topological spaces homeomorphism. Then $\psi_1(\overline{A_{R^1_N}}) \nsubseteq A_{R^1_N}$ and $\psi_1(\overline{B_{R^1_N}}) \subseteq B_{R^1_N}$. Thus Neutrosophic $NOS\, B_{R^1_N}$ is an Inverting Neutrosophic set and $(B_{R^1_N},\ \psi_1) is$ an Inverting pair. Hence $(\mathfrak{R}^1_N,\ \Im^1_N)$ is an invertible Neutrosophic topological spaces with respect to $NOS\, B_{R^1_N}$ and is of Type 1.*

## 3. Characterizations Of Invertiable

In this section some characterizations of invertible Neutrosophic topological spaces are studied.

**Proposition 3.1.** *The following statements are equivalent:*

1. *$(\mathfrak{R}^1_N,\ \Im^1_N)$ is a completely invertible Neutrosophic topological space.*
2. *Given a proper $NCS\, A_{R^1_N}$ and a proper $NOS\, B_{R^1_N}$, there is an Neutrosophic homeomorphism $\psi_1 : (\mathfrak{R}^1_N,\ \Im^1_N) \to (\mathfrak{R}^1_N,\ \Im^1_N)$ such that $\psi_1(A_{R^1_N}) \subseteq (B_{R^1_N})$.*
3. *Given a proper $NCS\, A_{R^1_N}$ and a proper $NOS\, B_{R^1_N}$, there is an Neutrosophic homeomorphism $\psi_1 : (\mathfrak{R}^1_N,\ \Im^1_N) \to (\mathfrak{R}^1_N,\ \Im^1_N)$ such that $A_{R^1_N} \subseteq \psi_1(B_{R^1_N})$.*
4. *Given any proper $NS$ A such that c1$(A_{R^1_N}) \neq 1_N$ and a proper $NS\, B_{R^1_N}$ such that int$B_{R^1_N} \neq 0_N$, there exists an Neutrosophic homeomorphism $\psi_1 : (\mathfrak{R}^1_N,\ \Im^1_N) \to (\mathfrak{R}^1_N, \Im^1_N)$ such that $\psi_1(\mathrm{c1}\, A_{R^1_N}) \subseteq \mathrm{int}B_{R^1_N}$.*
5. *Given any proper $NS\, A_{R^1_N}$ such that $\mathrm{c1}A_{R^1_N} \neq 1_N$ and a proper $NS\, B_{R^1_N}$ such that int$B_{R^1_N} \neq 0_N$ there exists an Neutrosophic homeomorphism $\psi_1 : (\mathfrak{R}^1_N,\ \Im^1_N) \to (\mathfrak{R}^1_N, \Im^1_N)$ such that $\mathrm{c1}A_{R^1_N} \subseteq \psi_1(\mathrm{int}B_{R^1_N})$.*

*Proof.*
$(1) \Rightarrow (2)$: Let us assume that $(\mathfrak{R}^1_N,\ \Im^1_N)$ is completely invertible Neutrosophic topological space. Let $A_{R^1_N}$ be a proper $NCS$ and $B_{R^1_N}$ be any proper $NOS$ in

$(\mathfrak{R}_N^1,\ \Im_N^1)$.Then $\overline{A_{R_N^1}}$ is a proper $NOS.(\mathfrak{R}_N^1,\ \Im_N^1)is$ completely invertible $NTS$, then there is an Neutrosophic homeomorphism $\psi_2 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\psi_2(A_{R_N^1}) \subseteq \overline{A_{R_N^1}}$ If $\overline{A_{R_N^1}} \subseteq B_{R_N^1}$ then $\psi_2(A_{R_N^1}) \subseteq \overline{A_{R_N^1}} \subseteq B_{R_N^1}.If\overline{A_{R_N^1}} \supseteq B_{R_N^1}$ then $A_{R_N^1} \subseteq \overline{B_{R_N^1}}$. In this case let $\psi_1 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ be be Neutrosophic homeomorphism corresponding to proper $NOSB_{R_N^1}$, then $\psi_1(\overline{B_{R_N^1}}) \subseteq B_{R_N^1}$. Also $A_{R_N^1} \subseteq \overline{B_{R_N^1}}$ implies $\psi_1(A_{R_N^1}) \subseteq \psi_1(\overline{B_{R_N^1}}) \subseteq B_{R_N^1}$. This proves $(1) \Rightarrow (2)$.

$(2) \Rightarrow (1)$: Let $A_{R_N^1}$ be any proper $NOS$ in $(\mathfrak{R}_N^1, \Im_N^1)$Then $\overline{A_{R_N^1}}$ is proper $NCS$. Hence by (2), there is an Neutrosophic homeomorphism $\psi_1 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\psi_1(\overline{A_{R_N^1}}) \subseteq A_{R_N^1}$. This proves $(\mathfrak{R}_N^1,\ \Im_N^1)$ is completely invertible Neutrosophic TS. Hence $(2) \Rightarrow (1)$ is proved.

$(2) \Rightarrow (3)$: Let $A_{R_N^1}$ be any proper NCS and $B_{R_N^1}$ be any proper $NOS$. Then, by (2) there exists an Neutrosophic homeomorphism $\psi_2 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\psi_2(A_{R_N^1}) \subseteq B_{R_N^1}$. This implies $A_{R_N^1} \subseteq \psi_2^{-1}(\psi_2(A_{R_N^1})) \subseteq \psi_2^{-1}(B_{R_N^1})$. Put $\psi_2^{-1} = \psi_1$. Then $\psi_1$ is Neutrosophic homeomorphism from $(\mathfrak{R}_N^1,\ \Im_N^1)$ onto $(\mathfrak{R}_N^1,\ \Im_N^1)andA_{R_N^1} \subseteq \psi_1(B_{R_N^1})$. Hence $(2) \Rightarrow (3)$.

$(3) \Rightarrow (4)$: Let $A_{R_N^1}$ be any proper NSs such that $\mathrm{c1}A_{R_N^1} \neq 1_N$ and $B$ be any proper NSs such that $\mathrm{int}B_{R_N^1} \neq 0_N$. Then $\mathrm{c1}A_{R_N^1}$ and $\mathrm{int}B_{R_N^1}$ are proper NCS and proper NOS respectively. Hence by (3), there exists an N homeomorphism $\psi_2 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\mathrm{c1}A_{R_N^1} \subseteq \psi_2(intB_{R_N^1})$. Then $\psi_2^{-1}(\mathrm{c1}A_{R_N^1}) \subseteq \psi_2^{-1}(\psi_2(intB_{R_N^1})) = \mathrm{int}B_{R_N^1}$. Let $\psi_2^{-1} = \psi_1$. Clearly $\psi_1$ is N homeomorphism and $\psi_1(\mathrm{c1}A_{R_N^1}) \subseteq \mathrm{int}B_{R_N^1}$. Thus $(3) \Rightarrow (4)$.

$(4) \Rightarrow (5)$: Let $A_{R_N^1}$ be any proper NSs such that $\mathrm{cl}A_{R_N^1} \neq 1_N$ and $B_{R_N^1}$ be any proper NSs such that $\mathrm{int}B_{R_N^1} \neq 0_N$. Hence by (4), there exists an N homeomorphism $\psi_2 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\psi_2(\mathrm{c1}A_{R_N^1}) \subseteq \mathrm{int}B_{R_N^1}$. Then $\mathrm{c1}A_{R_N^1} = \psi_2^{-1}(\psi_2(\mathrm{cl}A_{R_N^1})) \subseteq \psi_2^{-1}(\mathrm{int}B_{R_N^1})$. Let $\psi_2^{-1} = \psi_1$. Clearly $\psi_1$ is N homeomorphism and $\mathrm{c1}A_{R_N^1} \subseteq \psi_1(\mathrm{int}B_{R_N^1})$. Thus $(4) \Rightarrow (5)$.

$(5) \Rightarrow (2)$: Let $A_{R_N^1}$ be a proper $NCS$ and $B_{R_N^1}$ be any proper $NOS$. Since $A_{R_N^1}$ is proper $NCS, \mathrm{c1}A_{R_N^1} \neq 1_N$. By (5), $\psi_2 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\mathrm{c1}A_{R_N^1} \subseteq \psi_2(\mathrm{int}B_{R_N^1})$. Then $A_{R_N^1} = \mathrm{c1}A_{R_N^1} \subseteq \psi_2(\mathrm{int}B_{R_N^1}) = \psi_2(B_{R_N^1})$. In other words, $\psi_2^{-1}(A_{R_N^1}) \subseteq B_{R_N^1}$. Put $\psi_2^{-1} = \psi_1$. Clearly $\psi_1$ is Neutrosophic homeomorphism and $\psi_1(A_{R_N^1}) \subseteq B_{R_N^1}$. This proves $(5) \Rightarrow (2)$. □

**Remark 3.1.** *An Neutrosophic TS*$(\mathfrak{R}^1_N,\ \Im^1_N)$ *is completely invertible if and only if for each proper Neutrosophic closed subset* $B_{R^1_N}$ *and each* $0_N \neq A_{R^1_N} \in \mathfrak{R}^1_N$*, there is an Neutrosophic homeomorphism* $\psi_1$ *of* $(\mathfrak{R}^1_N,\ \Im^1_N)$ *such that* $\psi_1(B_{R^1_N}) \subseteq A_{R^1_N}$*.*

## 4. Properties Of Invertible Neutrosophic topological Spaces

In this section some properties of invertible NTS are studied. Also Neutrosophicstrongly compact, Neutrosophic nearly crisp set are defined. Mutual relationships between invertible $NTS$ and Neutrosophic compact space are investigated.

**Proposition 4.1.** *Let* $(\mathfrak{R}^1_N,\ \Im^1_N)$ *and* $(\mathfrak{R}^2_N,\ \Im^2_N)$ *be completely invertible Neutrosophic topological spaces. Then* $(\mathfrak{R}^1_N \times \mathfrak{R}^2_N,\ \Im^1_N \times \Im^2_N)$ *is completely invertible* $NTS$*.*

*Proof.* Let $0_N \neq A_{R^1_N} \neq 1_N \in \Im^1_N$ and $0_N \neq B_{R^1_N} \neq 1_N \in \Im^2_N$. Then $0_N \neq A_{R^1_N} \times B_{R^1_N} \neq 1_N$ is an $NOS$ in $\mathfrak{R}^1_N \times \mathfrak{R}^2_N$. Since $A_{R^1_N}$ is a proper $NOS$ in $\Im^1_N$, there exists an Neutrosophic homeomorphism $\psi_{1_{A_{R^1_N}}} : \mathfrak{R}^1_N \to \mathfrak{R}^1_N$ such that $\psi_{1_{A_{R^1_N}}}(\overline{A_{R^1_N}}) \subseteq A_{R^1_N}$. As $B_{R^1_N}$ is a proper $NOS$ in $\Im^2_N$, there exists an Neutrosophic homeomorphism $\psi_{1_{B_{R^1_N}}} : \mathfrak{R}^2_N \to \mathfrak{R}^2_N$ such that $\psi_{1_{B_{R^1_N}}}(\overline{B_{R^1_N}}) \subseteq B_{R^1_N}$. Then $\psi_{1_{A_{R^1_N} \times B_{R^1_N}}} : \mathfrak{R}^1_N \times \mathfrak{R}^2_N \to \mathfrak{R}^1_N \times \Im^2_N$ is an Neutrosophic homeomorphism such that

$$\begin{aligned}(\psi_{1_{A_{R^1_N}}} \times \psi_{1_{B_{R^1_N}}})(\overline{A_{R^1_N} \times B_{R^1_N}}) &= (\psi_{1_{A_{R^1_N}}} \times \psi_{1_{B_{R^1_N}}})(\overline{A_{R^1_N}} \times \overline{B_{R^1_N}}) \\ &= \psi_{1_{A_{R^1_N}}}(\overline{A_{R^1_N}}) \times \psi_{1_{B_{R^1_N}}}(\overline{B_{R^1_N}}) \subseteq A_{R^1_N} \times B_{R^1_N}\end{aligned}$$

which implies $\mathfrak{R}^1_N \times \mathfrak{R}^2_N$ is completely invertible $NTS$. □

**Theorem 4.1.** *A Neutrosophic closed crisp subset of an Neutrosophic compact space is Neutrosophic compact relative to* $\mathfrak{R}^1_N$*.*

*Proof.* Let $A_{R^1_N}$ be Neutrosophic closed crisp subset of $\mathfrak{R}^1_N$. Let $M = \{G_i; i \in J\}$ be cover of $A_{R^1_N}$ by Neutrosophic open sets in $\mathfrak{R}^1_N$. Then the family $\{M, \overline{A_{R^1_N}}\}$ is an Neutrosophic open cover of $\mathfrak{R}^1_N$. Since $\mathfrak{R}^1_N$ is Neutrosophic compact, there is a finite subfamily $\{G_1,\ G_2, \ldots, G_n\}$ of Neutrosophic open cover which also covers $A_{R^1_N}$. If this cover contains $\overline{A_{R^1_N}}$, discard it.

Otherwise leave the subcover as it is. Thus a finite Neutrosophic open subcover of $A_{R_N^1}$ is obtained. So $A_{R_N^1}$ is Neutrosophic compact relative to $\mathfrak{R}_N^1$. □

**Remark 4.1.** *In the topological space if the invertible space $S$ contains a nonempty open set $U$ whose closure is compact, then $S$ is compact. However in the case of Neutrosophictopological space a similar version of the above said result is not true as the following example shows.*

**Example 3.** *Let $\mathfrak{R}_N^1 be$ the set of all natural numbers. Then $(\mathfrak{R}_N^1,\ \Im_N^1) is NTS$ where $\tau = \{0_N,\ 1_N, A_n\}, n = 1,2,3,$ where $A_n : \mathfrak{R}_N^1 \to [0,1]$ is defined by $A_n = \{< x, 1 - \dfrac{1}{n(n+1)}, \frac{1}{n(n+1)}, \frac{1}{n(n+1)} >; x \in \mathfrak{R}_N^1\}; n = 1,2,3,$ Define $\psi_1 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ by $\psi_1(x) = x$. Clearly $\psi_1$ is Neutrosophic homeomorphism. Now $\psi_1(\overline{A_n}) \subseteq A_{n/}n = 1,2,3,$ Thus $(\mathfrak{R}_N^1,\ \Im_N^1) is$ completely invertible $NTS. A_1 = \{< x, \dfrac{1}{2}, \dfrac{1}{2} >; x \in \mathfrak{R}_N^1\}$ is both $NOS$ and $NCS$. Clearly $\mathrm{c1}(A_1) = \overline{A_1}$ and is Neutrosophic compact. The family $\{A_n\}, n = 1,2,3\ldots,$ is Neutrosophic open cover of $1_N$, but it has no finite subcover. Therefore $(\mathfrak{R}_N^1,\ \Im_N^1)$ is not Neutrosophic compact.*

**Remark 4.2.** *The following theorem shows that an $NTS(\mathfrak{R}_N^1,\ \Im_N^1)$ containing an invertible crisp subset $A_{R_N^1}$ such that $\mathrm{c1}A_{R_N^1}$ is Neutrosophic compact, is Neutrosophic compact.*

**Theorem 4.2.** *If an Neutrosophic TS $(\mathfrak{R}_N^1,\ \Im_N^1)$ contains an invertible crisp subset $A_{R_N^1}$ such that $(\mathrm{c1}A_{R_N^1},\ \psi_{\mathrm{c1}A_{R_N^1}})$ is Neutrosophic compact then $\mathfrak{R}_N^1$ is Neutrosophic compact.*

*Proof.* If an $NTS$ contains an invertible crisp subset of $(\mathfrak{R}_N^1,\ \Im_N^1)$ then there exists an Neutrosophic homeomorphism $\psi_1$ of $(\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\psi_1(\overline{A_{R_N^1}}) \subseteq A_{R_N^1}$. Also $\psi_1(\overline{A_{R_N^1}})$ is a closed crisp subset of $A_{R_N^1}$, where $\mathrm{c1}A_{R_N^1}$ is Neutrosophic compact. Hence $\psi_1(\overline{A_{R_N^1}})$ is Neutrosophic compact. As $\psi_1$ is Neutrosophic homeomorphism $\overline{A_{R_N^1}}$ is Neutrosophic compact. $\mathfrak{R}_N^1 = \mathrm{cl}A_{R_N^1} \cup \overline{A_{R_N^1}}(\mathfrak{R}_N^1,\ \Im_N^1)$ is Neutrosophic compact. □

**Definition 4.1.** *An $NTS(\mathfrak{R}_N^1,\ \Im_N^1)$ is Neutrosophic strongly compact if every Neutrosophic closed subset of $\mathfrak{R}_N^1$ is Neutrosophic compact.*

**Definition 4.2.** *An Neutrosophicsubset* $A_{R_N^1}$ *of an* $NTS(\mathfrak{R}_N^1,\ \Im_N^1)$ *is said to be Neutrosophicnearly crisp if* $\mathrm{c1}A_{R_N^1} \cap \overline{\mathrm{c1}A_{R_N^1}} = 0_N$.

**Theorem 4.3.** *If an* $NTS(\mathfrak{R}_N^1, \Im_N^1)$ *contains an invertible Neutrosophic nearly crisp subset* $A_{R_N^1}$ *such that* $(\mathrm{c1}A_{R_N^1},\ F_{\mathrm{c1}A_{R_N^1}})$ *is Neutrosophic strongly compact, then* $\mathfrak{R}_N^1$ *is Neutrosophic compact.*

*Proof.* Let $(A_{R_N^1},\ \psi_1)$ be an Inverting pair of $(\mathfrak{R}_N^1,\ \Im_N^1)$ so that $\psi_1(\overline{A_{R_N^1}}) \subseteq A_{R_N^1}$. Also $\psi_1(\overline{A_{R_N^1}})$ is a closed subset of $\mathrm{c1}A_{R_N^1}$, where $\mathrm{c1}A_{R_N^1}$ is Neutrosophicstrongly compact. Therefore $\psi_1(\overline{A_{R_N^1}})$ is Neutrosophic compact. Consequently $\overline{A_{R_N^1}}$ is Neutrosophic compact. Since $A_{R_N^1}$ is Neutrosophic nearly crisp set and $\mathfrak{R}_N^1 = \mathrm{c1}A_{R_N^1} \cup \overline{A_{R_N^1}}$ it implies that $(\mathfrak{R}_N^1,\ \Im_N^1)$ ) is Neutrosophic compact. □

**Theorem 4.4.** *In a completely invertible Neutrosophic strongly compact* $NTS(\mathfrak{R}_N^1, \Im_N^1)$ *every non-zero open Neutrosophic subset contains an Neutrosophic compact subset.*

*Proof.* Since $\mathfrak{R}_N^1$ is Neutrosophic strongly compact, each Neutrosophic closed set is Neutrosophic compact. As $\mathfrak{R}_N^1$ is completely invertible, given a proper $NCS\, A_{R_N^1}$ and a non-zero $NOS\, B_{R_N^1}$ there is an Neutrosophic homeomorphism $\psi_1 : (\mathfrak{R}_N^1,\ \Im_N^1) \to (\mathfrak{R}_N^1,\ \Im_N^1)$ such that $\psi_1(A_{R_N^1}) \subseteq B_{R_N^1}$. Since $(\mathfrak{R}_N^1,\ \Im_N^1)$ is Neutrosophic strongly compact $NTS$, every Neutrosophic closed subset in $\mathfrak{R}_N^1$ is Neutrosophic compact. Whence $A_{R_N^1}$ is Neutrosophic compact. The continuous image of compact $NTS$ is compact. $\psi_1(\overline{A_{R_N^1}})$ is Neutrosophic compact. □

## 5. Conclusion

The concepts of invertible and completely invertible neutrosophic topological spaces are introduced. Also Type 1 and Type 2 invertible $NTS$ are defined with examples. Some properties and characterizations of invertible $NTS$ are studied. Also the relationships between invertible $NTS$ and neutrosophic compact and neutrosophic strongly compact neutrosophic TS are investigated.

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