

FIXED POINT THEOREMS IN SOFT PARAMETRIC METRIC SPACE

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ABSTRACT. In the present paper, some fixed-point theorems are proved in soft complete soft parametric metric space for rational expressions.

1. PRELIMINARIES

In this section we present some basic definition of soft parametric space. Details can be found in [1-10].

Definition 1.1. A mapping $\tilde{\rho} : SP(\tilde{X}) \times SP(\tilde{X}) \times (0, \infty) \rightarrow \mathbb{R}(E)^*$, is said to be a soft parametric metric on the soft set $\tilde{X}$ if $\tilde{\rho}$ satisfies the following conditions:

- (M1) $\tilde{\rho}(\tilde{x}_{e_1}, \tilde{y}_{e_2}, t) = \bar{0}$ if and only if $\tilde{x}_{e_1} = \tilde{y}_{e_2}$ for all $t > 0$;
- (M2) $\tilde{\rho}(\tilde{x}_{e_1}, \tilde{y}_{e_2}, t) = \tilde{\rho}(\tilde{y}_{e_2}, \tilde{x}_{e_1}, t)$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2} \in \tilde{X}$ and $t > 0$, $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \in \tilde{X}$, $t > 0$;
- (M3) $\tilde{\rho}(\tilde{x}_{e_1}, \tilde{z}_{e_3}, t) \leq \tilde{\rho}(\tilde{x}_{e_1}, \tilde{y}_{e_2}, t) + \tilde{\rho}(\tilde{y}_{e_2}, \tilde{z}_{e_3}, t)$ for all $\tilde{x}_{e_1}, \tilde{y}_{e_2}, \tilde{z}_{e_3} \in \tilde{X}$, $t > 0$ and the pair $(\tilde{X}, \tilde{\rho}, E)$ is called soft parametric metric space.

Definition 1.2. Let $\{\tilde{x}_{\lambda_n}^n\}_{n=1}^\infty$ be a sequence in a Soft parametric metric space $(\tilde{X}, \tilde{\rho}, E)$.

- (i) $\{\tilde{x}_{\lambda_n}^n\}_{n=1}^\infty$ is said to be convergent to $\tilde{x}_\lambda \in \tilde{X}$, if $\lim_{n \rightarrow \infty} \tilde{\rho}(\tilde{x}_{\lambda_n}^n, \tilde{x}_\lambda, t) = 0$ written as $\lim_{n \rightarrow \infty} \tilde{x}_{\lambda_n}^n = \tilde{x}_\lambda$, for all $t > 0$;

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- (ii) $\{\tilde{x}_{\lambda_n}^n\}_{n=1}^\infty$ is said to be a Cauchy sequence in $\tilde{X}$, if $\lim_{n,m \rightarrow \infty} \tilde{\rho}(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_m}^m, t) = 0$ for all $t > 0$;
- (iii) $(\tilde{X}, \tilde{\rho}, E)$ is said to be complete if every Cauchy sequence is a convergent sequence.

Definition 1.3. Let $(\tilde{X}, \tilde{\rho}, E)$ be a Soft parametric metric space and a function $(f, \varphi) : (\tilde{X}, \tilde{\rho}, E) \rightarrow (\tilde{X}, \tilde{\rho}, E)$ is continuous at $\tilde{x}_\lambda \in \tilde{X}$, if for any sequence $\{\tilde{x}_{\lambda_n}^n\}_{n=1}^\infty$ in $\tilde{X}$ such that $\lim_{n \rightarrow \infty} \tilde{x}_{\lambda_n}^n = \tilde{x}_\lambda$, then $\lim_{n \rightarrow \infty} (f, \varphi)\tilde{x}_{\lambda_n}^n = (f, \varphi)\tilde{x}_\lambda$

Lemma 1.1. Let $\{\tilde{x}_{\lambda_n}^n\}_{n=1}^\infty$ be a sequence in a Soft parametric metric space $(\tilde{X}, \tilde{\rho}, E)$ such that

$$\tilde{\rho}(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t) \leq r \tilde{\rho}(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t),$$

where $r \in [0, 1)$ and $n = 1, 2, \dots$. Then $\{\tilde{x}_{\lambda_n}^n\}_{n=1}^\infty$ is a Cauchy sequence in $(\tilde{X}, \tilde{\rho}, E)$

2. MAIN RESULTS

Theorem 2.1. Let $(\tilde{X}, \tilde{\rho}, E)$ be a complete soft parametric metric space and let (f, φ) a continuous mapping. Let mapping $(f, \varphi) : (\tilde{X}, \tilde{\rho}, E) \rightarrow (\tilde{X}, \tilde{\rho}, E)$ is defined such that for all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}$, $\tilde{x}_\lambda \neq \tilde{y}_\mu$, and for all $t > 0$, where $\tilde{a} \in [0, \frac{1}{2})$. Then (f, φ) has a unique fixed point in $\tilde{X}$, if satisfies the following condition:

$$\tilde{\rho}((f, \varphi)\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t) \leq \tilde{a} \max \left[\begin{array}{l} \tilde{\rho}(\tilde{x}_\lambda, \tilde{y}_\mu, t), \tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{x}_\lambda, t) \\ \tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{y}_\mu, t), \tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t), \\ \tilde{\rho}((f, \varphi)\tilde{x}_\lambda, \tilde{y}_\mu, t), \frac{\tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{x}_\lambda, t) \cdot \tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{y}_\mu, t)}{1 + \tilde{\rho}((f, \varphi)\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t)} \end{array} \right].$$

Proof. Let $\tilde{x}_\lambda^0$ be any soft point in $SP(X)$. Set

$$\begin{aligned} \tilde{x}_{\lambda_1}^1 &= (f, \varphi)(\tilde{x}_\lambda^0) = (f(\tilde{x}_\lambda^0))_{\varphi(\lambda)} \\ \tilde{x}_{\lambda_2}^2 &= (f, \varphi)(\tilde{x}_{\lambda_1}^1) = (f^2(\tilde{x}_\lambda^0))_{\varphi^2(\lambda)} \\ \tilde{x}_{\lambda_{n+1}}^{n+1} &= (f, \varphi)(\tilde{x}_{\lambda_n}^n) = (f^{n+1}(\tilde{x}_\lambda^0))_{\varphi^{n+1}(\lambda)}. \end{aligned}$$

Now consider

$$\tilde{\rho}(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t) = \tilde{\rho}\left((f, \varphi)(\tilde{x}_{\lambda_{n-1}}^{n-1}), (f, \varphi)(\tilde{x}_{\lambda_n}^n), t\right)$$

$$\begin{aligned}
& \leq \tilde{a} \max \left[\begin{array}{c} \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, (f, \varphi) \tilde{x}_{\lambda_{n-1}}^{n-1}, t \right) \\ \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, (f, \varphi) \tilde{x}_{\lambda_n}^n, t \right) \\ \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, (f, \varphi) \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left((f, \varphi) \tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right) \\ \frac{\tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, (f, \varphi) \tilde{x}_{\lambda_{n-1}}^{n-1}, t \right) \cdot \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, (f, \varphi) \tilde{x}_{\lambda_n}^n, t \right)}{1 + \tilde{\rho} \left((f, \varphi) \tilde{x}_{\lambda_{n-1}}^{n-1}, (f, \varphi) \tilde{x}_{\lambda_n}^n, t \right)} \end{array} \right] \\
& \leq \tilde{a} \max \left[\begin{array}{c} \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right) \\ \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \\ \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_n}^n, t \right) \\ \frac{\tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right) \cdot \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right)}{1 + \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right)} \end{array} \right] \\
& \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \leq \tilde{a} \max \left[\begin{array}{c} \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \\ \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \end{array} \right] \\
\textbf{Case I:} & \text{ If } \max \left[\begin{array}{c} \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \\ \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \end{array} \right] = \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right) \text{ then} \\
& \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \leq \tilde{a} \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \\
& \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \leq \tilde{a}^n \tilde{\rho} \left(\tilde{x}_{\lambda_0}^0, \tilde{x}_{\lambda_1}^1, t \right).
\end{aligned}$$

It is clear by the basic lemmas sequence $\{\tilde{x}_{\lambda_n}^n\}$ is a Cauchy sequence in $\tilde{X}$. But $\tilde{X}$ is a complete soft parametric metric space, hence $\{\tilde{x}_{\lambda_n}^n\}$ is converges. here is $\tilde{x}_{\lambda}^* \in \tilde{X}$ such as $\tilde{x}_{\lambda_n}^n \rightarrow \tilde{x}_{\lambda}^*, n \rightarrow \infty$.

By continuity of t we have:

$$(f, \varphi) \tilde{x}_{\lambda}^* = (f, \varphi) \left(\lim_{n \rightarrow \infty} \tilde{x}_{\lambda_n}^n \right) = \lim_{n \rightarrow \infty} (f, \varphi) \tilde{x}_{\lambda_n}^n = \lim_{n \rightarrow \infty} \tilde{x}_{\lambda_{n+1}}^{n+1} = \tilde{x}_{\lambda}^*,$$

i.e., $(f, \varphi) \tilde{x}_{\lambda}^* = \tilde{x}_{\lambda}^*$. Thus, (f, φ) is a fixed point in $\tilde{X}$.

Case II:. If

$$\max \left[\tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \right] = \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right),$$

then

$$\tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \leq \tilde{a} \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right),$$

since $0 \leq \tilde{a} < \frac{1}{2}$, which gives contradiction.

Case III:. If

$$\max \left[\tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right), \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \right] = \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right),$$

then

$$\begin{aligned} \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) &\leq \tilde{a} \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \leq \tilde{a} \left[\tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right) + \tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \right] \\ &\leq \left(\frac{\tilde{a}}{1-\tilde{a}} \right) \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right) \leq k \tilde{\rho} \left(\tilde{x}_{\lambda_{n-1}}^{n-1}, \tilde{x}_{\lambda_n}^n, t \right), \end{aligned}$$

where $k = \left(\frac{\tilde{a}}{1-\tilde{a}} \right)$ and

$$\tilde{\rho} \left(\tilde{x}_{\lambda_n}^n, \tilde{x}_{\lambda_{n+1}}^{n+1}, t \right) \leq k^n \tilde{\rho} \left(\tilde{x}_{\lambda_0}^0, \tilde{x}_{\lambda_1}^1, t \right).$$

Sequence $\{\tilde{x}_{\lambda_n}^n\}$ is a Cauchy sequence in $\tilde{X}$. But $\tilde{X}$ is a complete soft parametric metric space, hence $\{\tilde{x}_{\lambda_n}^n\}$ is converges. Here is $\tilde{x}_\lambda^* \in \tilde{X}$ such that $\tilde{x}_{\lambda_n}^n \rightarrow \tilde{x}_\lambda^*, n \rightarrow \infty$.

By continuity of (f, φ) we have,

$$(f, \varphi) \tilde{x}_\lambda^* = (f, \varphi) \left(\lim_{n \rightarrow \infty} \tilde{x}_{\lambda_n}^n \right) = \lim_{n \rightarrow \infty} (f, \varphi) \tilde{x}_{\lambda_n}^n = \lim_{n \rightarrow \infty} \tilde{x}_{\lambda_{n+1}}^{n+1} = \tilde{x}_\lambda^*,$$

i.e., $(f, \varphi) \tilde{x}_\lambda^* = \tilde{x}_\lambda^*$. Thus, (f, φ) is a fixed point in $\tilde{X}$.

Uniqueness. Let $\tilde{y}_\mu^*$ is another fixed point of (f, φ) in $\tilde{X}$ such that $\tilde{x}_\lambda^* \neq \tilde{y}_\mu^*$, then we have

$$\begin{aligned} \tilde{\rho} \left((f, \varphi) \tilde{x}_\lambda^*, (f, \varphi) \tilde{y}_\mu^*, t \right) &\leq \tilde{a} \max \left[\begin{array}{l} \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right), \tilde{\rho} \left(\tilde{x}_\lambda^*, (f, \varphi) \tilde{x}_\lambda^*, t \right) \\ \tilde{\rho} \left(\tilde{y}_\mu^*, (f, \varphi) \tilde{y}_\mu^*, t \right), \tilde{\rho} \left(\tilde{x}_\lambda^*, (f, \varphi) \tilde{y}_\mu^*, t \right) \\ \tilde{\rho} \left((f, \varphi) \tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right), \frac{\tilde{\rho} \left(\tilde{x}_\lambda^*, (f, \varphi) \tilde{x}_\lambda^*, t \right) \cdot \tilde{\rho} \left(\tilde{y}_\mu^*, (f, \varphi) \tilde{y}_\mu^*, t \right)}{1 + \tilde{\rho} \left((f, \varphi) \tilde{x}_\lambda^*, (f, \varphi) \tilde{y}_\mu^*, t \right)} \end{array} \right] \\ &\leq \tilde{a} \max \left[\begin{array}{l} \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right), \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{x}_\lambda^*, t \right) \\ \tilde{\rho} \left(\tilde{y}_\mu^*, \tilde{y}_\mu^*, t \right), \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right) \\ \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right), \frac{\tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{x}_\lambda^*, t \right) \cdot \tilde{\rho} \left(\tilde{y}_\mu^*, \tilde{y}_\mu^*, t \right)}{1 + \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right)} \end{array} \right] \end{aligned}$$

and

$$\tilde{\rho} \left((f, \varphi) \tilde{x}_\lambda^*, (f, \varphi) \tilde{y}_\mu^*, t \right) \leq \tilde{a} \tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right).$$

This is true only when $\tilde{\rho} \left(\tilde{x}_\lambda^*, \tilde{y}_\mu^*, t \right) = 0 \Rightarrow \tilde{x}_\lambda^* = \tilde{y}_\mu^*$. Hence, fixed point of (f, φ) is unique. $\square$

Theorem 2.2. Let $(\tilde{X}, \tilde{\rho}, E)$ be a complete soft parametric metric space and let (f, φ) a continuous mapping.

Let mapping $(f, \varphi) : (\tilde{X}, \tilde{\rho}, E) \rightarrow (\tilde{X}, \tilde{\rho}, E)$ satisfies the following condition:

$$\begin{aligned} \tilde{\rho}((f, \varphi)\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t) &\leq \tilde{a} [\tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{x}_\lambda, t) + \tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{y}_\mu, t)] \\ &\quad + \tilde{b} [\tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t) + \tilde{\rho}((f, \varphi)\tilde{x}_\lambda, \tilde{y}_\mu, t)], \end{aligned}$$

for all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}, \tilde{x}_\lambda \neq \tilde{y}_\mu$ and for all $t > 0$, where $\tilde{a} + \tilde{b} < \frac{1}{2}, \tilde{a}, \tilde{b} \in [0, \frac{1}{2})$. Then (f, φ) has a unique fixed point in $\tilde{X}$.

Theorem 2.3. Let $(\tilde{X}, \tilde{\rho}, E)$ be a complete soft parametric metric space and let (f, φ) a continuous mapping.

Let mapping $(f, \varphi) : (\tilde{X}, \tilde{\rho}, E) \rightarrow (\tilde{X}, \tilde{\rho}, E)$ satisfies the following condition:

$$\begin{aligned} &\tilde{\rho}((f, \varphi)\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t) \\ &\leq \tilde{a} \left[\frac{\tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{y}_\mu, t) \cdot \{\tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{x}_\lambda, t) + \tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t)\}}{1 + \tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{x}_\lambda, t) + \tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{y}_\mu, t)} \right] \\ &\quad + \tilde{b} [\tilde{\rho}(\tilde{x}_\lambda, \tilde{y}_\mu, t)], \end{aligned}$$

for all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}, \tilde{x}_\lambda \neq \tilde{y}_\mu$, and for all $t > 0$, where $\tilde{a} + \tilde{b} < 1, \tilde{a}, \tilde{b} \in (0, 1)$. Then (f, φ) has a unique fixed point in $\tilde{X}$.

Theorem 2.4. Let $(\tilde{X}, \tilde{\rho}, E)$ be a complete soft parametric metric space and let (f, φ) a continuous mapping.

Let mapping $(f, \varphi) : (\tilde{X}, \tilde{\rho}, E) \rightarrow (\tilde{X}, \tilde{\rho}, E)$ satisfies the following condition:

$$\begin{aligned} &\tilde{\rho}((f, \varphi)\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t) + \tilde{a} \max \{ \tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{y}_\mu, t), \tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{x}_\lambda, t) \} \\ &\geq \tilde{b} \frac{\tilde{\rho}(\tilde{x}_\lambda, (f, \varphi)\tilde{x}_\lambda, t) [1 + \tilde{\rho}(\tilde{y}_\mu, (f, \varphi)\tilde{y}_\mu, t)]}{1 + \tilde{\rho}(\tilde{x}_\lambda, \tilde{y}_\mu, t)} + \tilde{c} \tilde{\rho}(\tilde{x}_\lambda, \tilde{y}_\mu, t), \end{aligned}$$

for all $\tilde{x}_\lambda, \tilde{y}_\mu \in \tilde{X}, \tilde{x}_\lambda \neq \tilde{y}_\mu$ and for all $t > 0$, where $\tilde{a}, \tilde{b}, \tilde{c} \geq 0, \tilde{b} + \tilde{c} - 2\tilde{a} > 1, \tilde{c} - \tilde{a} > 1$. Then (f, φ) has a unique fixed point in $\tilde{X}$.

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